Application of dynamical eikonal approximation in elastic scattering reaction within 10-60 MeV/nucleon

Authors: Yun, Dr. Xiaoyan, Xu, Dr. Yi-Ping, Dr. Danyang Pang, Xu, Dr. Yi-Ping 许祎萍

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Abstract

Application of the dynamical eikonal approximation(DEA) to elastic scattering for Coulomb-dominated reactions at low energy is studied. Our test case consists of elastic scattering for 8B, 9C and 11Be on 208Pb at 21.3, 25.2 and 12.7 MeV/nucleon, respectively. We introduce an empirical correction to the DEA method to account for Coulomb deflection, which significantly improves the description of elastic scattering of weakly bound nuclei on heavy target. The angular distributions of elastic scattering obtained using the empirical correction show a good agreement with experimental data down to around 10 MeV/nucleon. Furthermore, we study the the effect of relativistic kinematics corrections on the angular distributions of elastic scattering at incident energies between 20 and 60 MeV/nucleon. The results show that relativistic kinematics corrections are crucial for describing the angular distributions of elastic scattering as low as around 40 MeV/nucleon.

Full Text

Preamble

We investigate the application of the dynamical eikonal approximation (DEA) to elastic scattering in Coulomb-dominated reactions at low energies. Our test cases consist of elastic scattering of ^{8}B, ^{9}C, and ^{11}Be on ^{208}Pb at 21.3, 25.2, and 12.7 MeV/nucleon, respectively. We introduce an empirical correction to the DEA method to account for Coulomb deflection, which significantly improves the description of elastic scattering for weakly bound nuclei on heavy targets. The angular distributions obtained using this empirical correction show good agreement with experimental data down to approximately 10 MeV/nucleon. Furthermore, we examine the effect of relativistic kinematics corrections on the

angular distributions of elastic scattering at incident energies between 20 and 60 MeV/nucleon. Our results demonstrate that relativistic kinematics corrections are crucial for accurately describing the angular distributions of elastic scattering at energies as low as approximately 40 MeV/nucleon.

Keywords: elastic scattering, empirical correction, relativistic kinematics correction

Introduction

With the development of radioactive ion beam physics in the mid-1980s, Isotope Separator On-Line (ISOL) facilities have been able to produce an increasing number of intermediate- and high-energy beams, as well as short-lived nuclides. The number of observed nuclides has soared from approximately 300 to over 3000, while the number of theoretically predicted nuclides is estimated to be as high as 8000–10000, the vast majority of which are unstable. The study of exotic nuclei far from the β stability line has emerged as one of the primary objectives in astrophysics, intrinsically linked to the synthesis of elements and the evolution of celestial bodies following the Big Bang. Numerous novel phenomena have been uncovered in unstable nuclei, including the existence of neutron halo or neutron skin in some nuclei [**?**, **?**, **?**], the emergence of new magic numbers [**?**, **?**, **?**, **?**], and shape coexistence [**?**, **?**]. These new phenomena continue to provide challenges to nuclear theory.

These exotic nuclear structures also manifest themselves in reactions induced by radioactive nuclei. Elastic scattering induced by weakly bound nuclei is of paramount importance, as it contains crucial information about the exotic structure and reaction mechanisms of these nuclei [**?**]. One particularly noticeable phenomenon is the significant reduction of the Coulomb-nuclear interference peak in the elastic scattering angular distributions of weakly bound nuclei such as ^{6}He and ^{11}Be. This reduction has been found to be caused by coupling effects from breakup reaction channels [**?**, **?**, **?**, **?**, **?**, **?**, **?**].

To obtain reliable information on the structure of weakly bound nuclei, a precise reaction method is essential. Numerous theories have been proposed to achieve this goal, among which the Continuum Discretized Coupled-Channels (CDCC) method has proven to be a highly successful tool for describing reactions induced by weakly bound systems [**?**, **?**, **?**, **?**]. Since CDCC treats the collision in a fully quantum mechanical manner, it can be computationally demanding. Additionally, it faces challenges in achieving convergent results in the low-energy region, while relativistic effects need to be considered in the high-energy region [**?**].

Several cutting-edge facilities, such as HIE-ISOLDE at CERN, are either already capable of or will soon be able to provide Radioactive Ion Beams (RIBs) at energies as low as 10 MeV/nucleon. Recently, the High Intensity Heavy-ion Accelerator Facility (HIAF) has been established in Huizhou, Guangdong Province. This state-of-the-art facility is capable of delivering highly intense beams of various stable and radioactive ions, with energies spanning from MeV/u to GeV/u. At

HIAF, a wealth of elastic scattering experiments across a broad energy range are anticipated to be carried out in the near future. Therefore, a reaction method applicable across a wide energy range, capable of effectively handling elastic scattering of weakly bound nuclei, and featuring high numerical stability and computational efficiency is essential.

