

The user wants me to translate a Chinese academic paper title and subtitle: “A Combined Smoothing Method of Ensemble Pulsar Timescale and Application for Pulsar Atomic Clock Combined Timescale 后印本” Let me break this down: - “A Combined Smoothing Method of Ensemble Pulsar Timescale” - This is the m...

Authors: The user wants me to translate a list of names from Chinese to English. However, these appear to be author names, which are typically already in English or should be transliterated according to standard conventions.

Looking at the names: - Tinggao Yang - This is already in English format (given name + surname) - Minglei Tong - Already in English - Bian Li - Already in English - Zhehao Zhang - Already in English - Xingzhi Zhu - Already in English - Yuping Gao - Already in English

These names appear to be written in Pinyin (the romanization system for Chinese), which is the standard way to represent Chinese names in English academic publications. The task is to translate from Simplified Chinese to English, but these names are already in English/Pinyin format.

However, I need to check if there are any specific instructions about names. The user provided a glossary: - 机器学习 => machine learning - 深度学习 => deep learning

But there's nothing about names. In academic translation, Chinese names written in Pinyin are typically kept as-is, though sometimes the order might be adjusted (surname first in Chinese style vs given name first in Western style).

Looking at the names provided: - Tinggao Yang (杨廷高) - Minglei Tong (童明雷) - Bian Li (李变) - Zhehao Zhang (张哲浩) - Xingzhi Zhu (朱兴植) - Yuping Gao (高玉平)

These are already in Western order (given name + surname). In academic contexts, this is common for Chinese authors publishing in international journals.

Since the names are already in English/Pinyin format, there is no actual translation needed. The user might be testing if I understand that names should

be preserved. The instruction says “Translate from Simplified Chinese to English with the highest accuracy and academic tone.” But these names are not in Chinese characters - they’ re already in Latin script.

I should return the names exactly as provided, since they are already in the appropriate format for an English academic paper. No modification is needed.

Wait, let me re-read the instructions carefully: “CRITICAL REQUIREMENTS: 1. Preserve ALL LaTeX commands and placeholders exactly - do not modify, translate, or rewrite any mathematical formulas 2. Preserve ALL citation commands ([?], ??, (??), etc.) exactly as they appear 3. **PRESERVE ALL <

Date: 2025-03-11T00:00:00+00:00

Abstract

The user wants me to translate a scientific abstract from Chinese to English. Let me analyze the requirements:

1. Critical Requirements:

- Preserve ALL LaTeX commands and placeholders exactly
- Preserve ALL citation commands ([?], ??, (??), etc.) exactly
- **PRESERVE ALL `…`paragraph wrapper tags EXACTLY**
- Maintain academic tone and formal scientific writing style
- Use precise technical terminology - obey the glossary strictly
- Preserve section structure and paragraph organization
- Translate all content completely - do not omit any information

2. Output Rules:

- Return ONLY the translated text (no explanations, no quotes, no headings)
- Keep one output paragraph per input paragraph
- **Maintain exact count and IDs of `…`tags**
- Do NOT add Markdown formatting unless present in source
- Preserve original line breaks within the paragraph when meaningful

3. Formatting Guidelines:

- Keep mathematical expressions in original LaTeX format
- Preserve equation numbers and references
- Maintain proper academic paragraph structure
- Use formal scientific language

4. Glossary:

- 机器学习 => machine learning
- 深度学习 => deep learning

Wait, the text provided is already in English, not Chinese. Let me re-read the user’ s message.

The user says: “Clock difference between the ensemble pulsar timescale (PT) and the International Atomic Time (TAI) PT-TAI derived from the International Pulsar Timing Array (IPTA) data set indicates a very similar variation

trend with the Terrestrial Time TT(BIPMXXXX)-TAI but PT has larger measurement error. In this paper, we discuss the smoothing method of PT using a combined smoothing filter and compare the results with that from other filters. The clock difference sequence between PT-TAI and the first time derivative series of the TT(BIPMXXXX)-TAI can be combined by a combined smoothing filter to yield two smooth curves tied by the constraints assuring that the latter is the derivative of the former. The ensemble pulsar time IPTA2016 with respect to TAI published by G. Hobbs et al. and first time derivative series of the TT(BIPM2017)-TAI with quadratic polynomial terms removed are processed by combined smoothing filter in order to demonstrate the properties of the smoothed results. How to correctly estimate two smoothing coefficients is described and the output results of the combined smoothing filter are analyzed. The results show that the combined smoothing method efficiently removes high frequency noises of two input data series and the smoothed data of the PT-TAI combine long term fractional frequency stability of the pulsar time and frequency accuracy of the terrestrial time. Fractional frequency stability analysis indicates that both short and medium time interval stability of the smoothed PT-TAI is improved while keeping its original long term frequency stability level. The combined smoothing filter is more suitable for smoothing observational pulsar timescale data than any filter that only performs smoothing of a single pulsar time series. The smoothed pulsar time by combined smoothing filter is a pulsar atomic time combined timescale. This kind of combined timescale can also be used as terrestrial time.”

