Advances in Simulation and Calibration Methods for Ionospheric Effects in Low-Frequency Radio Interferometric Array Imaging: Postprint

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Abstract

High-precision calibration of ionospheric effects is not only crucial for detecting signals from the cosmic reionization era, but also a key challenge facing all low-frequency (typically less than 300 MHz) radio interferometric array observations. To improve imaging dynamic range and ultimately achieve the scientific objectives of the telescope, it is necessary to investigate the characteristics of the ionosphere, its impact on radio interferometric array imaging, and corresponding calibration methods. Beginning with the principles of radio interferometric array imaging, we first analyze the phase errors induced by the ionosphere on baselines and the resulting source position shifts. Building upon this foundation, we further systematically elaborate on existing ionospheric monitoring data, prediction models, and current ionospheric simulation methods and calibration techniques in the field of radio interferometric arrays. This comprehensive review and in-depth analysis of ionospheric effects will provide fundamental support for data simulation and calibration in the domain of low-frequency radio interferometric array imaging.

Full Text

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Research Progresses on Ionospheric Effect Simulation and Calibration Methods in Low-Frequency Radio Interferometric Imaging

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Abstract

High-precision calibration of ionospheric effects is not only crucial for detecting cosmic reionization signals but also a key challenge for all low-frequency radio interferometric observations (typically below 300 MHz). To improve imaging dynamic range and ultimately achieve the scientific goals of telescopes, it is necessary to investigate ionospheric characteristics, their impact on radio interferometric arrays, and calibration methods. This paper first analyzes the phase errors induced by the ionosphere on baselines and the resulting source position offsets from the principles of radio interferometric imaging. Based on this foundation, we systematically review existing ionospheric monitoring data, prediction models, and current ionospheric simulation and calibration techniques in the radio interferometric array field. This comprehensive review and in-depth analysis of ionospheric effects will provide fundamental support for data simulation and calibration in low-frequency radio interferometric imaging.

Keywords: radio interferometer array; ionospheric effect; simulation; calibration

1 Introduction

With advances in telescope equipment and technology, a number of low-frequency radio interferometric arrays have been established worldwide. These include the Low Frequency Array (LOFAR) [1], Precision Array for Probing the Epoch of Reionization (PAPER) [2], Hydrogen Epoch of Reionization Array (HERA) [3], Murchison Widefield Array (MWA) [4], and the under-construction Square Kilometre Array (SKA) [5]—the world' s largest radio telescope. These next-generation telescopes aim to explore unknown frontiers in astronomy, such as detecting cosmic reionization epoch (EoR) signals, galaxy formation and evolution, and the origin and evolution of cosmic magnetic fields. With these advanced instruments, astronomers hope to unravel more mysteries about the origin and evolution of the universe, further advancing our understanding of the cosmos.

Ionospheric effects represent one of the main challenges in low-frequency radio astronomy data processing. The ionosphere is the ionized portion of Earth' s upper atmosphere, located approximately 50–1,000 km above the surface [6]. Its activity intensity is typically defined based on the electron column density or total electron content (TEC) along the line of sight (LoS), measured in TEC units (1 TECU = 10^{16} m^{-2}). These electrons affect radio signals passing through the ionosphere through absorption, scattering, and Faraday rotation, causing phase variations in radio interferometer visibility data and ultimately resulting in source position offsets during imaging (as shown in Figure 1 [Figure 1: see original paper]).

Depending on the telescope' s baseline length (A), field of view (V), and ionospheric scale size (S), radio interferometric arrays can be categorized into four observation scenarios [8], as illustrated in Figure 2 [Figure 2: see original paper].

Traditionally, for observations with small fields of view and higher frequencies (A S), ionospheric effects can be largely handled through a single correction per station within the FoV (direction-independent calibration). This applies to both compact (A S) and extended (A S) arrays, corresponding to scenarios 1 and 2 in Figure 2, where conventional self-calibration techniques are applicable.

However, next-generation radio interferometers feature both wide fields of view and long baselines (widely distributed arrays), corresponding to scenario 4 in Figure 2 (V > S). In this case, radio signals received by different antennas may pass through different ionospheric regions (direction-dependent effects, DDE), requiring independent calibration solutions for each direction and station. Moreover, ionospheric TEC varies dynamically with time of day, season, geographic location, and solar magnetic storms, leading to complex phase errors that pose considerable difficulties for observation data simulation and calibration.

