

## Detuning Analysis and Design Optimization of an 1.3 GHz 3-cell Superconducting Cavity

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### Abstract

Energy recovery linac (ERL) offers a promising path to superior beam quality and substantial energy savings at high average beam currents. This paper concentrates on optimizing the structure of a 1.3 GHz 3-cell superconducting cavity for high-current ERL injectors. It particularly examines helium pressure sensitivity and the detuning mechanism in the cavity. To mitigate detuning, strategies such as optimizing stiffening rings, reducing the helium vessel diameter, and enhancing cavity stiffness are proposed. Additionally, the paper outlines the mechanical design of the cavity and evaluates its performance through tuning force, Lorentz force detuning, stress, and modal analyses to ensure robustness and reliability.

### Full Text

#### Preamble

#### Detuning Analysis and Design Optimization of a 1.3 GHz 3-cell Superconducting Cavity

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Energy recovery linac (ERL) offers a promising path to superior beam quality and substantial energy savings at high average beam currents. This paper concentrates on optimizing the structure of a 1.3 GHz 3-cell superconducting cavity for high-current ERL injectors, with particular emphasis on helium pressure sensitivity and the detuning mechanism. To mitigate detuning, strategies such as optimizing stiffening rings, reducing the helium vessel diameter, and enhancing cavity stiffness are proposed. Additionally, the paper outlines the mechanical design of the cavity and evaluates its performance through tuning force, Lorentz force detuning, stress, and modal analyses to ensure robustness and reliability.

**Keywords:** Energy Recovery Linac, High-Current Superconducting Cavity, Detuning, Stiffening Rings, Microphonics

## Introduction

The core concept of Energy Recovery Linac (ERL) is that, after initial acceleration, the electron beam emits high-brightness radiation while a specially designed loop guides it back to the accelerating section in a decelerated state. This process recovers the electron beam's kinetic energy, converting it into high-frequency electromagnetic waves that are then used to accelerate new electron beams, while the decelerated old beam is discarded. This design significantly reduces energy consumption and ensures the quality of the electron beam, making ERL an effective method for generating high-quality electron beams [?]. ERL projects worldwide have demonstrated remarkable advancements in beam energy recovery and operational efficiency. The CEBAF experiment achieved a maximum energy of 1055 MeV in single-pass configurations with a continuous-wave (CW) beam current of 80  $\mu\text{A}$  [?, ?]. The CBETA project demonstrated multi-turn energy recovery using superconducting radio-frequency (SRF) cavities and a fixed-field alternating-gradient (FFAG) system. The permanent magnets are arranged in the FFAG system to construct a single return loop that successfully transports electron bunches of 42, 78, 114, and 150 MeV in one common vacuum chamber [?, ?]. The cERL facility at KEK has successfully demonstrated stable CW operation at 1 mA with beam energies ranging from 17.6 to 20 MeV, achieving energy recovery efficiency exceeding 99.9% [?, ?]. The recirculating superconducting linear accelerator S-DALINAC at TU Darmstadt's Institute for Nuclear Physics operates in CW mode, achieving beam currents up to 20  $\mu\text{A}$  and energies as high as 130 MeV, employing a three-pass recirculation scheme [?]. The Novosibirsk ERL in BINP is a multi-turn ERL based on normal conducting accelerating structures [?].

Future projects such as MESA at Mainz University, PERLE at IJC Lab, the five-pass CEBAF at JLAB, the EIC Cooler at BNL, and high-energy ERL colliders will rely on high-current SRF accelerator technologies due to their ability to achieve high beam intensities, efficient energy recovery, and stable continuous-wave operation [?]. Together, these projects underscore the transformative potential of ERL technology in achieving high-energy efficiency and beam quality. Based on recent developments in continuous-wave SRF accelerator technology

[?], this paper investigates ERL-based linear accelerators and proposes a concept to achieve an average operational beam current of 10 mA. This study presents an in-depth analysis and optimization of the mechanical structure of a 1.3 GHz high-current injector SRF cavity, significantly enhancing its frequency stability and mechanical performance. Additionally, a comprehensive simulation methodology for evaluating the stability and strength of SRF cavities has been developed, establishing a solid theoretical foundation for future cavity manufacturing and tuning processes while providing valuable engineering insights for research.