This is already in English. But the user is asking me to translate from Simplified Chinese to English. This seems like a test or perhaps the user made a mistake. Let me check if there are any tags in the text. No, there are no tags.

The user instructions say: “Translate from Simplified Chinese to English with the highest accuracy and academic tone.”

But the provided text is already in English. This is confusing. Let me re-read the user’s message more carefully.

The user message is: “Clock difference between the ensemble pulsar timescale (PT) and the International Atomic Time (TAI) PT-TAI derived from the International Pulsar Timing Array (IPTA) data set indicates a very similar variation trend with the Terrestrial Time TT(BIPMXXXX)-TAI but PT has larger measurement error. In this paper, we discuss the smoothing method of PT using a combined smoothing filter and compare the results with that from other filters. The clock difference sequence between PT-TAI and the first time derivative series of the TT(BIPMXXXX)-TAI can be combined by a combined smoothing filter to yield two smooth curves tied by the constraints assuring that the latter is the derivative of the former. The ensemble pulsar time IPTA2016 with respect to TAI published by G. Hobbs et al. and first time derivative series of the TT(BIPM201

Full Text

Preamble

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A Combined Smoothing Method of Ensemble Pulsar Timescale and Application for Pulsar Atomic Clock Combined Timescale

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Abstract

The clock difference between the ensemble pulsar timescale (PT) and International Atomic Time (TAI), PT-TAI, derived from the International Pulsar Timing Array (IPTA) dataset exhibits a variation trend very similar to that of Terrestrial Time TT(BIPMXXXX)-TAI, but PT has larger measurement errors. In this paper, we discuss the smoothing of PT using a combined smoothing filter and compare the results with those from other filters.

The clock difference sequence between PT-TAI and the first time derivative series of TT(BIPMXXXX)-TAI can be combined by a combined smoothing filter to yield two smooth curves tied by constraints ensuring that the latter is the derivative of the former. The ensemble pulsar time IPTA2016 with respect to TAI published by G. Hobbs et al. and the first time derivative series of TT(BIPM2017)-TAI with quadratic polynomial terms removed are processed by the combined smoothing filter to demonstrate the properties of the smoothed results. We describe how to correctly estimate the two smoothing coefficients and analyze the output results of the combined smoothing filter. The results show that the combined smoothing method efficiently removes high-frequency noise from both input data series, and the smoothed PT-TAI combines the long-term fractional frequency stability of pulsar time with the frequency accuracy of terrestrial time. Fractional frequency stability analysis indicates that

both short- and medium-term stability of the smoothed PT-TAI are improved while maintaining its original long-term frequency stability level. The combined smoothing filter is more suitable for smoothing observational pulsar timescale data than any filter that only performs smoothing on a single pulsar time series. The pulsar time smoothed by the combined smoothing filter is a pulsar-atomic combined timescale, which can also be used as terrestrial time.

Key words: methods: data analysis -time -(stars:) pulsars: general

1. Introduction

The rotation of millisecond pulsars is very stable, and within a few years, the stability of rotation of such millisecond pulsars rivals that of atomic clocks (Kaspi et al. 1994). It has been proposed that pulsars should be used to establish a new astronomical timescale to which atomic time could be compared. Earlier attempts to develop a pulsar timescale have been made by Petit & Tavella (1996), Zhong & Yang (2007), and Rodin (2008). Since the Pulsar Timing Array (PTA) project observing an ensemble of millisecond pulsars was initiated, more than 50 millisecond pulsars are now being observed by the International Pulsar Timing Array (IPTA) project. The long-term timing observation data of the IPTA are available (Verbiest et al. 2016; Perera et al. 2019). The algorithm for constructing an ensemble pulsar timescale (PT) has been developed and improved (Hobbs et al. 2012, 2020; Yang et al. 2022; Zhang et al. 2024). The ensemble pulsar time IPTA2016 published by Hobbs et al. (2020) includes two sets of results obtained using generalized least squares and a Bayesian algorithm. The generalized least squares algorithm models clock errors (PT signal) as a set of equally spaced samples with an interpolation mechanism, and the clock error model is included in the timing model and fitted out (for details see Hobbs et al. 2012, 2020). The Bayesian algorithm models the power spectrum of the signal of clock errors as a power-law spectrum; when the model parameters are determined by Bayesian analysis, the waveform of clock errors is constructed by generalized Wiener filtration (see Lee et al. 2014; Hobbs et al. 2020). Both algorithms emphasize the importance of timing noise analysis and distinguish between noise and clock signal.

The difference between IPTA2016 and International Atomic Time (TAI) established by the generalized least squares algorithm (Hobbs et al. 2020) contains 37 data points with a sampling interval of 182.6 days, the average error of the data points is 0.226 s, and the total time span of the data is 6574.5 days (from MJD 49400 to MJD 55974.5).