To achieve the scientific goals of next-generation wide-field, high-dynamic-range imaging radio telescopes, current core challenges include undersampling and deconvolution methods, non-coplanar issues, and direction-dependent effect calibration [10]. The ionosphere is one of the major contributors to DDE [11–15]. Taking SKA as an example, its primary scientific goal is to image the cosmic dawn and reionization using neutral hydrogen 21 cm radiation and measure power spectra. To obtain high-dynamic-range images and achieve its ambitious scientific objectives, it is essential to account for DDE in imaging and deconvolution algorithms and perform high-precision calibration of ionospheric phase errors.

This paper derives the impact of the ionosphere on radio interferometric imaging from first principles, including induced phase errors and source position offsets in images (Section 2). Building on this foundation, we describe the basic characteristics of the ionosphere and existing ionospheric models, prediction models, and simulation methods in the radio interferometric array field (Section 3). Section 4 provides a thorough analysis of direction-dependent effect calibration methods and specific techniques for calibrating ionospheric phase errors, along with relevant tools. Finally, we summarize the work and present future prospects.

2.1 Basic Principles

The first form of the Radio Interferometer Measurement Equation (RIME), i.e., the single-point-source form [16], is:

$$V_{pq} = J_p B J_q^H$$

where B is the brightness matrix, and J is a 2$\times$2 complex matrix called the Jones matrix [17], which represents the product of Jones matrices corresponding to system and environmental effects along the signal propagation path. The multiplication order is determined by the sequence in which effects appear during signal propagation:

$$J = J_n J_{n-1} \cdots J_1$$

Extending from a single point source to the entire sky, the relationship between observed complex visibility data V and true sky brightness B becomes:

$$V_{pq} = \int J_p(l,m) B(l,m) J_q^H(l,m) e^{-2\pi i(ul+vm+wn)}\, dl\, dm$$

The refractive index n of ionospheric plasma along the signal propagation path relates to signal frequency ν as [18]:

$$n = \sqrt{1 - \left(\frac{\nu_p}{\nu}\right)^2}$$

where ν_p is the plasma frequency. The refractive index varies with signal frequency, which may cause the signal path to bend or undergo other changes during propagation. The integrated total propagation delay for a signal of frequency ν along the LoS causes phase rotation:

$$\phi_{\text{ion}} = -\frac{2\pi}{\nu} \int (n-1)\, dl$$

Assuming the ionospheric magnetized plasma is cold and collisionless, the refractive index can be estimated using the generalized Appleton-Hartree equation and expanded as a Taylor series [19]:

$$n \approx 1 - \frac{q^2}{8\pi^2 m_e \epsilon_0} \cdot \frac{n_e}{\nu^2} \pm \frac{q^3}{16\pi^3 m_e^2 \epsilon_0 c} \cdot \frac{n_e B \cos\theta}{\nu^3} - \frac{q^4}{128\pi^4 m_e^2 \epsilon_0} \cdot \frac{n_e^2}{\nu^4} + \cdots$$

where n_e is the free electron density, B is the magnetic field strength, θ is the angle between the magnetic field B and the electromagnetic wave propagation direction, q is the electron charge, m_e is the electron mass, and ϵ_0 is the vacuum permittivity.

LOFAR-based research [20] shows that different orders of the Taylor series correspond to different types of Jones matrices, with varying impacts on visibility amplitude and phase, frequency and time dependencies, and direction dependence. The first term corresponds to dispersive delay, which is the dominant ionospheric effect. For most radio astronomical observations above a few hundred megahertz, higher-order terms can be neglected. The second term corresponds to Faraday rotation, depending on TEC and Earth' s magnetic field.

The third term corresponds to ionospheric scintillation. The last two terms need consideration at frequencies below 40 MHz [12]. Detailed ionospheric effects in LOFAR are shown in Table 1 .