To meet the 10 mA average beam current requirement of the ERL project, a complete radio-frequency (RF) parameter design of the injector cavity was implemented and a 1.3 GHz 3-cell injector superconducting cavity configuration was selected [?, ?]. The vacuum model of the SRF cavity is shown in Fig. 1 [Figure 1: see original paper]. This study focuses on the design and performance evaluation of a 1.3 GHz 3-cell injector superconducting cavity for an energy recovery linac aimed at achieving an average current of 10 mA. Based on the physical design, the mechanical structure of the high-current 1.3 GHz SRF cavity was designed and evaluated, with emphasis on key parameters such as mechanical strength, helium pressure detuning, Lorentz detuning, and modal analysis. A multiphysics optimization approach was applied to the cavity, with particular attention given to detuning and stability. Finite element simulation software was used to optimize and reinforce the cavity structure, which is crucial for the fabrication of high-current SRF cavities. Through design optimization, the cavity's frequency and phase deviations caused by small deformations during operation were effectively controlled. This research provides valuable insights into the mechanical properties of SRF cavities and offers important theoretical support for the fabrication process.

[Figure 1: see original paper] Vacuum model of the 1.3 GHz 3-cell SRF injector cavity. The green area represents the designed transition section. The yellow area maintains the same mid-cell and end-cell dimensions as the TESLA cavity.

Chapter II details the theoretical framework of key detuning mechanisms in superconducting cavities, establishing a foundation for subsequent analysis. Chapter III conducts a systematic investigation of helium pressure sensitivity ( $df/dp$ ) across different tuner stiffness configurations, specifically aimed at quantifying the cavity's mechanical response to pressure fluctuations. This analysis utilizes multiphysics-coupled finite element simulations to correlate structural deformations with frequency detuning, providing critical insights into the cavity's stability. Chapter IV systematically formulates the primary factors contributing to cavity detuning, quantifies their individual effects, and validates the results through comparisons with full-model simulations. Chapter V focuses on the mechanical design of the cavity, covering the evaluation of tuning forces and Lorentz force detuning, and concludes with a thorough assessment of the cavity's structural strength to ensure its operational reliability.

## II. Detuning Theory of Superconducting Cavity

### A. Microphonics

Superconducting cavities are typically designed with thin walls, usually less than 3 mm [?], to meet heat dissipation and tuning requirements. This design makes them highly susceptible to deformation from external disturbances. Mechanical resonance occurs when the frequency of external vibrations closely matches the natural mechanical resonance frequency of the cavity. This phenomenon, termed microphonics, leads to substantial structural deformation and frequency instability. External sources of vibration, including ambient noise, equipment operation, and structural vibrations, are the primary causes of this effect.

To mitigate the impact of external vibrations on high-frequency superconducting cavities and prevent resonance, three strategies are commonly used. The first involves adding vibration isolation systems to reduce the transmission of environmental vibrations, the second utilizes low-level control compensation to counteract vibrations, and the third optimizes the cavity's mechanical design to increase its natural mechanical resonance frequency, reducing its susceptibility to resonance [?]. The first two methods focus on optimizing the external environment and active control, while this paper focuses on the third approach—improving the cavity's structural design to enhance its natural mechanical resonance frequency.

During the design phase, analyzing the mechanical vibration modes of the cavity helps identify its natural mechanical frequencies. External vibrations are typically below 50 Hz and have relatively large amplitudes. As a result, the low-frequency modes within the superconducting cavity are more likely to resonate with environmental vibration sources. Increasing the cavity's mechanical resonance frequency minimizes the risk of resonance with low-frequency environmental vibrations, reducing microphone detuning effects.

### B. Helium Pressure Fluctuation

In superconducting accelerator systems, the liquid helium tank plays a crucial role in maintaining the low-temperature operating state of the superconducting cavity. Fluctuations and pressure changes in the liquid helium not only affect the cooling performance of the cavity but can also have a significant impact on its frequency stability. In particular, helium pressure fluctuations within the tank, especially in cryogenic environments, can lead to detuning of the cavity's frequency. This phenomenon is known as helium pressure fluctuation detuning.

Liquid helium fluctuations cause pressure variations within the helium tank, leading to surface deformation of the cavity, which in turn results in changes to the cavity's RF resonant frequency [?]. This deformation is directly proportional to the helium pressure fluctuations. The mathematical relationship between the cavity frequency and the helium pressure variation can be expressed as [?]:

$$\Delta f_1 = \frac{df}{dp} \cdot \Delta P$$

Here  $df/dp$  denotes the helium pressure sensitivity coefficient, which characterizes the sensitivity of the superconducting cavity to fluctuations in the helium bath pressure within the cryostat.  $\Delta P$  represents the magnitude of the fluctuations in helium pressure.

### C. Lorentz Force Detuning

Lorentz force detuning (LFD) arises from the inelastic deformation of the superconducting cavity material under the influence of a strong RF field [?]. The interaction between the RF field and the cavity material induces small deformations, particularly in the superconducting material (e.g., niobium). As the field strength increases, elastic deformation alters the geometry of the cavity, changing its RF resonant frequency. The effect of the Lorentz force on an elliptical cavity is shown in Fig. 2 [Figure 2: see original paper] [?]. When the accelerating gradient is 4.77 MV/m, the Lorentz force distribution calculated by simulation is shown in Fig. 3 [Figure 3: see original paper].