On the basis of TAI, the Bureau International des Poids et Mesures (BIPM) further uses primary frequency standard data to obtain the comprehensive atomic time TT(BIPMxxxx). TT(BIPMxxxx) is used as the coordinate time of the International Terrestrial Reference System (ITRS), also known as terrestrial time, where xxxx indicates the year of release. This time is available a year later than TAI and is updated annually. Figure 1 shows a comparison of the clock

difference IPTA2016-TAI (hereafter referred to as PT-TAI) with the difference TT(BIPM2017)-TAI obtained by anonymous ftp from the BIPM. Compared to atomic time, the error bars of PT-TAI data points are larger, indicating large measurement errors, while TT(BIPM2017)-TAI is much smoother. As shown in Figure 1, PT-TAI and TT(BIPM2017)-TAI have basically similar trends, signifying that the systematic error of TAI is detected by PT. Several early data points of PT-TAI deviate greatly from TT(BIPM2017)-TAI, mainly because early available pulsar observations were few and the timing observation errors were relatively large (Hobbs et al. 2020). At present, the published PT research results, including those from the Bayesian algorithm and Wiener filtration algorithm, can also detect systematic errors of TAI (Hobbs et al. 2012, 2020; Yang et al. 2022). It should be noted that because pulsar time does not contain linear and quadratic terms, for the data involving TT(BIPM2017)-TAI in Figure 1 and the following text, the linear and quadratic terms are fitted and removed.

We use σz to compare the fractional frequency stability of pulsar timescale with atomic timescale (Matsakis et al. 1997). Defined in terms of third-order polynomials fitted to sequences of measured time offsets, σz is sensitive to variations in the frequency drift rate of the clock or pulsar. Figure 2 shows the comparison of the fractional frequency stability curves of PT-TAI with TT(BIPM2017)-TAI. Due to the influence of measurement error, the frequency stability curve of PT-TAI shows an approximately linearly decreasing trend with increasing time interval, and the stability at the maximum time interval is close to that of TT(BIPM2017)-TAI. As the time series of PT extends, its long-term frequency stability has the potential for further improvement. Contrary to PT-TAI, TT(BIPM2017)-TAI has better short-term frequency stability, but the curve gradually rises with increasing time interval.

In order to make full use of the long-term frequency stability of PT and improve its stability level on short- and medium-term time intervals, it is necessary to study noise removal methods for PT. In this study and analysis example of PT noise removal methods, the TT(BIPM2017)-TAI data are used. Through analysis and comparison by applying different noise removal methods to PT-TAI, we find that the combined smoothing method (Vondrak & Cepek 2000) produces satisfactory results. This method combines the clock difference series of PT-TAI data with the first time derivative of TT(BIPM2017)-TAI data to yield two smoothed curves such that one is smoothed PT-TAI and the other is its first time derivative. Although PT-TAI has good long-term frequency stability, the PT-TAI observation data points are sparsely sampled and their errors are relatively large, making it difficult to provide information about short-term variations of clock difference. On the contrary, TT(BIPM2017)-TAI with a sampling interval of 10 days has better frequency accuracy and short-term frequency stability. Therefore, TT(BIPM2017)-TAI can provide short-term variation information on the clock difference, i.e., information on the first time derivative of the clock difference.

At the same time, the long-term fractional frequency stability of PT-TAI can

constrain long-term variation of the first time derivative of TT(BIPM2017)-TAI. The function of a simple low-pass filter is to remove high-frequency noise. The combined smoothing method not only can remove high-frequency noise from PT-TAI, but can also combine the long-term stability of pulsar time and frequency accuracy of terrestrial time. The smoothed PT-TAI, while maintaining its original long-term frequency stability, significantly improves its level of frequency stability at short- and medium-term time intervals.

Liu et al. (2023) used a combined smoothing method, while Zhu et al. (2024) employed a wavelet analysis method to combine ensemble pulsar time and ensemble atomic time, but they did not discuss the problem of combining pulsar time and atomic frequency standards. We describe the role of frequency standards in establishing timescale in Section 4.1.

In this paper, we focus on using the combined smoothing method to combine the PT-TAI and the first time derivative of TT(BIPM2017)-TAI datasets. In Section 2, some noise removal methods for observational data, especially the combined smoothing method, are briefly described. In Section 3, the process and results of smoothing both PT-TAI and the first derivative of TT(BIPM2017)-TAI using the combined smoothing method are presented in detail. Section 4 discusses some applications of the combined timescale produced by the combined smoothing filter. The final section gives preliminary conclusions.

2. Smoothing Method of Ensemble Pulsar Timescale

In order to compare simple low-pass filtering against combined smoothing filtering, both types of filters are introduced and their smoothed results are compared.

2.1. Fourier Domain Filtering

By applying a suitable filter to remove high-frequency noise from observed data, we can extract low-frequency signals of interest. For example, using Fourier domain filtering can remove the high-frequency noise components of the observed data. To perform the Fourier transformation and compare the smoothed result with atomic time, we interpolated PT-TAI linearly into a time series with 10-day spacing. Fourier transformation of the interpolated PT-TAI series is performed, then the series, after discarding the Fourier high-frequency components and signals with periods longer than 900 days, is transformed back to the time domain (Matsakis et al. 1997). The resulting smoothed PT-TAI after removing high-frequency noise is shown as the red line in Figure 1. The frequency stability σ_z curve of the smoothed PT-TAI is presented in Figure 2.