Combining equations (5) and (6), the phase error caused by the first term of the Taylor series can be expressed as:

$$\phi_{\text{ion}} = \frac{4\pi}{\epsilon_0 m_e c\nu} \int n_e \, dl = 8.4479745 \times 10^9 \times \text{TEC}/\nu$$

The ionosphere-induced phase offset is inversely proportional to frequency ν, meaning low-frequency radio observations suffer more severe ionospheric effects. During observations, subtracting the phase delays of two antennas (see equation (7)) yields the ionospheric phase error. Adding this to the original interferometric imaging principle formula (see equation (3)), the visibility data corrupted by the ionosphere becomes:

$$V_{pq} = \int e^{-i(\phi_{p,\text{ion}})} I(l,m) e^{-2\pi i(ul+vm+wn)} \, dl \, dm$$

2.2 Analysis Methods of Ionospheric Effects

Ionospheric characteristics are relevant not only to ionospheric science but also to radio astronomy. Ionospheric-induced source position offsets and flux density fluctuations can complicate cross-matching and classification of radio sources. The ionosphere can also distort the telescope’ s point spread function (PSF) in a time- and position-dependent manner, imposing higher demands on deconvolution algorithms and calibration methods [21, 22]. Currently, there are two main approaches for analyzing ionospheric effects.

2.2.1 Antenna-Based Method

This method tracks a single bright source to measure phase fluctuations on each baseline over time. Ionospheric phase errors are proportional to the TEC difference between the lines of sight to two receivers, enabling detection of delta TEC (dTEC) variations over time. The Giant Metrewave Radio Telescope (GMRT), located between the northern peak of the equatorial ionization anomaly and the magnetic equator (making it sensitive to harsh ionospheric conditions), is a unique radio interferometer for studying ionospheric properties [9].

Using GMRT observations of source 3C 68.2 from midnight to sunrise on August 6, 2012, Mangla et al. [9] analyzed ionospheric-induced phase offsets. After data processing and calibration (flagging, antenna-based complex gain calibration, instrument noise reduction), they obtained ionospheric phase offsets for each antenna. Using the theoretical formula (equation (7)), they derived dTEC for each baseline, with partial results shown in Figure 3a [Figure 3: see original paper]. In LOFAR data simulation and calibration, Edler et al. [20] analyzed simulated ionospheres based on baselines, verifying dTEC variations for different baseline lengths when observing target sources, as shown in Figure 3b. These dTEC variations obtained from real data analysis and antenna-based methods provide valuable references for ionospheric simulation and calibration.

In antenna-based studies, ionospheric spatial sampling is closely related to array configuration, meaning spatial sampling is direction-dependent. Therefore, this method typically applies to radio interferometric arrays with small fields of view, where ionospheric effects are considered uniform across the FoV. For arrays with larger fields of view, this limitation can be effectively addressed by simultaneously observing multiple radio sources and evaluating ionospheric effects in different directions separately.

2.2.2 Field-Based Method

This approach focuses on the synthesized radio image from the entire array rather than individual antenna information. It analyzes and corrects data by examining source position offsets in the image. Field-based methods require RFI flagging, imaging, deconvolution, and source finding before analysis can proceed. Consequently, this method demands high-performance algorithms for imaging, calibration, and source extraction.

Loi et al. [23] conducted power spectrum analysis based on real ionospheric data from MWA. Analysis of source position offsets revealed typical positional displacements of 10 -20 at 154 MHz and 183 MHz. As a pathfinder for SKA, MWA' s observation data provides reference for ionospheric impact types and intensities expected in SKA-LOW data analysis. Based on MWA simulation data analysis and evaluation, only observations with source position offsets smaller than 10 -15 can be used for EoR science [14].

MWA' s ionospheric simulation tool SIVIO (IOnospherically contaminated Visibilities SImulator) [7, 14] simulates ionospheric TEC and adds its effects to visibility data, analyzing source position offsets after imaging. By fitting the frequency-dependent source position offsets caused by the ionosphere, they confirmed that the relationship between ionospheric offsets and frequency matches theory (Δ $^{-2}$, see Figure 4 [Figure 4: see original paper]), validating the reasonableness of the simulated ionospheric TEC. The basic data processing workflow is: (1) Determine source catalogs and FoV size in the sky model to obtain undisturbed visibility data; (2) Simulate ionospheric TEC; (3) Obtain pierce-point coordinates for each source and antenna on the TEC screen; (4) Calculate ionosphere-corrupted visibility data; (5) Image, clean, and extract sources (using the AEGEAN [24] source extraction tool); (6) Calculate RA and Dec offsets based on true source positions from the catalog.

Antenna-based calibration assigns a single time-varying phase error to each antenna in the array. This approach is sufficient for eliminating tropospheric phase errors at short wavelengths, as tropospheric phases vary rapidly with array position and are nearly constant within each antenna' s FoV. Albert et al. [25, 26]

proposed a probabilistic description of antenna-based ionospheric phase errors using Gaussian processes to calibrate ionospheric phase errors in low-frequency interferometric data.

