

Study on High-energy X-ray Source Reconstruction Method Using L-shaped Imaging Device

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Abstract

The L-shaped imaging devices are used to reconstruct the intensity distribution of the high-energy X-ray source. The physical model considering the penetration effect of X-ray on the imaging device is established, the transmission imaging matrix is constructed, and the algebraic solution method of source intensity reconstruction is presented. The X-ray source with Gaussian distribution is reconstructed. The reconstruction results show that the artifacts and discontinuities in the center of the reconstructed image using L-Edge device can be improved by L-Rolled Edge device, while L-Cylinder device can further improve the reconstruction quality.

Full Text

Preamble

Study on High-energy X-ray Source Reconstruction Method Using L-shaped Imaging Device

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Abstract

L-shaped imaging devices are employed to reconstruct the intensity distribution of high-energy X-ray sources. We establish a physical model that accounts for X-ray penetration effects in the imaging device, construct the transmission imaging matrix, and present an algebraic solution method for source intensity reconstruction. X-ray sources with Gaussian distribution are reconstructed, and the results demonstrate that artifacts and discontinuities in the center of images

reconstructed using the L-Edge device can be mitigated by the L-Rolled Edge device, while the L-Cylinder device can further improve reconstruction quality.

Keywords: High-energy X-ray source, Source intensity reconstruction, L-shaped imaging device, Penetration, Transmission imaging matrix

1. Introduction

High-energy X-ray flash radiography can penetrate the structure, state, and evolution process of high-speed moving objects, making it widely used to study the transient evolution of internal structures under impact loading. It serves as an indispensable diagnostic tool for fast transient processes such as hydrodynamic experiments [?, ?, ?, ?, ?]. However, high-energy X-ray flash imaging systems are complex, involve numerous physical processes, and their imaging quality is constrained by many factors. Geometrical blur caused by the focal spot size of high-energy X-ray sources represents one of the primary sources of image degradation in radiographic images [?]. Consequently, the intensity distribution and focal spot size of high-energy X-ray sources are critical parameters in image analysis, and the accuracy of these focal spot parameters directly determines whether high-precision reconstructed images can be obtained theoretically [?, ?].

Focal spot measurement constitutes an essential aspect of high-energy X-ray flash radiography research [?]. Major laboratories engaged in this field have conducted related research on spot size measurement, with representative methods including the pinhole method, slit method, edge method, rollbar method, and others [?, ?]. The Atomic Weapons Establishment (AWE) employs the cylindrical edge method to measure focal spots on its SuperSwarf flash imaging device [?]. Sandia National Laboratory (SNL) and Lawrence Livermore National Laboratory (LLNL) in the United States use conical pinhole imaging devices to measure focal spots on the ETA-II accelerator, while LLNL has also performed measurements using the cylindrical edge method on the ETA-II device [?]. Los Alamos National Laboratory (LANL) utilizes the pinhole method to measure focal spots on the DARHT-I device [?], and SNL has conducted focal spot measurements on the RITS-3 accelerator using the Rolled Edge imaging device [?].

The Institute of Fluid Physics of the China Academy of Engineering Physics initiated research on focal spot measurement technology in the 1990s, measuring focal spot size using the pinhole method on a 10 MeV linear induction accelerator (LIA) and the slit method on a 12 MeV LIA. After 2010, they developed the Rolled Edge method to measure the focal spot size of flash imaging devices. The Northwestern Institute of Nuclear Technology has also investigated measuring focal spots of Rod-Pinch Diodes (RPD) using the pinhole method [?] and the lamination method [?] (also known as the bread slice method).

These methods can be collectively referred to as the Modulation Transfer Function (MTF) method based on information transmission theory. This approach first calculates the MTF of the source's image distribution on the imaging plane

and then derives the equivalent focal spot size. Such methods are also known as indirect measurement techniques. While mature, convenient, and applicable, they can only provide size information about the source, not its shape information.