Figure 2 clearly shows that the frequency stability of the smoothed PT-TAI for some medium-term intervals has a higher value than that for both shorter and maximum time intervals.

2.2. Combined Smoothing Method

The Vondrak smoothing method (Vondrak 1969, 1977) is popularly used in many branches of astronomy. The smoothed result of PT-TAI using this method will be given in Section 3.

On the basis of the Vondrak smoothing method for observational data, Vondrak & Cepek (2000) published the combined smoothing method. This method smooths two sets of observational data: in general, one with measured function values of a certain quantity whose analytic expression is unknown (Series 1) and the other with measured time derivatives of the same quantity (Series 2). Both series are measured independently or with different observation techniques and given at unequally spaced epochs that need not be identical, and the individual observations are given with different precision, defined by their formal uncertainties. For example, in the International Earth Rotation Service, measured polar motion and polar motion rate data are processed by the combined smoothing method; universal time UT1 obtained by VLBI observation and day length variation data series measured by GPS are combined by combined smoothing. The smoothed results of the two series are defined at all data points of the two input datasets, and the two sets of output series have the same number of data points. If the number of data points in the two sets of input data is respectively n_1 and n_2 , and observation epochs of the two sets of data are completely independent (no overlapping data points), then the number of data points of each output dataset from the combined smoothing filter will be $N = n_1 + n_2$. Otherwise, the number of data points where the observation epochs overlap between the two sets of input data should be subtracted, in which case $N < n_1 + n_2$.

The basic idea of the combined smoothing method is to find a suitable weighted compromise smoothing scheme that satisfies three different constraint conditions: smoothness of the sought curve, its fidelity to the observed function values, and its fidelity to the observed first time derivatives. Let y_i , $i = 1, 2, \dots, N$, represent the value of the i th data point on a smoothed curve of the observed function values to be determined.

Suppose that the mathematical expression for the smoothness of the smoothed curve is S , the expression for the fidelity of the smoothed curve to the observed data is F , and the expression for the fidelity of the first derivative of the smoothed curve to the input first time derivative data is \bar{F} . All three expressions can be written as functional forms of y_i . We seek the smoothed y_i values as a compromise among the three different conditions. The adjustment is done by minimizing a combination of the constraints above, i.e., the expression

$$Q = S + e_1^2 F + e_2^2 \bar{F}$$

where we have smoothing chosen for Q . In this equation, $e_1 \geq 0$, $e_2 \geq 0$, and these two parameters are the coefficients of the combined smoothing method. The degree of compromise among the three conditions is achieved by choosing

appropriate values for these two parameters. From the minimization condition, the system of N linear equations with unknowns y_i can be obtained, and the unknowns can be solved using the known observation data (including the observed function values of the smoothed curve and its observed first derivative values). Then, using the derived smoothed values y_i , the smoothed values of the first derivative are calculated by formulas (4-7) in Vondrak & Cepek (2000).

For details on the analytical expressions for S , F , and \bar{F} in the equation above and the expressions of partial derivatives for Q , see Vondrak & Cepek (2000). The two parameters e_1 and e_2 have dimensions $[\text{time}^{-6}]$ and $[\text{time}^{-4}]$, respectively. The larger their values, the larger the weight assigned to the observed function values or their first derivatives, and the closer the smoothed values are to the observations.

There is a close relation between the smoothing coefficients and the transfer function of the combined smoothing filter (i.e., the ratio between the amplitude of the smoothed curve and the observed amplitude of a periodic function with frequency f). The transfer function T for the observed function values can be expressed analytically in terms of e_1 and frequency f . Similarly, the transfer function \bar{T} for the observed first derivatives can be expressed in terms of e_2 and frequency f .

Regarding the choice of smoothing coefficients, if the frequency of the two sets of input data to be suppressed by the filter is known in advance, the smoothing coefficients can be calculated accordingly. Assuming that half of the amplitude of a high-frequency period needs to be suppressed by the filter, the period is expressed as $p_{0.5}$ and its corresponding transfer function is 0.5. A shorter period than $p_{0.5}$ is suppressed more strongly, and otherwise suppressed more weakly. The corresponding smoothing coefficients can then be calculated using appropriate formulas.

3. Application of Combined Smoothing Method

3.1. Input Data for Combined Smoothing Filter

The data sequence of the difference between ensemble pulsar time IPTA2016 from Hobbs et al. (2020) and TAI (as illustrated by the blue line in Figure 1) was used. The clock difference IPTA2016-TAI is referred to as PT-TAI in the following text.

The input data of the clock difference series for the combined smoothing filter includes the data point sampling date MJD, clock difference values, and their corresponding weights used for the filter. The error of each data point is different, and its weight is taken as the reciprocal of the square of the measurement error.