However, for wide-field observations, ionospheric phase errors vary significantly across the FoV, prompting the development of field-based calibration methods [27]. This approach uses Zernike polynomials to model time-varying ionospheric phase screens across the FoV, correcting phase errors accordingly. Cotton et al. [27] successfully applied it to calibrate VLA 74 MHz observations, significantly improving image quality (especially for weak sources). Field-based calibration was the first method to consider DDE, and many DD-related algorithms have been optimized and improved based on this foundation. In summary, the main difference between antenna-based and field-based methods lies in their data analysis approach: the former focuses on individual antenna information, while the latter focuses on the synthesized array image. Both methods play important roles in ionospheric simulation data analysis, imaging result analysis, and ionospheric phase error calibration.

3.1 Basic Characteristics of the Ionosphere

Jordan et al. [14] found that the ionosphere consists of large-scale (anisotropic structural wave behavior) and small-scale (isotropic turbulence) structures. Using a real-time calibration system to calculate ionospheric-induced source offsets, they classified ionospheric activities based on offset scales and dominant directions, providing occurrence probabilities for each type in MWA EoR observations: Type I: quiet ionosphere, 74%; Type II: turbulent ionosphere, 15%; Type III: structurally anisotropic ionosphere, 3%; Type IV: structurally turbulent ionosphere, 8%.

Mevius et al. [28] observed quasar 3C 196 with LOFAR and found that nighttime ionospheric TEC exhibited anisotropy. They discovered that nighttime observations from winter 2012–2013 followed a power-law rule across all baseline lengths, confirming Kolmogorov turbulence in the ionosphere. De Gasperin et al. [13] demonstrated that ionospheric scintillation at very low frequencies interferes with visibility amplitudes, causing an average data loss of 30% at night (compared to daytime). They therefore encouraged daytime observations, particularly for LOFAR-EoR experiments.

Comprehensive analysis of LOFAR, Global Navigation Satellite System (GNSS), and ionosonde data indicates that vertical propagation of large-scale structures and turbulence in the ionosphere causes instabilities and breaks down large-scale structures into small-scale ones [29]. Therefore, studying and analyzing the large-scale structure and turbulence characteristics of the ionosphere is fundamental for ionospheric simulation and calibration.

In 1998, the International GNSS Service (IGS) launched an international project to calculate TEC from GNSS data. Since then, ionospheric analysis centers such as the Centre for Orbit Determination in Europe (CODE), Jet Propulsion Laboratory, European Space Agency, Energy Mines and Resources, and Universitat Politècnica de Catalunya have produced different Global Ionospheric Map (GIM) products. CODE’ s GIM, based on over 500 TEC observations, has a temporal resolution of 2 hours and spatial resolution of 5° (longitude) × 2.5° (latitude).

The automated software MAPGPS, developed by MIT Haystack Observatory, can obtain ionospheric TEC from GPS receiver data biases to estimate global TEC data, known as MIT-TEC. Since 1998, the institution has collected and processed TEC observations from over 6,000 stations worldwide, providing high spatiotemporal resolution TEC information updated every 5 minutes with spatial resolution reaching 1° (longitude) × 1° (latitude). Data is available through the Madrigal database. However, due to the limited number of receiver stations capable of satellite communication measurements, MIT-TEC has limitations in oceanic regions. Detailed methods and techniques can be found in Rideout and Coster’ s 2006 publication [30].

In summary, ionospheric monitoring data and various GIM products play important roles in improving navigation system accuracy, predicting space weather, and optimizing radio communications. GIM products also help researchers better understand the spatiotemporal variation characteristics of the ionosphere. In radio interferometric telescope data processing and simulation, existing GIM products can be referenced to help analyze phase delays caused by the ionosphere, thereby improving observation data accuracy.

3.3 Single-Layer Model of the Ionosphere and Its Limitations

For convenient estimation of signal propagation delays caused by the ionosphere, the single-layer model is widely used in radio astronomy [31]. This simplified model assumes that free electrons in the ionosphere are distributed in a uniform-thickness single layer. However, in the actual Earth’ s ionospheric environment, electrons are not uniformly distributed but vary dynamically with geographic location, time, and solar activity. When converting vertical TEC (VTEC) obtained from GNSS systems to slant TEC (STEC) along the line of sight, the single-layer model ignores the actual thickness and complex structure of the ionosphere, introducing calculation errors. For example, signals at different geographic locations and elevation angles pass through varying actual ionospheric thicknesses and electron distributions, causing STEC values calculated based on the single-layer model to deviate from reality. Therefore, in-depth research and improvement of the single-layer model are necessary for high-precision ionospheric impact analysis.