The Dynamical Eikonal Approximation (DEA) method relies on the eikonal approximation, which assumes that the projectile-target interaction occurs along a straight line [**?**, **?**]. Based on this assumption, the wave function can be factorized into a plane wave multiplied by a function that varies smoothly with the projectile-target relative coordinate. This factorization allows us to perform reaction calculations more efficiently, reducing computational time compared to the CDCC method. Moreover, the DEA method is an improvement over the traditional eikonal approximation, as it fully accounts for the dynamical effects of projectile excitation. This avoids the divergence issue in the integral over impact parameter b during cross-section calculations within the eikonal approximation. In previous studies, a detailed comparison between the CDCC and DEA methods for the breakup of the one-neutron halo nucleus ^{15}C on ^{208}Pb at an incident energy of 68 MeV/nucleon was conducted [**?**], showing excellent agreement between both methods. However, the DEA method fails to reproduce the CDCC results at 20 MeV/nucleon, as the eikonal approximation becomes invalid at such low energies due to Coulomb deflection. At low energies, this deflection significantly distorts the projectile-target relative motion from a pure plane wave. Therefore, it is crucial to find a way to correct for this low-energy limitation in DEA.

Recent results have shown that an empirical correction can markedly enhance the description of breakup reactions involving neutron-rich projectiles on heavy targets, down to incident energies of 20 MeV/nucleon [**?**]. The empirical correction replaces the impact parameter with the distance of closest approach of the corresponding classical trajectory. The excellent results obtained for neutron-rich nuclei lead us to consider extending this correction to proton-rich nuclei. However, the feasibility of applying DEA to the elastic scattering of proton-rich nuclei on heavy targets at these energies remains to be explored.

In this paper, we examine three specific reactions: ^{8}B, ^{9}C, and ^{11}Be impinging on ^{208}Pb at incident energies of 21.3, 25.2, and 12.7 MeV/nucleon, respectively. We apply the DEA method to elastic scattering reactions at incident energies of several tens of MeV/nucleon with both the empirical correction and relativistic kinematics correction. This energy range is typically considered one where relativistic effects are negligible. The purpose of this study is to investigate the feasibility of extending the DEA method across a wide energy range.

This paper is organized as follows: the theoretical framework of the dynamical eikonal approximation, empirical correction, and relativistic kinematics correction is introduced in Section II; the results with and without these corrections are presented in Section III; and a summary is given in Section IV.

II. Theoretical Descriptions

A. Dynamical Eikonal Approximation

We consider a collision between a two-body projectile (P) and a structureless target (T) with mass m_T and charge $Z_T e$. The two-body projectile consists of a structureless core (c) with mass m_c and charge $Z_c e$ and a fragment (f) with mass m_f and charge $Z_f e$. The projectile is described by an internal Hamiltonian H_0:

$$H_0 = -\frac{\hbar^2}{2\mu_{cf}} \triangle_r + V_{cf}(r)$$

where $\mu_{cf} = \frac{m_f m_c}{m_f + m_c}$ is the c-f reduced mass of the projectile, r is the relative coordinate of the fragment to the core. H_0 is composed of the kinetic energy operator for the relative motion between core and fragment and the core-fragment interaction potential V_{cf}. The potential V_{cf} contains an angular-momentum-dependent central term (including Coulomb interaction) and a spin-orbit term involving the fragment spin; the spin of the core is neglected.

With this two-body description for the projectile, the P-T collision reduces to a three-body problem whose Hamiltonian H reads:

$$H = \hat{T}_{PT} + H_0 + V_{cT} + V_{fT}$$

where $\hat{T}_{PT}$ is the kinetic energy operator of projectile-target relative motion, and V_{cT} and V_{fT} are the core-target and fragment-target interaction potentials, respectively.

To study the reactions of projectile (P) on target (T), we need to solve the three-body Schrödinger equation:

$$[\hat{T}_{PT} + H_0 + V_{cT} + V_{fT}]\Psi(R_{PT}; r) = E\Psi(R_{PT}; r)$$

where $\Psi(R_{PT}; r)$ is the three-body wave function, E is the total energy of the system, and R_{PT} is the coordinate of the projectile with respect to the target.