[Figure 2: see original paper] Schematic of Lorentz force in an elliptical cavity. The blue contour denotes the original cell shape before deformation, the red contour denotes the deformed shape due to LFD.

For superconducting cavities in pulsed mode, LFD has a more significant impact, while helium pressure fluctuations are less influential [?, ?]. In contrast, for high-current ERL cavities in CW mode, Lorentz detuning primarily affects the early field buildup, so the focus is on mitigating the effect of helium pressure fluctuations, with Lorentz detuning as a secondary concern [?].

## III. The Complete Model Detuning Analysis

Based on the internal profile parameters of the designed superconducting cavity [?], the cavity's outline can be sketched and modeled in CAD software. To minimize potential errors during the modeling process, the vacuum model established in the electromagnetic simulation phase is imported directly. Using the shell feature, the cavity's outer shell is automatically generated. The outer shell model is then precisely assembled with the vacuum model, effectively preventing interference issues in subsequent finite element analysis and ensuring the accuracy of the simulation.

The typical thickness for a 1.3 GHz superconducting cavity ranges from 2.5 to 3 mm [?]. Insufficient thickness compromises the structural strength of the cavity, while excessive thickness could lead to challenges in welding and material waste. The thickness of the 1.3 GHz 3-cell high-current superconducting cavity is set to 2.8 mm, which is similar to the TESLA cavity design [?]. The model of the

dressed SRF cavity is shown in Fig. 4 [Figure 4: see original paper]. The helium tank design and detailed parameters will be discussed in Chapter V.

[Figure 4: see original paper] Mechanical model (grey color) of 3-cell dressed SRF cavity with vacuum part (purple color) inside.

The Lorentz force is generated by the electromagnetic field within the cavity and acts on the inner surface of the cavity, leading to deformation-induced detuning. The magnitude of the Lorentz force ( $P$ ) is given by Equation [?]:

$$P = \frac{1}{2}(\mu_0 H^2 - \epsilon_0 E^2)$$

Here  $\mu_0$  is the vacuum permeability,  $\epsilon_0$  is the vacuum permittivity, and  $E$  and  $H$  are the magnetic field strength and electric field strength at the inner surface of the cavity, respectively.

Under different values of accelerating gradient  $E_{acc}$ , the deformation of the cavity and the corresponding frequency shift due to the Lorentz force vary. The frequency shift  $\Delta f$  is proportional to the square of the accelerating gradient  $E_{acc}^2$ :

$$\Delta f_2 = K_L \times E_{acc}^2$$

Here  $K_L$  is the Lorentz detuning coefficient, which represents the frequency effect of the Lorentz force detuning on the superconducting cavity.

The elliptical cavity's relatively weak stiffness is one of its disadvantages. Therefore, an analysis of the detuning caused by helium pressure fluctuations, mechanical vibrations, and Lorentz forces is a necessary step in the development of the elliptical cavity. The accuracy of simulations largely depends on the precision of material properties. Given that the superconducting cavity involves multi-physics coupling of electromagnetic, mechanical, and thermal fields, various material properties must be considered. In electromagnetic field simulations, two primary materials are involved: Perfect Electric Conductor (PEC) and vacuum, with  $\mu_0$  set at  $4\pi \times 10^{-7}$  H/m and  $\epsilon_0$  set at  $8.854 \times 10^{-12}$  F/m. For the mechanical structural simulations, the mechanical properties of materials such as niobium, niobium-titanium alloy, and titanium for the helium tank are required. The relevant mechanical properties of these materials, which can influence the cavity strength and stability, are summarized in Table 1 .

Properties of Superconducting Cavity Materials.

Material	Density [kg/m <sup>3</sup> ]	Young' s Modulus [GPa]	Poisson' s Ratio	Allowable Stress (S) at 4 K [MPa]	Allowable Stress (S) at 293 K [MPa]
Nb					

Material	Density [kg/m <sup>3</sup> ]	Young' s Modulus [GPa]	Poisson' s Ratio	Allowable Stress (S) at 4 K [MPa]	Allowable Stress (S) at 293 K [MPa]
55Ti					

The helium pressure detuning ( $df/dp$ ) of a 3-cell high-current superconducting cavity was simulated using COMSOL Multiphysics simulation software [?]. One end of the cavity' s beam tube was fixed, while the other end was allowed to expand freely. A pressure of 1 bar was applied to both the external surface of the bare cavity and the inner surface of the helium tank to simulate pressure fluctuations within the dressed cavity.

Meanwhile, in finite element analysis, parts with complex structures that have minimal impact on the simulation results should be appropriately optimized to save computational resources. For instance, the bellows with an intricate and convoluted design require a high mesh density to accurately capture their stiffness. To optimize computational efficiency, this study employs a material stiffness parameter equivalence approach, replacing the actual bellows model with an equivalent straight pipe of the same length.