Assuming a flat Earth with a two-dimensional phase screen at a fixed height h above the mean Earth’ s surface, the observer’ s line-of-sight zenith angle θ' equals the line-of-sight angle θ at the pierce point (as shown in Figure 5a [Figure 5: see original paper]). In this case, ignoring horizontal TEC variations, the single-layer model is perfectly accurate. STEC is independent of h, and the conversion formula is:

$$\mathrm{STEC} = \frac{\mathrm{VTEC}}{\cos\theta'}$$

For a more realistic spherical Earth, as shown in Figure 5b, even when ignoring horizontal TEC variations, the STEC obtained from equation (9) will differ from the true electron column density along the line of sight. In this spherical Earth case, STEC depends on the single-layer height h. The relationship between the line-of-sight angle θ at the pierce point (the angle between the line of sight and the layer normal) and the observer' s zenith angle θ' can be derived from the sine theorem. The corrected STEC conversion formula is:

$$\mathrm{STEC} = \mathrm{VTEC} \times \frac{R_E + h}{\sqrt{(R_E + h)^2 - (R_E \sin\theta')^2}}$$

where $R_E = 6,371$ km is Earth' s mean radius.

Martin et al. [32] compared the 2D ionospheric STEC calculated from equation (10) at height h with the 3D ionospheric STEC calculated from equation (11), demonstrating the limitations of the single-layer model:

$$\mathrm{STEC}_{3D} = \sum_{i=1}^{30,000} \mathrm{STEC}_i$$

where STEC_i is calculated from the VTEC value of an individual layer at altitude $h = i$ km using equation (10).

STEC_{3D} is calculated based on a realistic height profile of electron density, created by combining separate models for the ionosphere and plasmasphere. Yizengaw et al. [33] noted that the plasmasphere is included in this profile, typically contributing about 10% to daytime TEC. To generate the ionospheric profile for February 17, 2015, at 12:00 UT, with a single-layer pierce point at geomagnetic latitude 40° and geomagnetic longitude 0° (see Figure 6a [Figure 6: see original paper]), Martin et al. combined the IRI model with Gallagher et al.' s plasmasphere model [34]. For the ionosphere below 2,000 km altitude, the IRI model directly provides electron density distribution, while Gallagher et al.' s model is used for the plasmasphere above 2,000 km.

Figure 6b compares STEC_{2D} and STEC_{3D} as functions of zenith angle. At heights of 250 km and 350 km, the single-layer model overestimates STEC compared to the 3D model. When the single-layer height is set to 450 km, STEC_{2D} approximates STEC_{3D}.

3.4 Ionospheric Prediction Models

The Klobuchar [35–37], NeQuick [38], and BDGIM [39] models are all ionospheric correction methods designed to improve positioning accuracy in GNSS systems. They estimate and compensate for ionospheric effects through different algorithms and parameter settings. The Klobuchar model, proposed in the 1970s–80s [35] and initially designed for GPS, uses a simplified ionospheric model to estimate ionospheric effects on signal propagation, broadcasting parameters to users via satellites. Based on eight parameters, this model can reduce ionospheric positioning errors to some extent.

The NeQuick model is the ionospheric model adopted by Europe' s Galileo navigation system, providing global ionospheric correction services. Compared to the Klobuchar model, NeQuick offers more accurate ionospheric delay correction under certain conditions. BDGIM is the model used by China' s BeiDou satellite navigation system, independently designed and developed for the BeiDou-3 system. Compared to the improved Klobuchar model used in previous BeiDou generations, BDGIM employs a simplified spherical harmonic expansion referencing a solar-fixed geomagnetic coordinate system to describe global VTEC distribution, providing more precise ionospheric correction services. Under different solar activities, BDGIM reduces ionospheric errors by 25%–98% relative to GIM-IGSG [40].