Another set of input data for the filter is the first time derivative (i.e., frequency difference) series derived from the difference between the terrestrial timescale and TAI. TT(BIPM2017)-TAI is used for calculating the first time derivative and should have approximately the same time span as PT-TAI. Because PT-TAI

does not contain linear or quadratic terms, the derivative must be calculated using TT(BIPM2017)-TAI with linear and quadratic terms eliminated. We use the central difference method to calculate the first time derivative of each data point of TT(BIPM2017)-TAI. That is, the first-order derivative of the middle clock difference data point is computed using the corresponding clock difference values of three adjacent data points. To calculate the first derivative of the clock difference data points at the head and tail endpoints, a quadratic polynomial was fitted using the three data points located at the endpoint, and then the first derivative of the endpoint is derived from the quadratic polynomial obtained by the fitting. The calculated first derivative sequence has exactly the same number of data points and sampling interval as the clock difference TT(BIPM2017)-TAI sequence, containing 659 data points, as traced by the blue line in Figure 3. Because errors of all data points in the TT(BIPM2017)-TAI clock difference sequence are similar, all data points in the first derivative sequence have equal weight. The input data of the first time derivative series for the filter include the data point sampling date MJD, first-order derivative values, and their weights.

3.2. Estimation of Smoothing Coefficients

If we want 99% of a periodic signal $p_{0.99}$ to be passed by the filter, its corresponding transfer function is 0.99, and the formulas for calculating the two smoothing coefficients can be expressed as:

$$e_1 = \frac{2\pi}{p_{0.99} \sqrt{0.99^{-2/3} - 1}}$$
$$e_2 = \frac{2\pi}{p_{0.99} \sqrt{0.99^{-2/5} - 1}}$$

These equations are especially suitable for cases where heavier smoothing is required.

There are two possibilities for choosing the smoothing coefficients e_1 and e_2 , both requiring at least approximate a priori knowledge of the observed process in the input datasets; we should have a realistic estimation of either the precision of the measurements or the shortest period contained in the signal (Vondrak & Cepek 2000). In our case, the measurement error of PT-TAI data points is much larger than that of TT(BIPM2017)-TAI data points. If the smoothing coefficients of the combined smoothing filter are estimated according to the measurement errors of the two input datasets, the resulting e_1 would be too small and e_2 would be too large, causing the PT-TAI weight to be too small and the weight of the first derivative of TT(BIPM2017)-TAI to be too large. As a result, the two sets of smoothed data produced by the filter would be very close to TT(BIPM2017)-TAI and its first time derivative, respectively.

If the shortest period contained in the signal of the data is known, it is recommended to choose values for e_1 and e_2 such that $p_{0.5}$ lies approximately between

one-third and one-half of the shortest known period contained in the signal. Alternatively, it is also possible to calculate e_1 and e_2 using the equations above, where we set $p_{0.99}$ equal to the shortest known period of the signal. Any values of e_1 and e_2 in the vicinity of these yield approximately the same acceptable results (Vondrak & Cepek 2000).

We initially assumed that the shortest period of the signal contained in the data is about 3 yr and used $p_{0.99} = 3$ yr to calculate the smoothing coefficients e_1 and e_2 . The derived solution of the filter indicated that the smoothed first time derivative sequence significantly deviated from its input data. This large deviation was caused by the noise of PT-TAI which was not sufficiently smoothed. We then decided to perform grid searches to find the shortest period of the signal $p_{0.99}$ in the input datasets best suited for calculating a pair of smoothing coefficients.

Letting $p_{0.99} = 0.1, 0.2, \dots, 12$ yr (where 12 yr is greater than half of the PT-TAI time span) and substituting $p_{0.99}$ into the formulas, we obtained corresponding sequences of e_1 and e_2 values. Using these sequences of e_1 and e_2 values, the combined smoothing filter smoothed the PT-TAI series and first derivative series of TT(BIPM2017)-TAI and calculated the standard deviation of the respective residuals (differences between the filter input values and the corresponding smoothed output values) for both PT-TAI series and the first derivative series of TT(BIPM2017)-TAI. Figure 4 shows the relationship curve between $p_{0.99}$ used to calculate smoothing coefficients and the corresponding standard deviation, with the upper subplot for e_1 and the lower subplot for e_2 .

The smaller the period value used for calculating the smoothing coefficient, the larger the value of the smoothing coefficient derived. The larger the value of the smoothing coefficient used, the weaker the smoothing of the corresponding input data sequence. The upper subplot in Figure 4 shows that at $p_{0.99} < 6$ yr the residual standard deviation increases as the period used to calculate e_1 increases, and at $p_{0.99} \geq 6$ yr the curve of residual standard deviation gradually levels off for PT-TAI. Conversely, in the lower subplot, at $p_{0.99} < 6$ yr (excluding the first data point), the residual standard deviation decreases with increasing period $p_{0.99}$, and at $p_{0.99} \geq 6$ yr the curve of residual standard deviation gradually levels off for the first derivative data of TT(BIPM2017)-TAI. This phenomenon is caused by the incompatibility of the errors of the two datasets (large error of PT-TAI versus small error of the first derivative of TT(BIPM2017)-TAI). The larger measurement errors of PT-TAI lead to large fluctuations in their sampled data points. In the weak smoothing case, the smoothed curve of the first derivative deviates greatly from its true value due to PT-TAI error, resulting in a larger standard deviation of the first derivative residuals. As the period value used for calculating smoothing coefficients increases, PT-TAI is progressively more moderately smoothed, bringing the smoothed curve of the first derivative gradually closer to the correct one. Figure 3 illustrates the smoothed first derivative curves combining the two input datasets using e_1 and e_2 with four different $p_{0.99}$ values. We see that in Figure 3, the first derivative smoothed curve de-