The bellows segment was replaced with a straight cylindrical tube having the same length (35.5 mm) and outer diameter (120 mm), with a wall thickness of 4 mm. To ensure that this simplification does not affect the mechanical behavior of the assembly under axial loads, an equivalent Young' s modulus was calculated for the straight segment such that its axial stiffness matches that of the original bellows. The axial stiffness of the original titanium bellows was determined via simulation to be 0.919 kN/mm. For a cylindrical tube of cross-sectional area  $A = 742.3 \text{ mm}^2$  and length  $L = 35.5 \text{ mm}$ , the axial stiffness is given by the equation:

$$E_{eq} = \frac{K_{bellow} \cdot L}{A}$$

Hence, in the simplified model, the equivalent straight pipe was assigned a Young' s modulus of 43.95 GPa to preserve the same axial deformation behavior as the original bellows. The simulation conditions involved in this paper do not require high accuracy for the equivalent shear stiffness, and it contributes minimally to dressed cavity system behavior. Consequently, detailed shear load equivalence analysis was omitted to streamline the model without compromising its axial fidelity. This method preserves the original stiffness characteristics while significantly reducing mesh resource consumption. The simulation boundary condition and the equivalent straight pipe model are shown in Fig. 5 [Figure 5: see original paper].

[Figure 5: see original paper] A and B represent the beam tubes at both ends, helium pressure is inside helium vessel, and the green section represents the

equivalent straight pipe model of the bellows.

The  $df/dp$  was evaluated as a function of the stiffening ring radius through multiphysics finite element simulations, with the analysis spanning three tuner stiffness conditions: 10 kN/mm, 40 kN/mm, and 80 kN/mm. The simulation results are shown in Fig. 6 [Figure 6: see original paper]. The outer diameter of the bare cavity iris is 38 mm, and the outer diameter of the equator is 105 mm, so the range of possible stiffening ring radii is between 38 mm and 105 mm, with  $r_{ring} = 38$  mm representing the case with no stiffening ring.

The helium pressure sensitivity initially increases with the stiffening ring radius and then begins to decrease significantly. In the TESLA cavity, the stiffening ring is located at the region where  $df/dp$  reaches its approximate maximum value. This is because the primary optimization goal for the stiffening ring in the TESLA cavity is to minimize Lorentz detuning rather than the microphone effect [?]. In contrast, when the stiffening ring is positioned at the equator,  $df/dp$  reaches its minimum. For a tuner stiffness of 40 kN/mm [?], the optimal stiffening ring configuration to minimize  $df/dp$  was identified at the maximum radius position, followed by the configuration with no stiffening ring. For all stiffening ring radii, increasing the tuner stiffness results in a reduction of  $df/dp$ . This simulation provides a good assessment of the  $df/dp$  for the entire cavity, but it does not directly explain the variation of  $df/dp$  with stiffening ring radius and tuner stiffness, nor does it account for the influence of other specific components. Next, the factors influencing  $df/dp$  were analyzed and simulated individually.

#### IV. Deconstructive Analysis and Validation of Helium Pressure Detuning

To investigate the factors contributing to the variation of  $df/dp$  in detail, the total frequency variation of the cavity is divided into two components: the frequency shift caused by changes in cavity length and the frequency shift resulting from changes in the shape of the cavity cells:

$$\Delta f_{total} = \Delta f_{cell} + \Delta f_{length}$$

Here  $\Delta f_{cell}$  is the frequency shift caused by the shape variations of the cavity cells, and  $\Delta f_{length}$  is the frequency shift caused by the cavity length change. The above equation divided by the change in helium pressure [?] yields:

$$\frac{df}{dp} = \frac{df_{cell}}{dp} + \frac{df_{length}}{dp}$$

The frequency shift  $\Delta f_{cell}$  results from the direct effect of liquid helium pressure on the outer wall of the cells. The helium bath pressure causes deformation of the cell walls. The frequency shift  $\Delta f_{length}$  due to the cavity length change is

attributed to the liquid helium's effect on the conical discs at both ends of the helium tank. The pressure from the helium bath exerts a stretching force at the ends, which leads to a change in the overall cavity length, thereby affecting the tuning of the cavity. The length variation can further be decomposed into two components: the original cavity length  $x$  and the force  $F$  applied to the cavity due to the elongation.

$$\frac{df_{length}}{dp} = \frac{df_{length}}{dx} \cdot \frac{dx}{dF} \cdot \frac{dF}{dp}$$

Here  $df_{length}/dx$  represents the tuning sensitivity of the bare cavity, and  $K$  denotes the axial stiffness of the dressed cavity. These parameters will be simulated and analyzed to evaluate the specific factors influencing  $df/dp$ . The simulation conditions for each parameter are listed in Table 2, and the corresponding loads applied can be seen in Fig. 5.