The International Reference Ionosphere (IRI) [41] is a widely used empirical model in ionospheric research. It integrates various observational data and theoretical research results to provide a standardized framework for predicting ionospheric characteristics (electron density, ion density, electron temperature, and other important parameters) as functions of space and time under different geographic locations, solar activity levels, seasons, and geomagnetic conditions. Since its first release in 1979, the IRI model has undergone multiple updates and revisions to reflect the latest advances in ionospheric research and new observational data. IRI-2016 includes updated ionospheric parameterization models and more empirical data, while IRI-2020, the latest version, integrates the most recent scientific results and observational data to provide more accurate ionospheric predictions.

On the other hand, with the development of artificial intelligence technology, deep learning algorithms have made progress in ionospheric prediction. Many researchers have applied the Long Short-Term Memory (LSTM) algorithm, a time-series data processing method, to ionospheric prediction with good performance [42–49]. The pix2pixhd deep learning algorithm based on Generative Adversarial Networks (GAN) has been used to fill missing data in global TEC maps [50]. Unlike traditional GANs, pix2pixhd has two generators and three discriminators, enhancing model completion capability and enabling it to fill large-scale missing image regions.

Yang et al. [50] showed that this model can better simulate ionospheric peak structures in low-latitude regions but performs poorly at the edges of ionospheric peak regions. Comparing different scales of missing data areas, the model performs best in regions with 0%-15% missing data; in cases with larger missing data areas (>30% and >45%), satisfactory results can still be achieved. Additionally, the model' s completion performance is less affected by geomagnetic and solar activities. Jeong et al. [51] demonstrated that convolutional LSTM (ConvLSTM) outperforms LSTM in ionospheric prediction, generating reliable ionospheric predictions with smaller root-mean-square errors.

3.5 Ionospheric Simulation Methods in Radio Interferometry

Many researchers have conducted work on ionospheric data analysis and simulation [9, 14, 15, 19, 20, 23]. Albert et al. [25] introduced Kriging for three-dimensional ionospheric tomographic inference, providing more accurate modeling through a regression kernel compared to other kernels commonly used in Gaussian process regression. Ionospheric phase screens inferred using this method have been applied to direction-dependent effect calibration in LoTSS (LOFAR Two-metre Sky Survey), effectively reducing image-plane artifacts caused by the ionosphere [26].

Srinath et al. [52] proposed a sample-based autoregressive method (ARatmospy) for calculating effective atmospheric phase screens and their evolution over time, which has been used for ionospheric simulation in OSKAR. Chege et al. [7] developed the ionospheric simulation tool SIVIO for MWA, using thin-screen approximation (single-layer model) and providing two turbulence models —s-screen (tube turbulence) and k-screen (Kolmogorov turbulence)—to simulate ionospheric TEC and add its effects to visibility data, directly assessing ionospheric impacts on observed EoR data and corresponding power spectra. The LOFAR simulation tool (LoSiTo) uses the woofer-tweeter algorithm to simulate turbulence and adds a uniform TEC background value for ionospheric simulation and subsequent calibration method research. Loi et al. [23] conducted ionospheric monitoring and characteristic analysis using GMRT and theoretically calculated ionospheric effects on future SKA1-MID and SKA1-LOW low-frequency observations based on ionospheric characteristics estimated from uGMRT (upgraded GMRT), LOFAR, MWA, and VLA (Very Large Array) observations.

In this section, we review current observational analyses of basic ionospheric characteristics, the single-layer model and its limitations. Based on this, we survey various algorithmic models for predicting ionospheric TEC, particularly simulation models in the radio interferometric array field. These contents will provide valuable references for subsequent ionospheric simulations.

4.1 Third-Generation Calibration Techniques

Before 1980, first-generation calibration (1GC) primarily relied on "open-loop" methods, observing calibration sources before and after the target observation. This method depended on instrument stability during the observation period.

Although simple, it was sufficient to support radio astronomy observations at that time.

Around 1980, the invention of self-calibration [54] marked the beginning of second-generation calibration (2GC). This "closed-loop" method continuously estimates complex station gain factors for one or more bright sources in the field of view, dramatically improving dynamic range to typically 10^4–10^5. However, 2GC techniques only apply to direction-independent effects (DIE).

With new technologies and telescope construction, data processing faces dual challenges. On one hand, the sensitivity of next-generation telescopes has significantly improved, requiring consideration of weaker instrumental effects, with new technologies like phased arrays making these effects more complex. On the other hand, wide fields of view and broad bandwidths introduce DDE, where errors vary with observation direction and cannot be corrected with a simple global factor. Consequently, the concept of third-generation calibration (3GC) was proposed to achieve a more complex and general self-calibration method, ultimately enabling higher dynamic range improvements. The Radio Interferometer Measurement Equation (RIME) provides the mathematical foundation for describing radio interferometric measurements [55, 56]. According to RIME theory, various instrumental and environmental effects along the signal propagation path correspond to multiplication of Jones (matrix) chains. Therefore, the calibration problem essentially involves estimating the direction-dependent Jones matrix chain for each antenna.