In the DEA method, the resulting three-body Schrödinger equation is solved using the eikonal ansatz for the wave function:

$$\Psi(R_{PT}; r) = e^{iKZ}\Phi(R_{PT}; r)$$

where K is the wave number, which is related to the total energy. At high energies, one expects a weak dependence on R_{PT} of Φ. Using the factorization in the Schrödinger equation and neglecting second-order derivatives of Φ that are small at high velocities, we obtain:

$$i\hbar v \frac{\partial}{\partial Z}\Phi(b;Z;r) = [(H_0 - E_0) + V_{cT} + V_{fT}]\Phi(b;Z;r)$$

where Z is the longitudinal component of R_{PT}, and the vector $b = (b,\phi)$ represents the transverse part of R_{PT}. v is the relative velocity of projectile and target, and E_0 corresponds to the projectile ground state energy.

In the standard eikonal implementation, the adiabatic approximation is performed to solve this equation. That approximation corresponds to neglecting the excitation energy of the projectile compared to the beam energy. In DEA, no such adiabatic approximation is made, and the equation is solved numerically for each impact parameter b imposing the initial condition that initially, the projectile is in its ground state $n_{0l_{0j}}0$ of energy E_0 and has an initial P-T relative momentum $\hbar K_0$. j_0 is the total angular momentum resulting from the coupling of the orbital angular momentum l_0 and the spin of the fragment, and m_0 is its projection. ϕ corresponds to the projectile ground state wave function:

$$\Phi^{(m_0)}(b;t \to -\infty;r) = \phi_{l_{0j_{0m}}0}(E_0;r)$$

where the variable t ($Z = vt$) is linked to the longitudinal part of R_{PT}.

Let $\Phi^{(m_0)}(b;t;r)$ be a particular solution (for the particular orientation $b = (b,\phi = 0)$) corresponding to the initial condition $\Phi^{(m_0)}(b;t \to -\infty;r) = \phi_{l_{0j_{0m}}0}(r)$. Note that b is the impact parameter related to a classical trajectory, whereas in the previous equation b is the transverse part of a quantal coordinate.

The elastic scattering differential cross section can be derived from the wave function (see Ref. [**?**] for more details):

$$\frac{d\sigma}{d\Omega}_{\text{el}} = \left|\langle\phi_{l_{0j_{0m}}0}(E_0;r)|\Phi^{(m_0)}(R;r)\rangle\right|^2$$

B. Empirical Correction

As previously discussed, the DEA method is based on the eikonal approximation, which assumes that the interaction between the projectile and the target occurs along a straight line [**?**, **?**]. However, in reality, the trajectory deviates from a straight line due to deflection caused by interaction with the target. At sufficiently high energies, the assumption of straight-line trajectories becomes more valid, as the deflection of the projectile by the target can be considered negligible. The eikonal approximation becomes invalid at low energies, primarily due to Coulomb deflection, which significantly distorts the projectile-target relative motion from a pure plane wave.

Fukui et al. discovered that DEA no longer aligns with CDCC calculations for the breakup of ^{15}C on ^{208}Pb at 20 MeV/nucleon. They observed that the discrepancies between DEA and CDCC appear not only in the magnitude of the angular distributions but also in their shape. Specifically, the oscillatory pattern of DEA is shifted toward more forward angles compared to the CDCC calculation. To pinpoint the source of this discrepancy, they analyzed the contribution of each projectile-target relative angular momentum L to the total breakup cross section. They found that the DEA method tends to favor larger L values compared to the full CDCC calculation. To address this issue, they replaced the transverse component of the projectile-target relative coordinate b by an empirical value. For a collision dominated by repulsive Coulomb interaction, that distance will be larger than b. The distance of closest approach b_c can be derived analytically [**?**, **?**]:

$$b_c = \frac{\eta + \sqrt{\eta^2 + \eta_0^2}}{K_0}$$

where η is the Sommerfeld parameter, $\eta_0 = Z_P Z_T e^2/\hbar v$, and Z_P and Z_T are the charges of the projectile and target, respectively. K_0 is the wave number for the initial projectile-target relative motion, which is related to the total energy $E = \hbar^2 K_0^2/2\mu_{PT} + E_0$. Ref. [**?**] has shown that this simple empirical correction could significantly improve the description of breakup for neutron-rich nuclei on heavy targets at about 20 MeV/nucleon.