Simulations for determining  $df/dp$  model parameters.

Parameter	Boundary conditions	Calculation results
$df_{cell}/dp$	A, B tubes fixed, 1 bar helium pressure	frequency variation
$df_{length}/dx$	A tube fixed, 1 mm displacement (B)	frequency variation
$dF/dp$	A, B tubes fixed, 1 bar helium pressure, 2.5 kN axial force (B)	Length change, Average force

### A. Helium Pressure Sensitivity of the Bare Cavity

First, the helium pressure detuning component  $\Delta f_{cell}$  is analyzed. In COMSOL, both ends of the beam pipe are fixed, and a pressure of 1 bar is applied to simulate the helium pressure acting solely on the outer wall of the bare cavity, to investigate the helium pressure detuning caused by the deformation of the cavity wall, denoted as  $df_{cell}/dp$ . The results are shown in Fig. 7 [Figure 7: see original paper].

[Figure 7: see original paper]  $df_{cell}/dp$  of cavity versus stiffening rings radius.

The results indicate that the variation in the bare cavity wall has an impact on  $df/dp$ , fluctuating between -5 Hz/mbar and 10 Hz/mbar. In the configuration without a stiffening ring, the  $df_{cell}/dp$  value is lower compared to the configuration with the stiffening ring placed at a radius of approximately 55 mm. This is because, as the helium pressure increases, the negative frequency shift caused by the deformation of the iris counteracts the positive frequency shift caused by the deformation of the equator and the increase in length.

## B. The Tuning Sensitivity of the Bare Cavity

The tuning sensitivity  $df_{length}/dx$  of the bare cavity is influenced by the cavity shape and dimensions. Using COMSOL simulations, the relationship between the tuning sensitivity  $df_{length}/dx$  of the 3-cell cavity and the stiffening ring radius is simulated and presented in Fig. 8 [Figure 8: see original paper].

[Figure 8: see original paper] Tuning sensitivity of cavity versus stiffening rings radius.

The results indicate that  $df_{length}/dx$  of the 3-cell superconducting cavity varies fractionally as the stiffening ring radius changes, fluctuating between 0.7 MHz/mm and 1 MHz/mm. Overall, the stiffening ring radius does have some effect on  $df_{length}/dx$ , but this influence is relatively small.

## C. Axial Stiffness and Helium Pressure Tuning Force

The cavity axial stiffness is an essential factor in the study of helium pressure detuning. Stiffness simulations were conducted for both the bare cavity and the dressed cavity, with the comparative results presented in Fig. 9 [Figure 9: see original paper].

[Figure 9: see original paper] Stiffness of bare cavity and dressed cavity versus stiffening rings radius.

In addition to exerting pressure on the cavity walls, the liquid helium in the helium bath also induces deformation of the bath wall itself due to the pressure, thereby generating a tensile force in the axial direction on the cavity. For the 3-cell cavity, the total axial stiffness of the helium cryostat assembly is 26 kN/mm from Fig. 9, while the bellows contributes only 0.919 kN/mm, representing a mere 3.5% of the overall stiffness. The pressure on the end wall of the helium bath has the most significant impact on the  $dF/dp$ . As shown in Fig. 10 [Figure 10: see original paper], for a cavity with a stiffening ring radius of 70 mm as an example, the pressure applied on the end wall of the helium bath, which has a small outer diameter of 120 mm, was simulated and integrated in COMSOL to calculate the resulting force. When the internal pressure was 1 bar, the integral result of the force component in the x-direction (axial direction),  $dp_x$ , was 3499.4 N. Therefore, for the 3-cell cavity,  $dF/dp = 3499.4$  N/bar.

[Figure 10: see original paper] Integral region of pressure on the helium tank end wall.

To further investigate the effect of helium pressure on the end wall of the helium bath on  $df/dp$ , a cavity model with an end wall outer diameter expanded to 150 mm was established while keeping other parameters unchanged. The integral result of the force component  $dp_x$  was 6023.3 N. The detuning simulation results are shown in Table 3. It can be observed that as the outer diameter of the helium bath's end wall increases, the  $df/dp$  of the cavity rises rapidly.

Simulation comparison of dressed cavities with different outer diameters for stiffening rings radius of 70 mm.

Detuning	120 mm	150 mm
$df/dp @ K_{tuner} = 10 \text{ kN/mm}$		
$df/dp @ K_{tuner} = 40 \text{ kN/mm}$		
$df/dp @ K_{tuner} = 80 \text{ kN/mm}$		

The main source of the cavity length change caused by helium pressure is the pressure exerted by the helium bath on the end walls of the helium tank. When using the same type of tuner, the ratio of  $dF/dp$  for different cavities is approximately equal to the ratio of the endwall areas of the helium bath. Therefore, reducing the diameter of the helium bath is a feasible approach to optimize helium pressure detuning [?, ?].