4.2 Research Progress on Ionospheric Calibration Methods

Models for correcting ionospheric delays (such as Klobuchar [35–37], NeQuick [38], and IRI [41]) are widely used for ionospheric prediction and error calibration. These models meet user needs for high-precision single-frequency positioning [37, 38, 41], but their calibration accuracy is unstable due to factors like sudden changes in solar activity and Earth's environment.

Over the past decade, new direction-dependent calibration strategies and related software environments have been developed, making it possible to use interferometers at low frequencies and largely achieving calibration of ionospheric effects. Noordam and Smirnov [57] designed the MeqTrees software for simulation and calibration, a specialized package for handling 3GC problems. MeqTrees considers a mathematical model where some function $M(v,t)$ depends on parameters $p = (p_1, p_2, p_3, \cdots, p_k)$, denoted as $M(v,t,p)$. The model solving process finds a set of p values that minimize the difference between measured data and the model. In radio astronomy, model M is given by the parameterized form of RIME, and calibration is the process of model fitting and solving. MeqTrees has proven effective in measurements, simulation, and calibration for new radio telescopes, achieving noise-limited dynamic ranges exceeding 10^6 on WSRT data [16, 58].

Rioja et al. [59] developed the Low-frequency Excision of Atmosphere in Parallel (LEAP) method, which can treat each direction as an independent parallel analysis, outperforming SPAM (Source Peeling and Atmospheric Modeling [22]), RTS (Real-Time System [21], which corrects each source in order of intensity), and SageCal (which solves all directions with matrices and does not scale well to large numbers of sources or stations).

Tasse [60] applied Wirtinger complex differentiation to solve RIME Jones matrices (CohJones algorithm) and used Voronoi tessellation (see Figure 7 [Figure 7: see original paper]) to cluster sources in the sky model for direction-dependent calibration in each small region, an image segmentation method also used in DDFacet Imager [61]. The KillMS software implements two complex optimization algorithms—CohJones and the non-linear Kalman filter algorithm (KAFCA [62])—using approximate matrix inversion to simultaneously solve calibration for multiple directions. Compared to traditional field-based calibration methods like SPAM and SageCal, it solves many problems.

For handling DDE, Tasse et al. [61] proposed the Subspace Deconvolution using Genetic Algorithm (SSDGA) and HMP (Hybrid Matching Pursuit) algorithm for wide-field broadband imaging. Imaging results show that SSDGA produces the smallest imaging residuals (see Figure 8 [Figure 8: see original paper]), and compared to direction-independent calibration algorithms, HMP or SSDGA deconvolution algorithms can effectively increase dynamic range and reduce errors in flux density and spectral index. DDFacet Imager estimates sky spectral properties during imaging and deconvolution and discusses issues caused by facets, such as image-plane normalization and position-dependent PSF. This algorithm has proven to work well with KillMS, producing excellent calibration results, and has been tested on data from various telescopes including LOFAR, VLA, MeerKAT, and ATCA (Australia Telescope Compact Array), making it the most promising algorithm for SKA data processing.

Albert et al. [25, 26] applied their proposed physics-based Gaussian process model to predict ionospheric phase screens for LOFAR and perform DDE calibration on sources within the FoV. Results from processing LoTSS observations show that DDE for most bright sources was calibrated, with root-mean-square residuals around sources reduced by an average of 32%. Roth et al. [63] designed a Bayesian imaging and calibration algorithm that uses image-domain gridding to numerically apply direction-dependent antenna gains efficiently. Successfully applied to VLA observations of radio galaxy Cygnus A, the algorithm produced sky images with higher resolution and fewer artifacts compared to compressed sensing [64], and significantly improved reconstructed image dynamic range compared to traditional calibration [65] and compressed sensing reconstruction [64] (see Figure 9 [Figure 9: see original paper]).