In recent years, to obtain two-dimensional focal spot distributions or achieve better engineering adaptability, Wang et al. [?] proposed a thick pinhole imaging scheme with double conical holes, with numerical simulations demonstrating that this design can effectively reduce image distortion caused by misalignment of the focal spot and deviation of the X-ray source from the axis. Although the thick pinhole method can theoretically provide shape information, it is difficult to fabricate, and edge penetration may affect imaging quality. To address the issue of actual X-ray focal spot drift after slit collimation, Gao et al. [?] adopted the lamination device, which consists of traditional slit devices overlapped to form a periodic structure for measuring focal spot size. Results showed that compared with the conventional slit method, the lamination method offers a wide field of view, high irradiation intensity on the image plane, high signal-to-noise ratio, and convenient collimation. In 2016, Fowler et al. [?] designed the L-Rolled Edge device for imaging based on the “opaque” physical model proposed by Barnea [?], using only one corner of a square-hole imaging device to obtain two-dimensional light and dark imaging information and deriving the two-dimensional intensity distribution of the focal spot through image reconstruction. This method can provide not only two-dimensional source information but also features an easily fabricated imaging device with convenient imaging and high environmental adaptability.

In this paper, based on the “L” configuration device, we propose an imaging physical model that considers transmission effects. First, using the L-Edge imaging device, we obtain the two-dimensional distribution of the source penetrating the imaging device and derive the intensity distribution of the source through image reconstruction. Through analysis of the reconstructed images, we sequentially propose two improved imaging device designs—L-Rolled Edge and L-Cylinder—to perform source intensity reconstruction.

The remainder of this paper is organized as follows. Section 2 presents the mathematical physical modeling and solution. Source intensity reconstruction using L-shaped imaging devices is described in Section 3. Finally, conclusions are summarized in Section 4.

2.1. Mathematical Model of Source Intensity Reconstruction

In high-energy X-ray flash radiography, X-ray energies reach up to 20 MeV, exhibiting strong penetrability [?, ?]. Therefore, it is necessary to consider the penetration of high-energy X-rays when measuring source intensity using imaging devices. The attenuation of X-rays penetrating matter follows the Lambert-Beer law. Assuming the X-ray source is an ideal isotropic monoenergetic point

source, the intensity of X-rays emitted from the source, passing through an object, and reaching each point on the detector plane can be expressed as:

$$I(x, y) = I_0 e^{-\int_{d(x,y)} \mu(l) \rho(l) dl}$$

where I and I_0 represent the X-ray intensities received by the detector with and without objects, respectively; $d(x, y)$ is the distance from point (x, y) on the imaging plane to the source; and $\rho(l)$ and $\mu(l)$ are the material density and mass attenuation coefficient of X-rays in the material at distance l from the source, respectively.

If the source intensity I_0 follows a certain distribution $I_0(x', y')$, the intensity of X-rays at any point (x, y) on the imaging plane after penetrating an object can be expressed as:

$$I(x, y) = \iint I_0(x', y') e^{-\int_{(x,y)}^{(x',y')} \mu(l) \rho(l) dl} dx' dy'$$

Note that the integration area covers the entire X-ray source region, and the integration path l is the straight line from source plane point (x', y') to imaging plane point (x, y) .

If the X-ray source region is discretized into pixels, the integral form of Eq. (2) can be rewritten in summation form:

$$I(x, y) = \sum_{x'} \sum_{y'} I_0(x', y') e^{-\int_{(x,y)}^{(x',y')} \mu(l) \rho(l) dl}$$

Furthermore, if both the source plane and imaging plane are discretized into pixels and regarded as one-dimensional vectors, Eq. (3) can be written in matrix-vector multiplication form:

$$Ax = b$$

where element a_{ij} of matrix A reflects the overall attenuation of the j -th source intensity reaching the i -th pixel on the imaging plane after passing through the imaging device, hence called the transmission imaging matrix; x is the source intensity to be reconstructed; and b is the X-ray intensity measured on the imaging plane.

The above equation represents an ideal case without considering imaging system noise. If the coefficient matrix A is full rank, the unknown source intensity vector can be obtained by solving linear equations. In practice, noise generally exists, and the equation becomes:

$$Ax = b + n$$

where n is the imaging system noise.

2.2. Construction of Transmission Imaging Matrix

In high-energy X-ray source measurement, an imaging layout with geometric magnification ratio $M > 1$ is generally adopted. In this paper, the X-ray source is positioned 15 cm from the center of the L-shaped imaging device, and the detector is 120 cm from the center of the L-shaped imaging device, yielding a geometric magnification ratio $M = 8$.