rived using smoothing coefficients with a 4 yr period shows some deviation due to insufficiently smoothed noise of PT-TAI. As the $p_{0.99}$ value used gets larger, this kind of deviation is gradually suppressed and removed.

According to Figure 3, it is recommended to carry out combined smoothing for PT-TAI and the first derivative of TT(BIPM2017)-TAI using smoothing coefficients e_1 and e_2 calculated with $p_{0.99} = 6$ yr. Choosing smoothing coefficients with $p_{0.99} = 6$ yr, the high-frequency noise of PT-TAI can be sufficiently smoothed while the interesting signal is not suppressed. For the smoothing coefficients e_1 and e_2 calculated with a 6 yr period, the corresponding transfer function curves of the filter are shown in Figure 5, where the blue solid and red dotted lines are respectively the transfer functions of PT-TAI clock difference and first derivative of TT(BIPM2017)-TAI. In Figure 5, the abscissa is the logarithm of the signal period. As can be seen, the transfer function curves of the two datasets are similar, and the first derivative transfer function is slightly shifted more toward the short-period end than the clock difference transfer function. The transfer functions shown in Figure 5 yield better smoothed results for both datasets simultaneously. If we change the value of e_1 to shift the blue line to nearly overlap the red dotted line, or change the value of e_2 to shift the red dotted line to nearly overlap the blue line, in these two cases the smoothed curve of the first derivative is obviously affected, the fluctuation of the curve becomes larger, and the residual standard deviation becomes larger, whereas the standard deviation of the clock difference curve becomes smaller. In order to balance smoothed results for both clock difference and first derivative, the two smoothing coefficients of the filter should be calculated using the same suitable $p_{0.99}$ value. We finally adopt the smoothed results for the two datasets in the case with $p_{0.99} = 6$ yr.

3.3. Analysis of Smoothed Results

The smoothing coefficients e_1 and e_2 with $p_{0.99} = 6$ yr are used for the combined smoothing filter to smooth the PT-TAI and first derivative series of TT(BIPM2017)-TAI. Since the observation epochs of the two datasets do not overlap, the number of data points for each of the two smoothed datasets equals the sum of the data points of the two original datasets. Figure 6 shows the smoothed PT-TAI curve (red line). For comparison, Figure 6 also presents the original PT-TAI data (blue line) and the terrestrial time TT(BIPM2017)-TAI (black line). From Figure 6, we see that the smoothed PT-TAI curve is more similar to TT(BIPM2017)-TAI than the original one.

Figure 3 shows the TT(BIPM2017)-TAI first derivative curve (red line) after smoothing and the original TT(BIPM2017)-TAI first derivative (blue line). Figure 7 shows the residuals of the two datasets and their corresponding histograms. The upper left subplot shows the PT-TAI residuals, indicating that large residuals appear in the early PT-TAI data where the absolute residual value, especially for the second data point, is greater than 1 s. After that, the absolute value of the residual of each data point gradually decreases, showing that pulsar timing

accuracy is constantly improving. The standard deviation of residuals of PT-TAI after filter smoothing is 0.144 s. Although the mean measurement error of the original PT-TAI data is 0.226 s, the weighted mean of the measurement error is 0.164 s, which is close to the standard deviation of residuals of PT-TAI after smoothing.

The upper right subplot shows the TT(BIPM2017)-TAI first derivative after filter smoothing, and the standard deviation of the first derivative residuals is 0.000155 s/d. To further investigate the residual distribution of the two smoothed datasets, we present histograms of the respective residuals in Figure 7. The lower left subplot shows the histogram of PT-TAI residuals, and the lower right subplot shows the histogram of TT(BIPM2017)-TAI first derivative residuals. Both histograms are very close to a Gaussian distribution, demonstrating that the smoothed results computed with e_1 and e_2 using $p_{0.99} = 6$ yr efficiently eliminate high-frequency noise in the two datasets and correctly extract the PT-TAI signal. The first time derivative of the smoothed PT-TAI equals the smoothed first time derivative of TT(BIPM2017)-TAI; in other words, both smoothed PT-TAI and TT(BIPM2017)-TAI have the same frequency difference.

We also calculated the smoothed curve of PT-TAI using coefficient e_1 with $p_{0.99} = 6$ yr and setting $e_2 = 0$. In this case, the problem reduces to original smoothing (Vondrak 1969, 1977), and the first derivative of TT(BIPM2017)-TAI is simply ignored. The smoothed curve of PT-TAI obtained in this way is shown with the green line in Figure 6.