#### D. Summary of Helium Pressure Sensitivity Model

Building on the previous simulations, an independent analysis was conducted for each influencing factor outlined in Eq. (6), and the impact of each factor was quantified through simulation. Fig. 11 [Figure 11: see original paper] shows the results of  $df_{length}/dp$  under the condition of a tuner stiffness of 40 kN/mm. Furthermore, the influence of cell cavity wall deformation,  $df_{cell}/dp$ , is also plotted. The sum of these two components is compared with the complete simulation result of  $df/dp$  for the entire cavity [?].

[Figure 11: see original paper] Comparison of model combinations and complete cavity simulations.  $df_{cell}/dp$  is the frequency shift caused by the shape variations of the cavity cells, and  $df_{length}/dp$  is the frequency shift caused by the cavity length change. The combined model result is derived by formulating the individual simulation results from above, and it closely matches the curve obtained from the completed dressed cavity simulation.

The decomposition prediction of the model is in close agreement with the complete simulation of the entire cavity. The difference between the two is less than 5% at most data points, validating the reliability of the helium pressure detuning model and its conclusions. Therefore, increasing  $K$ , or reducing  $df_{cell}/dp$ , tuning sensitivity  $k_{tune}$ , or helium tank end-wall area  $S$ , are all effective methods to reduce  $df/dp$ .

For the 3-cell cavity without a stiffening ring, the impact of cavity wall deformation on frequency is relatively small. This is because the positive frequency shift caused by the equatorial deformation counteracts the negative frequency shift caused by the iris deformation [?]. For the 3-cell cavity with the stiffening ring, the stiffening ring changes this balance and limits the deformation of the bare cavity, thereby affecting  $df_{length}/dp$  and  $df_{cell}/dp$ .

When selecting the location of the stiffening ring, multiple factors need to be considered. The cavity's strength cannot be too weak, so a stiffening ring is required to ensure its structural integrity. If the stiffening ring radius of the cavity is set in the region where  $df/dp$  is relatively low, meaning the stiffening ring radius is larger, this effectively reduces the detuning caused by helium pressure variations. However, it also results in greater tuning difficulty. Therefore, after balancing the reduction of  $df/dp$  with tuning flexibility, this study preliminarily sets the stiffening ring radius at 70 mm to optimize the performance of the superconducting cavity.

## V. Mechanical Structure Analysis

Building upon the findings from the previous chapter, the fundamental structural parameters of the superconducting cavity are further refined. The thickness of the bare cavity wall is set at 2.8 mm, and the stiffening ring radius is established at 70 mm to enhance structural stability. Additionally, to ensure deformation consistency between the end cell and the middle cell during tuning, a stiffening ring is also installed at the position of the U-shaped beam pipe. The thickness of the stiffening ring is set to 3 mm, and to maintain helium flow, six evenly distributed holes are placed on the stiffening ring. The hole diameter between the cells is set to 10 mm, while the holes connecting the end cells and the U-shaped beam pipe are larger, with a diameter of 16 mm. Fig. 12 [Figure 12: see original paper] shows the sectional view of the bare cavity.

[Figure 12: see original paper] Cross-sectional view of the bare cavity with a 70 mm stiffening ring radius, highlighting the structural configuration.

This study also includes an in-depth investigation of the mechanical performance of the 1.3 GHz superconducting cavity, covering the simulation of the tuning force and microphonics, the analysis of the Lorentz force detuning, and the evaluation of the pressure tolerance.

### A. The Analysis of the Tuning Force

In the cryomodule, the frequency fine-tuning of the superconducting cavity is achieved through a precise piezo tuner. However, before the cavity is assembled into the cryomodule, coarse frequency tuning must be performed using a mechanical tuner. For a 1.3 GHz superconducting cavity, the typical tuning range is 500 kHz [?]. To ensure that the cavity is not damaged within the tuning range, it is necessary to simulate the force required for a 500 kHz frequency shift during the tuning process.

Based on this, a modal analysis was carried out on a 1.3 GHz superconducting cavity equipped with a stiffening ring of 70 mm radius, under two distinct boundary conditions: one with a single-end fixed and the other with both ends fixed. The simulation results for the single-end fixed condition are illustrated in Fig. 14 [Figure 14: see original paper].

[Figure 13: see original paper] Tuning force for 500 kHz versus stiffening rings radius.