Given the current beam control level of accelerators, the radial size of the X-ray source can be less than 0.5 cm, so the source region is set to 1 cm \times 1 cm. To address practical source drift issues, the imaging region is expanded from 8 cm \times 8 cm to 15 cm \times 15 cm. Considering computational capabilities, both the source region and imaging region are discretized to a size of 80 \times 80, making the transmission imaging matrix A of size $(80 \times 80) \times (80 \times 80)$, i.e., 6400 \times 6400.

Assuming the L-shaped imaging device is a homogeneous single medium, the element a_{ij} of the imaging matrix based on the transmission model can be written as:

$$a_{ij} = e^{-\mu\rho L((x',y') \rightarrow (x,y))}$$

where $L((x',y') \rightarrow (x,y))$ is the geometric length of the X-ray path through the L-shaped imaging device from point (x',y') in the source plane to point (x,y) in the imaging plane. This length can be derived according to the geometric relative position relationship.

Taking the L-Edge imaging device as an example (see Figure 1: see original paper), the calculation of $L((x',y') \rightarrow (x,y))$ proceeds as follows. First, the geometric description is defined. Using the central ridge line of the L-Edge (the intersection line of two vertical planes) as the Z-axis, the plane where X-rays are incident on the L-Edge (the side near the source) is defined as the X_1Y_1 plane, which is divided into zones I, II, III, and IV according to the two vertical edges of the L-Edge. Similarly, the plane where X-rays exit the L-Edge (the side near the imaging plane) is defined as the X_2Y_2 plane, also divided into zones 1, 2, 3, and 4, as shown in [Figure 2: see original paper].

If an X-ray intersects the X_1Y_1 plane at point P_1 and the X_2Y_2 plane at point P_2 , the geometric length of the X-ray in the L-Edge can be obtained by examining the zones where P_1 and P_2 are located in the X_1Y_1 and X_2Y_2 planes, respectively. As shown in , various cases of X-rays penetrating (or not penetrating) the L-Edge are enumerated. Based on the complexity of treatment, these can be divided into four categories:

1. The simplest case: the X-ray does not intersect the L-Edge, or the distance within the L-Edge is calculated directly;
2. The X-ray passes through either the $Y_1O_1O_2Y_2$ plane or the $X_1X_2O_2O_1$ plane, requiring calculation of the intersection point between the X-ray and the respective plane;
3. More complicated: the X-ray passes through the $Y_1O_1O_2Y_2$ plane or the $X_1X_2O_2O_1$ plane, which requires judgment;
4. The most complicated case: the X-ray passes through both the $Y_1O_1O_2Y_2$ plane and the $X_1X_2O_2O_1$ plane simultaneously. In this scenario, the X-ray path in the L-Edge is divided into two segments, and the distance in air must be subtracted in the calculation, as shown in [Figure 3: see original paper].

For the “opaque” model adopted by Barnea et al. [?], this is equivalent to μ being infinite, and the matrix element based on the “opaque” model can be written as:

$$a_{ij} = \begin{cases} 0, & L > 0 \\ 1, & L = 0 \end{cases}$$

[Figure 4: see original paper] shows the imaging matrices of these two models under the same imaging conditions. The opaque model exhibits a sharp transition between light and dark, with values of either 0 or 1 (see Figure 4: see original paper), whereas the imaging matrix in Figure 4: see original paper changes gradually when transmission effects are considered, more accurately reflecting the gradual intensity variation of X-rays penetrating the L-shaped imaging device.

2.3. Numerical Solution Method

The source intensity reconstruction problem represented by Eq. (5) is a typical inverse problem. Due to the ill-posed nature of inverse problems, it is impossible to directly solve the linear equations (5) or its corresponding least squares formulation. Only by establishing appropriate constraint criteria can we converge to a reasonable solution [?, ?]. Regularization techniques are introduced to constrain the possible solutions of ill-posed inverse problems. By constructing a stable functional, the solution of the ill-posed inverse problem is reduced to a functional extremum problem with small regularization parameters. Although the solution may not be unique after introducing regularization, a relatively well-posed problem is realized for solving the original ill-posed problem. Regularization techniques can not only alleviate the ill-posedness of inverse problems but also weaken the influence of noise in the solution and filter noise, thereby improving reconstruction results [?, ?, ?, ?, ?]. After introducing regularization, solving the algebraic equations (5) is transformed into solving the following optimization problem:

$$\min_x \|Ax - b\|_2^2 + \lambda R(x)$$

where $\|Ax - b\|_2^2$ is called the fidelity term, describing the degree of approximation between the reconstructed source intensity and the original measurement data, and $\lambda R(x)$ is the regularization term, representing the artificially imposed constraint condition.