C. Relativistic Kinematics Correction

For high-energy reactions that transcend the non-relativistic energy regime, the influence of relativistic effects must be taken into account. To provide a reliable theoretical interpretation of experimental data, it is imperative to develop a nuclear reaction theory that incorporates relativistic effects. This section introduces relativistic kinematics corrections. When addressing relativity in nuclear reactions, several key aspects must be considered. For instance, the Schrödinger equation is not strictly valid in a relativistic context, and thus at least some reinterpretation of the nuclear optical potential is necessary [**?**]. Additionally, the parameters of reaction kinematics, including atomic masses and incident energies, require modification to account for relativistic effects. The latter aspect is rather simple, but it has been found to be important for some cases. In this paper, we study the latter aspect.

The velocity v of the projectile, which is utilized to solve the time-dependent Schrödinger equation, is calculated using the relativistic formula:

$$v = c\sqrt{1 - \left(\frac{m_P c^2}{m_P c^2 + T_P}\right)^2}$$

where T_P is the initial kinetic energy of the projectile, and m_P is the mass of the projectile.

We also use the relativistic kinematics correction method proposed by Satchler [?]. Defining:

$$\gamma_L = \frac{E_{\mathrm{lab}}}{m_P c^2}$$

where E_{lab} is the laboratory energy. The relativistic correction factor γ_P corresponds to the projectile:

$$\gamma_P = x_P + \gamma_L + \frac{2x_P\gamma_L}{1 + x_P^2}$$

and γ_T corresponds to the target nucleus:

$$\gamma_T = x_T + \gamma_L + \frac{2x_T\gamma_L}{1 + x_T^2}$$

where $x_P = m_P c^2/m_T c^2$ and $x_T = m_T c^2/m_P c^2$. Then the masses of the projectile and target nuclei after relativistic correction are $m'_P = \gamma_P m_P$ and $m'_T = \gamma_T m_T$.

The reduced mass after correction is:

$$\mu'_{PT} = \frac{x_P + \gamma_L + 2x_P\gamma_L}{x_P + \gamma_L + 2x_P\gamma_L} \cdot \frac{x_T + \gamma_L + 2x_T\gamma_L}{x_T + \gamma_L + 2x_T\gamma_L} \mu_{PT}$$

Inputs to the Reaction Method

We analyze the elastic scattering induced by ^{8}B, ^{9}C, and ^{11}Be on ^{208}Pb at 21.3, 25.2, and 12.7 MeV/nucleon, respectively. The nucleus ^{8}B is usually considered the archetypal one-proton halo nucleus, modeled as a valence proton and a ^{7}Be core. The spectrum of ^{8}B includes only one bound state with $J^\pi = 2^+$, which is obtained from the coupling of a $0p_{3/2}$ proton with the $3/2^-$ spin of the ground state of ^{7}Be. It is bound by a mere 137 keV with respect to one-proton separation. In this work, we use the description of ^{8}B developed by Bertsch in Ref. [?]. ^{9}C is treated as a valence proton (in the $0p_{3/2}$ orbital) and an ^{8}B core, using the description from Ref. [?]. ^{11}Be is seen as an inert ^{10}Be core in its 0^+ ground state, to which a neutron is bound by 0.5 MeV in the $1s_{1/2}$ orbit. The ^{11}Be description in this paper corresponds to a simplified version from Refs. [?, ?].

For computational reasons, we use a simple model for ^{8}B, ^{9}C, and ^{11}Be in which the spin and internal structure of the core are neglected.

The DEA equation is solved with the algorithm presented in Refs. [?, ?]. The wave function is expanded over a mesh on the unit sphere containing $N_\theta \times$

N_ϕ points, going up to 10×19 points for these cases. The radial mesh is quasi-uniform, contains $N_r = 800$ points, and extends up to $r_{N_r} = 800$ fm. These calculations are performed for impact parameters $b = 0 - 150$ fm with a discretization step that varies between 0.25 and 5 fm. As explained in Ref. [**?**], the angular distributions of elastic scattering are obtained with an extrapolation up to $b_{\max} = 800$ fm.

In these calculations, the core-target interactions V_{cT} are obtained with the systematic single-folding method of nucleus-nucleus potentials [**?**], which accounts well for nucleus-nucleus elastic scattering not only for stable nuclei but also for some unstable nuclei within the energy range of approximately 10-100 MeV/nucleon. The fragment-target interactions V_{fT} are taken from the CH89 systematics [**?**]. The Coulomb potentials for the c-T, f-T, and c-f systems are calculated with a uniform charge sphere distribution with radius $R_C = r_C(A_T^{1/3} + A_c^{1/3})$, where $r_C = 1.25$ fm, and A_T and A_c are the mass numbers of the target and core nuclei, respectively.