The force required to tune a frequency shift of 500 kHz in the 3-cell superconducting cavity as a function of the stiffening ring radius is shown in Fig. 13. The results indicate that as the radius of the stiffening ring increases, the required tuning force rises exponentially. When the radius exceeds 85 mm, the tuning force becomes very large (greater than 30 kN), which poses significant challenges for the tuner design. Therefore, the cavity with a 70 mm stiffening ring radius demonstrated compliance with operational specifications, while the required tuning force of less than 15 kN significantly reduced mechanical adjustment complexity and simplified the tuner design requirement.

## B. The Analysis of Microphonics

The microphone effect is closely related to the mechanical vibration modes of the superconducting cavity. When the cavity's natural mechanical modes are influenced by external vibrations, it can trigger mechanical vibrations within the superconducting cavity, sometimes causing detuning of the RF resonant frequency. The structure of the superconducting cavity has a specific response pattern to external disturbances, with mechanical vibrations being transmitted to the cavity through transmission lines, low-temperature systems, supports, and the ground. When the frequency of the external vibration matches the inherent mechanical resonance frequency of the cavity, mechanical resonance can occur, leading to deformation of the cavity and subsequently affecting the stability of the RF resonant frequency. To ensure the stable operation of the superconducting cavity and prevent low-frequency resonance caused by environmental vibrations, it is crucial to understand and enhance the intrinsic vibration frequency of the cavity.

[Figure 14: see original paper] The lowest six mechanical modes of the cavity with stiffening rings with the single-end fixed. For the cavity with a 70 mm radius and a stiffening ring, the lowest natural mechanical frequencies of (a) Mode 1 and (b) Mode 2 are significantly higher than 50 Hz.

To evaluate the effectiveness of the stiffening ring design, comparative analyses were also conducted on the cavity without a stiffening ring and with the 70 mm stiffening ring. The corresponding modal frequencies under both boundary conditions are summarized in Table 4 and Table 5, respectively. The results reveal that the lowest mechanical resonance frequency of the cavity with the 70 mm stiffening ring reaches approximately 88 Hz. This value is significantly higher than the typical environmental vibration frequency of 50 Hz, which is commonly encountered in standard laboratory and operational equipment [?]. Such a frequency margin is crucial for avoiding resonance-induced performance degradation or detuning.

Mechanical modes of 3-cell superconducting cavity with the single-end fixed.

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Mode	w/o stiffening ring [Hz]	70 mm stiffening ring [Hz]
Mode 1		
Mode 2		
Mode 3		
Mode 4		
Mode 5		
Mode 6		

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Mechanical modes of 3-cell superconducting cavity with both ends fixed.

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Mode	w/o stiffening ring [Hz]	70 mm stiffening ring [Hz]
Mode 1		
Mode 2		
Mode 3		
Mode 4		
Mode 5		
Mode 6		

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These findings demonstrate that the inclusion of the stiffening ring effectively enhances the structural rigidity of the cavity and shifts its fundamental vibration modes beyond the range of dominant external disturbances. Therefore, the mechanical design satisfies the dynamic stability requirements for superconducting cavity operation.

### C. The Analysis of Lorentz Force Detuning

In the CW operation of the ERL high-current superconducting cavity, Lorentz detuning primarily affects the cavity during the initial field buildup phase. Simulation results are shown in Fig. 15, under the design accelerating gradient of 12 MV/m and a tuner stiffness of 40 kN/mm, the frequency shift caused by Lorentz forces ranges between 138 Hz and 197 Hz. The 70 mm stiffening ring radius is optimized for minimizing  $df/dp$ , and at this point, the superconducting cavity exhibits a relatively high Lorentz detuning coefficient, with a detuning amount of 162 Hz. Despite the relatively large detuning, this does not imply that the cavity cannot operate normally. By employing feedforward control, the effects of Lorentz detuning can be effectively compensated and eliminated [?].

[Figure 15: see original paper] Lorentz detuning factor of cavity versus stiffening rings radius.

Feedforward control is a technique that preemptively compensates for detuning before it occurs. The feedforward control system continuously monitors the cavity's operational parameters through sensors, predicts the potential detuning, and generates corresponding corrective signals. These signals are then used

to adjust the tuner, temperature control, or power supply systems to immediately correct the frequency shift [?]. In this way, feedforward control allows for proactive measures to maintain frequency stability before detuning occurs. This control method is particularly suitable for CW operation, which requires high stability, as it ensures the cavity's frequency remains stable without interfering with the acceleration process, thus preventing performance degradation due to frequency shifts [?].

#### D. The Analysis of Stress Under Different Loading Conditions

As mentioned in previous conclusions, a smaller helium vessel diameter positively impacts the reduction of  $df/dp$ , thus the diameter of the helium vessel is designed to be as small as possible. To avoid interference between the bellows and the bare cavity, considering the outer diameter of the bare cavity is 106 mm, the inner diameter of the helium vessel was set to 116 mm, with a wall thickness of 4 mm. The INFN blade tuner is selected, with the bellows positioned at the center of the helium vessel. While this layout compromises the strength of the helium tank due to the presence of the bellows, it eliminates the need for additional longitudinal clearance, allowing for effective space savings [?, ?]. The cavity model with a helium tank and bellows is shown in Fig. 16.