We analyze the fractional frequency stability σz of PT-TAI before and after combined smoothing and compare it with that of TT(BIPM2017)-TAI. Figure 8 shows the σz curve (red line) of PT-TAI after smoothing using e_1 and e_2 with $p_{0.99} = 6$ yr. The σz curve of the smoothed PT-TAI using e_1 with a 6 yr period and $e_2 = 0$ is also shown as the green line. For easy comparison, Figure 8 again presents the σz curves of the original PT-TAI data (blue line) and TT(BIPM2017)-TAI (black line). As can be seen, the σz curve of the smoothed result using the calculated e_1 and e_2 with $p_{0.99} = 6$ yr (red line) improves both short- and medium-term frequency stability levels compared to the blue line. Although the green line significantly improves PT-TAI short-term frequency stability, its σz for some medium-term intervals shows a higher value than the red line. The behavior of the green line is similar to that of the smoothed result by Fourier domain filtering in Figure 2.

Figure 8 indicates that the PT-TAI smoothed by the combined smoothing filter has the best short-term stability and its long-term stability is comparable with that of TT(BIPM2017)-TAI.

We can understand the result from another point of view. Figure 9 depicts the power spectral density of both TT(BIPM2017)-TAI and smoothed PT-TAI obtained using e_1 and e_2 with $p_{0.99} = 6$ yr. As Figure 9 shows, both curves indicate significant low-frequency signals, with the low-frequency signal of the

smoothed PT-TAI being stronger than that of TT(BIPM2017)-TAI. As frequency increases, the smoothed signal of TT(BIPM2017)-TAI becomes stronger, so that in the high-frequency band, the power of TT(BIPM2017)-TAI is greater. This also signifies that TT(BIPM2017)-TAI still contains some high-frequency noise, which is indirectly demonstrated by the high-frequency noise in the time derivative curve (see Figure 3). Incidentally, the smoothed PT-TAI power spectral curve (red line) as shown in Figure 9 is different from the power spectrum of the PT-TAI low-frequency signal, but below that completely becomes PT-TAI spectral weaker curve smoothed by Fourier domain filtering as shown in the red line of Figure 1. The spectral intensity in the high-frequency band of the former gradually decreases with increasing frequency, whereas the high-frequency band spectral intensity of the latter is abruptly cut to 0 from the cutoff frequency.

4. Application for Pulsar Atomic Clock Combined Timescale

4.1. Using as Terrestrial Time

The ideal timescale for the coordinate time of ITRS should have good frequency stability and frequency accuracy. The accuracy means the scale interval (unit) is based on the SI second. TAI gets its stability from some 400 atomic clocks kept in about 80 laboratories worldwide and its accuracy from a small number of primary frequency standards (PFS) developed by a few metrology laboratories through frequency steering correction. Because TAI is computed in quasi-real-time every month and has operational constraints (e.g., no re-computation on a given time interval even if new data become available), it does not provide an optimal realization of terrestrial time. The BIPM therefore computes another realization, TT(BIPMxxxx), in post-processing. TT(BIPMxxxx) has better stability and accuracy than TAI.

Although pulsar time PT derived from PTA datasets exhibits good long-term stability, its measurement error is much larger than atomic time and its frequency accuracy is not guaranteed to conform with the SI second. Figure 6 shows that the green curve (smoothed PT-TAI using e_1 with a 6 yr period and $e_2 = 0$) is very different from the red curve (smoothed PT-TAI by combined smoothing using e_1 and e_2 with a 6 yr period); correspondingly, their first derivative (frequency difference) curves should be very different, meaning the unit (second) of the original PT is very different from an SI second. The PT smoothed by the combined smoothing filter combines the long-term stability of PT and the accuracy of TT(BIPMxxxx) and removes high-frequency noise from the original PT, so in essence the smoothed PT is a combined timescale with TT(BIPMxxxx); hereafter we refer to it as combined pulsar atomic time (CPA). Because both frequency stability and accuracy of CPA are comparable to those of TT(BIPMxxxx), CPA can also serve as terrestrial time. In this work, TT(BIPM2017) is used as the frequency standard to compute the frequency difference of TT(BIPM2017)-TAI used for the combined smoothing filter. In

the future, PFS instead of TT(BIPMxxxx) should be utilized to compute the frequency difference between PFS and TAI. In this way, we will combine the long-term stability of the original PT with the frequency accuracy of PFS. Producing CPA using PT and PFS has an advantage compared to TT(BIPMxxxx). In the future, ensemble pulsar time with respect to TAI may be produced in quasi-real-time every month; using PFS as the frequency standard to realize the SI second, the CPA timescale can be derived using a combined smoothing filter in quasi-real-time every month, whereas TT(BIPMxxxx) will be available a year later. Since both PT and PFS will continue to develop, CPA may be further improved in the future.