In this paper, by assuming that the unknown represents the sampled values of a slowly varying function and using the minimum of its first derivative magnitude as the criterion, we obtain the classical Tikhonov regularization model [?]:

$$\min_x \|Ax - b\|_2^2 + \lambda \|\nabla x\|_2^2$$

This is a nonlinear constrained optimization problem, which we solve using the practical Constrained Conjugate Gradient (CCG) method [?, ?, ?].

3. Reconstruction of Source Intensity

In this section, we investigate the proposed source reconstruction model considering attenuation by reconstructing a Gaussian-distributed X-ray source. In nuclear engineering, Monte Carlo (MC) simulation of particle transport is considered the method closest to real experimental conditions [?, ?, ?]. Therefore, we reconstruct MC-simulated images to test our mathematical and physical modeling and reconstruction method. The MC simulation is implemented based on the JMCT program [?], using the ENDF B7 database and considering physical processes including photoelectric effects, Compton scattering, and electron-positron pair production. This program has undergone rigorous verification and has been widely used in domestic engineering design.

The MC simulation discretizes the imaging plane into a grid of pixels and counts photons arriving at the imaging plane. When X-rays pass through a particular pixel area, the pixel generates a count and simultaneously records the fluctuation information of the count. When the standard deviation is less than 0.01, the count is considered equivalent to the true value. As shown in [Figure 5: see original paper], the full-width at half-maximum (FWHM) of the Gaussian-distributed X-ray source used in the MC simulation is set to 0.2 cm.

To evaluate reconstruction quality, we can obtain a visual impression and also calculate the relative error between the reconstructed image and the ground truth:

$$\text{Error} = \frac{\|x - x_0\|_2}{\|x_0\|_2}$$

where x is the reconstructed image and x_0 is the ground truth. Additionally, quantitative comparison of the FWHM between the reconstructed image and the ground truth can help us evaluate the reconstruction effectiveness.

3.1. Source Reconstruction Using L-Edge

First, we use the L-Edge imaging device to acquire MC-simulated radiographs. As shown in Figure 1: see original paper, the L-Edge imaging device has “two arms” with length $a = 10$ cm, height $c = 2$ cm, and thickness $d = 3$ cm, made of tungsten with a density of 19.0 g/cm^3 . The line connecting the center of the X-ray source and the center of the imaging plane coincides with the central ridge line of the L-Edge. [Figure 6: see original paper] shows the simulated image and its reconstructed intensity distribution, with cross-sectional lines across the center of the reconstructed image in different directions shown in [Figure 7: see original paper].

From the central transverse lines in [Figure 7: see original paper], the size of the reconstructed X-ray source can be obtained, with an FWHM of 0.2 cm in both horizontal and vertical directions, consistent with the true value. The relative error between the reconstructed image and the ground truth is 0.0841. However, obvious “discontinuities” appear in the center of the reconstructed source intensity, and the trailing portion of the Gaussian peak does not agree well with the ground truth.

3.2. Source Reconstruction Using L-Rolled Edge

Due to the right-angle edge of the L-Edge imaging device, there is a sudden intensity change near the edge, leading to serious artifacts in the reconstructed image. This “discontinuity” characteristic of edge devices has been recognized by researchers in the field of high-energy X-ray source measurement [?], who note that Roll-bar imaging devices can be used to improve this discontinuity. Referring to the concept of developing Edge devices into Roll-bar devices, we improved the L-Edge imaging device into the L-Rolled Edge imaging device, as shown in Figure 8: see original paper. Specifically, the side surface changes from a planar surface to a circular arc surface, avoiding step discontinuities in source intensity near the side surface.

For the selection of the radian of the L-Rolled Edge device, as shown in [Figure 9: see original paper], the height between the highest point of the arc segment and the Edge segment is set as h , equivalent to the focal spot size of the X-ray source, shown as a red circle in [Figure 9: see original paper]. According to the geometric relationship, it is easy to obtain:

$$R = \frac{L^2 + h^2}{2h}$$

In this paper, h is set to 0.2 cm, and the arc radius of the L-Rolled Edge

is 5.725 cm. The transmission imaging matrix of the L-Rolled Edge imaging device is shown in Figure 8: see original paper. Compared with Figure 4: see original paper, it is evident that the transition areas of the L-Edge device are mainly distributed on both sides of the central region of the transmission imaging matrix, with obvious discontinuities in the central region of the matrix. In contrast, the L-Rolled Edge device improves the discontinuity of the L-Edge, with the entire transition area distributed in a strip shape that is more uniform. Moreover, the transition region of the transmission imaging matrix of the L-Rolled Edge device is significantly wider and has better continuity, which can provide more information for source intensity reconstruction and is theoretically more conducive to the continuity of reconstructed images.