[Figure 16: see original paper] Schematic of 3-cell cavity helium tank with bellows. Red area represents the loads of the stress analysis, and the yellow area represents the boundary conditions.

The purpose of the stress analysis is to ensure that the mechanical design of the superconducting cavity can withstand the most challenging conditions encountered during its fabrication, testing, and operation. A comprehensive stress analysis of the dressed cavity was performed under four distinct loading conditions, including the effects of the helium vessel and gravity. The loading scenarios examined include leak tests, vertical tests, horizontal testing at cryogenic temperature, and extreme situations. The allowable stress limits for each material at room temperature and cryogenic temperatures are provided in Table 1. Based on the simulation results, the calculated stresses for all four loading conditions are within the permissible safety thresholds [?], where  $P_1$  is inside cavity,  $P_2$  is inside helium vessel and  $P_3$  is outside helium vessel, as summarized in Table 6, confirming that the cavity meets the stress safety criteria for operation. The stress distribution under vertical test is shown in Fig. 17. The maximum stress in the cavity is much lower than the material's allowable stress, and the cavity meets the strength requirements for stable operation under vertical test.

Summary of Stress Analysis Results Under Different Working Conditions.

Condition	Loads	Boundary	Temperature	Peak Stress on Cavity	Allowable Stress
Leak check (bare cavity)	i) Gravity ii) $P_1 = 0$ iii) $P_2 = 0.1$ MPa	Both ends fixed	293 K	47.0 (S)	70.5 (1.5S)
Leak check (cavity-string)	i) Gravity ii) $P_1 = 0$ iii) Entire exterior 0.1 MPa	free ends	293 K	47.0 (S)	70.5 (1.5S)
Vertical test (bare cavity)	i) Gravity ii) $P_1 = 0$ iii) $P_2 = 0.15$ MPa	Both ends fixed	293 K	212 (S)	318 (1.5S)
Horizontal test (dressed cavity)	i) Gravity ii) $P_1 = 0$ iii) $P_2 = 0.2$ MPa	Single-end fixed		212 (S)	318 (1.5S)

Condition	Loads	Boundary	Temperature	Peak Stress on Cavity	Allowable Stress
Operation (dressed cavity)	i) Gravity ii) $P_1 = 0$ iii) $P_2 = 0.2$ MPa	Single-end fixed		212 (S)	318 (1.5S)
Extreme situation (dressed cavity)	i) Gravity ii) $P_1 = 0$ iii) $P_2 = 0.225$ MPa	Single-end fixed		212 (S)	318 (1.5S)

[Figure 17: see original paper] (a) Stress and (b) deformation of the cavity under vertical test. Boundary and loading conditions depicted in Fig. 16 are used.

## VI. Conclusion

This study presents a comprehensive simulation and analysis methodology for helium pressure-induced detuning in high-current ERL superconducting cavities and structural optimizations to ensure long-term frequency and mechanical stability. A detailed investigation into the detuning mechanisms of the 1.3 GHz 3-cell injector cavity was conducted, utilizing multiphysics simulations to analyze the factors influencing helium pressure detuning. A predictive model for helium pressure detuning was established. To ensure stable operation of the superconducting cavity, the stiffening ring position was optimized to 70 mm. This modification significantly mitigated the impact of helium pressure detuning, reducing the frequency shift caused by helium bath pressure variations from 45.3

Hz/mbar to 33.4 Hz/mbar. This optimization of  $df/dp$  is not only intended to suppress microphonics detuning under nominal operating conditions but also aims to ensure cavity stability in low-current or low-power scenarios where the operational bandwidth becomes significantly narrower. Such robustness is essential for reliable long-term operation and to accommodate future upgrades or flexible beam modes in ERL-based accelerator facilities. Additionally, this design avoids the tuning challenges associated with excessively large stiffening ring radius, achieving a tuning range of 500 kHz with a tuning force of less than 15 kN, which significantly improves operational feasibility. Furthermore, critical structural components of the cavity were optimized, and their performance was rigorously evaluated. The optimized design demonstrated excellent results in stiffness, strength, and modal analysis, ensuring robust structural stability.

In summary, this paper provides a thorough analysis and design optimization of the mechanical structure of the 1.3 GHz 3-cell high-current superconducting cavity, significantly enhancing its frequency stability and mechanical performance. These advancements ensure its suitability for continuous-wave, high- $\beta$ , and high-current operation in ERL applications. The findings offer a valuable reference for the mechanical design of superconducting cavities, contributing to the development of more efficient and stable accelerator systems.

## VII. Acknowledgements

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