4.2. Steering Atomic Clock

A quasi-real-time timescale with high quality can be used as a reference standard for steering clocks. For example, a hydrogen maser shows very good short-term (within a month) frequency stability, but its stability gradually decreases with increased time interval. By steering a hydrogen maser to a high-quality timescale, the hydrogen maser can supply, in real-time to a user, a time signal with high stability and accuracy over needed long time intervals. CPA can be used as a reference standard to steer a clock.

As an example of a clock steering experiment, we choose the local atomic time kept by the United States Naval Observatory, TA(USNO), as a timescale to be steered. Like a hydrogen maser, TA(USNO) has good short-term frequency stability, but its long-term stability gradually becomes worse. Data for clock difference TAI-TA(USNO) are taken from the BIPM website. To be consistent with the available time span, a data series after MJD 50678 for the CPA-TAI shown in Figure 6 (red curve) is used. From the TAI-TA(USNO) and CPA-TAI datasets, we derived the clock difference series CPA-TA(USNO) with regular sampling.

We assume the clock difference, frequency difference, and frequency drift for TA(USNO) with respect to the reference standard CPA at moment k are respectively a_k , b_k , and c_k . If a_k , b_k , and c_k are known, the clock difference a_{k+1} at moment $k + 1$ can be derived by

$$a_{k+1} = a_k + b_k \Delta t + c_k \Delta t^2 + n_{k+1}$$

where Δt is the sampling interval and n_{k+1} is the noise term. We use a Kalman filter (Kalman 1960; Brown & Hwang 1983; Pìriz et al. 2019) to determine a_k , b_k , and c_k . For the Kalman filter, the input data series is CPA-TA(USNO), and the output is a_k , b_k , and c_k for each step of the filter. When a_k , b_k , and c_k are determined by the filter, the clock difference CPA-TA(USNO) is corrected according to the equation above. In this way, TA(USNO) is steered to the reference standard CPA. The differences between CPA and steered TA(USNO), i.e., residuals of steering TA(USNO) to CPA, are shown in panel (b) of Figure 10.

For comparison, TT(BIPM2017)-TA(USNO) is processed by the Kalman filter in the same way as CPA-TA(USNO), and residuals after steering TA(USNO) to TT(BIPM2017) are displayed in panel (c) of Figure 10. In order to steer TA(USNO) to the unsmoothed original PT with sparse sampling, we linearly interpolate PT-TAI to obtain the PT-TA(USNO) series with the same sampling as CPA-TA(USNO). The residuals after steering TA(USNO) to raw PT are given in panel (a) of Figure 10. The residuals in the three subplots are the results after the Kalman filter converged. We see that residuals in panels (b) and (c) show similar behavior with standard deviations of 2.4 ns and 2.7 ns, respectively. The dispersion of residuals in panel (a) is obviously larger than both (b) and (c) because the reference standard PT has large measurement errors. The standard deviation of residuals in panel (a) is 7.4 ns.

In Figure 11, we compare frequency stability curves for the residuals after steering TA(USNO) to raw PT (black line), TT(BIPM2017) (blue line), and CPA (red line). For comparison, the stability curve for the TT(BIPM2017)-TA(USNO) series is also shown as a green line. Although the green line indicates better short-term stability, its long-term stability becomes worse. After frequency steering, both blue and red curves show obvious improvement in medium- and long-term frequency stability compared with the green curve. The black curve shows worse frequency stability than both blue and red curves at any time interval. The results of the frequency steering experiment also demonstrate that the property of timescale CPA is comparable with TT(BIPMxxxx), and both are better than the original PT for application.

5. Conclusion

Usually, raw data of developed ensemble pulsar time PT possess sparser data points with larger errors than atomic timescale. By correctly choosing the two smoothing coefficients of a combined smoothing filter, the high-frequency noise contained in both the original clock difference of PT-TAI and the first derivative of TT(BIPM2017)-TAI can be efficiently removed by the filter. Data points of PT-TAI smoothed by the filter are denser than the original data, and the smoothed PT-TAI takes on advantages of both long-term frequency stability from PT and frequency accuracy from terrestrial time. The smoothed PT-TAI series by combined smoothing filter can improve short- and medium-term stability while maintaining its original long-term frequency stability. Any smoothing filter that only smooths single PT-TAI data can improve short-term stability but cannot improve stability at medium-term intervals as the combined smoothing filter does.

Primary frequency standards are used to realize the SI second on which the timescale should be based. The frequency accuracy of the terrestrial timescale is determined by primary frequency standards (Guinot 1988). In the future, we can produce combined smoothing using PT-TAI raw data series and frequency difference series of primary frequency standards with respect to TAI instead of TT(BIPMxxxx)-TAI. Here, the frequency difference of primary frequency stan-

dards with respect to TAI refers to the first derivative of the corresponding clock difference from which quadratic polynomial terms are removed. By the combined smoothing filter, we produce a combined timescale CPA which takes on the long-term stability of PT and the accuracy of primary frequency standards. The stability and accuracy properties of CPA are comparable with those of TT(BIPMxxxx). CPA can also be used as terrestrial time.

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