[Figure 1: see original paper]

III. Results and Discussion

Figures 1 and 2 show the analysis of ^{8}B and ^{9}C on ^{208}Pb at low energies (21.3 and 25.2 MeV/nucleon, respectively). These figures display the angular distributions of elastic scattering (ratio to Rutherford, σ/σ_R) as a function of the center-of-mass scattering angles, $\theta_{\mathrm{c.m.}}$. The pink dotted lines represent the results of the DEA method with empirical correction (labeled "Corr"), while the green dash-dotted lines correspond to the results without empirical correction (labeled "NoCorr"). As seen in Fig. 1, DEA ("NoCorr") and experimental data no longer agree at 21.3 MeV/nucleon. Not only do the DEA and experimental data differ in magnitude, but the DEA oscillatory pattern is also shifted to forward angles compared to the experimental data. When the empirical correction is switched on, the agreement becomes good. The results confirm that the empirical correction is very effective, significantly improving agreement with experimental data. The results in Fig. 2 are very similar to those observed in Fig. 1 for ^{8}B. Note that the DEA result using the correction shows good agreement with experimental data.

To further verify the applicability of the empirical correction to lower-energy reactions, we study the case of ^{11}Be on a lead target at 12.7 MeV/nucleon. In Fig. 3, the results show that the DEA calculations and experimental data agree very well at 12.7 MeV/nucleon. These calculations suggest that DEA could extend its range of validity down to approximately 10 MeV/nucleon in Coulomb-dominated collisions.

The difference observed between DEA ("NoCorr") and experimental data at low energy is related to the root cause: the lack of Coulomb deflection in DEA. Based on the eikonal approximation, DEA presumes that the incoming plane-wave motion of the projectile remains largely unperturbed by its interaction with the target. Nevertheless, the results show that the lack of Coulomb deflection in DEA calculations can be efficiently corrected by the empirical correction (see the curve labeled "Corr"). Although efficient, the empirical correction is not perfect. A slight shift of the oscillatory pattern to larger scattering angles can be seen compared to experimental data. However, its oscillatory pattern is now in phase with experimental data. This correction provides a simple and effective way to account for Coulomb deflection in the DEA method. In the following calculations, the empirical correction is included.

[Figure 2: see original paper]

[Figure 3: see original paper]

The calculations in Figs. 1, 2, and 3 are performed within the non-relativistic framework. However, it is well known that at sufficiently high incident energies, relativistic effects must be taken into account. We investigate reactions with incident energies in the range of several tens of MeV per nucleon. At these energy levels, relativistic effects may start to become significant. Consequently, we incorporate relativistic kinematics corrections into subsequent calculations to account for this influence.

To study how the importance of relativistic corrections evolves with the incident energy of the projectile, we examine the elastic scattering reaction of ^{8}B on ^{208}Pb at 20, 40, and 60 MeV/nucleon. The results are shown in Fig. 4. The grey curves represent results obtained without relativistic corrections. The pink curves represent results with the inclusion of the relativistic correction for velocity (Eq. 9), marked as "RV" . The green curves correspond to results incorporating both velocity and Satchler' s relativistic corrections, labeled "RV-Sat" . It is evident that the significance of relativistic corrections becomes apparent at 40 MeV/nucleon and increases with higher incident energies. As expected, the effect of relativistic corrections is minimal at lower incident energies. At 21.3 MeV/nucleon, the green curves closely align with the grey curves.

[Figure 4: see original paper]

IV. Summary

In this study, we investigate the application of the Dynamical Eikonal Approximation (DEA) method across a wide energy range. We examine the angular distributions of elastic scattering of ^{8}B, ^{9}C, and ^{11}Be on ^{208}Pb at incident energies from approximately 10 to 60 MeV/nucleon. Our results show the difficulty in properly describing elastic scattering at low energies within the DEA method. However, we find that an empirical correction significantly enhances the method' s ability to reproduce Coulomb deflection, which was previously identified as a missing component in DEA. With this correction, the agreement between DEA results and experimental data is markedly improved.

We also study the effect of relativistic kinematics corrections on the angular distributions of elastic scattering for ^{8}B on ^{208}Pb at incident energies ranging from 20 to 60 MeV/nucleon. Our calculations reveal that relativistic corrections play a crucial role in accurately describing the angular distributions of elastic scattering, even at relatively low incident energies of around 40 MeV/nucleon. From these results, we conclude that the DEA method, including both the empirical correction and relativistic corrections, can provide an efficient alternative tool for describing elastic scattering across a wide energy region.

To extend the DEA method to higher incident energies up to approximately 1 GeV/nucleon, a suitable phenomenological potential for reactions in the high-energy region is important. We plan to study this in future work.

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