Another calibration approach involves simultaneous observation with LBA (low-band antenna) and HBA (high-band antenna). Since both parts of the array observe the same ionospheric region, the fundamental parameters describing ionospheric activity are identical in the data, providing new prospects for ionospheric calibration. Edler et al. [20] compared ionospheric solutions derived separately from LOFAR LBA and HBA observations and from joint calibration of both. Results show: (1) The joint calibration method most accurately determines simulated ionospheric parameters, achieving root-mean-square errors at the mTEC level in 90% of cases. (2) For the "solution transfer" approach, where ionospheric solutions found in HBA calibration are applied to LBA data, results show good convergence. However, its main drawback is that errors in HBA data are significantly amplified when applying the solution to LBA data. Therefore, the "solution transfer" approach is only suitable when all non-ionospheric effects in HBA are accurately calibrated.

Meanwhile, to improve radio interferometric imaging and calibration efficiency, data processing pipelines based on container technology, parallelization, and GPUs have emerged. Stimela is a platform-independent radio interferometric scripting framework based on Python and container technology (such as Docker and Singularity). Stimela allows users to execute tasks in many Python data reduction packages without installing them separately (e.g., CASA, MeqTrees, AOFlagger). With Stimela, different software packages can be conveniently used in a unified environment. CARACal (Containerized Automated Radio Astronomy Calibration [66]) consists of a series of Stimela scripts that are connected and run sequentially. CUBICAL [67] is a Python-based software for direction-independent and direction-dependent self-calibration, utilizing CYTHON, multi-threading, and shared memory to fully exploit hardware capabilities. Application to simulated and real data shows that CUBICAL is competitive with existing calibration tools like MeqTrees. CUBICAL' s model prediction component is handled by Montblanc [68], which implements a GPU-accelerated version of RIME. Links to the radio interferometric data processing algorithms and software mentioned in this paper are listed in Table 2 .

5.1 Ionospheric Simulation

Next-generation radio telescopes feature both wide fields of view and long baselines, facing more complex ionospheric effects in observation data simulation and calibration. To eliminate ionospheric phase errors, high-precision calibration with high time resolution (on the order of 10 seconds), time- and baseline-dependent, and direction-dependent corrections is required for observations below 300 MHz.

Currently, validation of wide-field imaging and calibration algorithms for next-generation telescopes like SKA relies on simulated observation data, requiring the simulation to be as realistic as possible—that is, including various factors affecting the observation process.

Regarding real ionospheric monitoring data, GNSS provides Global Ionospheric Maps with spatial resolution of 5° latitude × 2.5° longitude and temporal resolution of 2 hours. MIT' s TEC data offers spatial resolution of 1° latitude × 1° longitude and temporal resolution of 5 minutes, though some regions have missing data. Overall, existing monitoring data lacks information on small spatial-and short temporal-scale TEC variations and cannot describe small-scale turbulent effects in the ionosphere.

Current ionospheric simulations only consider first- and second-order Taylor series ionospheric effects, while higher-order effects are non-negligible at the lowest frequencies observed by LOFAR (40 MHz). Taking SKA as an example, current ionospheric simulation methods are relatively crude in large-scale observation data simulation, considering only dispersive delay (first-order Taylor effect), which cannot adequately validate imaging and high-precision calibration algorithms. Moreover, in actual observations, ionospheric scintillation can affect the coherence of celestial radio signals under special ionospheric conditions. These scintillations, combined with RFI, can render observation data unusable for scientific research. Current radio interferometric data simulation tools such as OSKAR, SIVIO, and LoSiTo only consider turbulent effects in ionospheric simulation, without detailed treatment of higher-order terms like ionospheric scintillation. Research on simulation method implementation is needed to ultimately obtain high-precision ionospheric simulation data.

5.2 Calibration of Ionospheric Effects

Over the past decade, research on direction-dependent calibration algorithms (including ionospheric effect calibration) has deepened and has been applied to telescopes such as MWA, LOFAR, and MeerKAT, largely solving third-generation calibration problems. However, imaging and calibration algorithms still face enormous challenges for the international mega-science project SKA.

The applicability and performance of existing imaging, deconvolution, and calibration algorithms in SKA data processing urgently need validation. To achieve SKA' s primary scientific goals, the current focus is on how to obtain high-dynamic-range images through algorithm performance optimization and calibration method improvements. Meanwhile, SKA' s unprecedented wide field of view (resulting in large single image files) and massive observational data volumes pose significant challenges to computational costs and performance. Research on algorithm performance optimization, GPU acceleration, and distributed processing are all urgent technical issues.

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Note: Figure translations are in progress. See original paper for figures.