The MC-simulated image using the L-Rolled Edge imaging device and its corresponding reconstructed source image are shown in [Figure 10: see original paper]. Compared with Figure 6: see original paper, the “L”-shaped grid in the center of the reconstructed image using the L-Edge device is significantly improved after using the L-Rolled Edge device, with the latter being closer to the ground truth in the central region. Additionally, the relative error of the reconstructed source using the L-Rolled Edge device is 0.0765, which is smaller than the 0.0841 obtained with the L-Edge device.

[Figure 11: see original paper] shows the comparison between the reconstructed Gaussian source using the L-Rolled Edge device and the ground truth in horizontal and vertical directions, demonstrating good agreement. The FWHM of the reconstructed Gaussian source is 0.2 cm, consistent with the true value. Moreover, the reconstructed source using the L-Rolled Edge device significantly improves the discontinuity on the Gaussian peak compared with that using the L-Edge device.

3.3. Source Reconstruction Using L-Cylinder

After upgrading the L-Edge imaging device to L-Rolled Edge, the imaging quality is greatly improved, particularly eliminating the “discontinuity” in the central part of the reconstructed Gaussian source. However, careful observation of the transverse lines of the reconstructed image in horizontal and vertical directions reveals a slight “non-smoothness” at positions deviating from the central peak, as indicated by the blue boxes in [Figure 11: see original paper]. Preliminary analysis suggests that this non-smoothness may be related to the discontinuity at the junction between the arc section and the Edge section of the L-Rolled Edge device. Therefore, we propose an improved design: the L-Cylinder imaging device, formed directly by splicing two cylindrical sections into an “L” shape, as shown in Figure 12: see original paper.

The transmission imaging matrix of the constructed L-Cylinder device is shown in Figure 12: see original paper. Compared with Figure 4: see original paper and Figure 8: see original paper, the transition region of the transmission imaging matrix of the L-Cylinder device is wider and more uniform than those of the

L-Edge and L-Rolled Edge devices, which is theoretically more beneficial to the smoothness of the reconstructed image.

[Figure 13: see original paper] shows the MC radiographic image with the L-Cylinder device and its corresponding reconstructed intensity distribution, with cross-sectional lines of the reconstructed image in different directions shown in [Figure 14: see original paper]. It is evident that the L-Cylinder imaging device can accurately reconstruct the intensity distribution of the Gaussian source and precisely measure the FWHM of the Gaussian source as 0.2 cm. Furthermore, the relative error of the reconstructed Gaussian source is 0.0685, which is the smallest among the three imaging devices discussed in this paper.

[Figure 15: see original paper] compares the reconstruction results of the L-Rolled Edge and L-Cylinder devices. These two results appear very similar and show significant improvement compared with those in [Figure 7: see original paper]. However, as seen from the zoomed-in views, the L-Cylinder imaging device improves the non-smoothness of the reconstructed source with the L-Rolled Edge in the lower part of the Gaussian peak, and the reconstructed source with the L-Cylinder agrees better with the ground truth.

4. Conclusion

In this paper, we present a focal spot measurement method for high-energy X-ray sources based on source intensity reconstruction using L-shaped imaging devices. We establish a physical model that considers X-ray penetration effects in the imaging device and construct the transmission imaging matrix. By introducing regularization techniques, we provide an algebraic solution method for source intensity reconstruction. Using the simplest L-Edge imaging device, we reconstruct X-ray sources with Gaussian distribution. However, serious artifacts appear in the reconstructed image. To improve reconstruction quality, we sequentially propose the L-Rolled Edge and L-Cylinder imaging devices. The reconstruction results show that artifacts and discontinuities in the center of the reconstructed image can be improved using the L-Rolled Edge imaging device, while discontinuities can be further improved using the L-Cylinder imaging device. Consequently, the L-Cylinder imaging device is more suitable for reconstructing high-energy X-ray sources.

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