Constraint on the symmetry energy at high densities from neutron star observations using relativistic mean-field models

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Abstract

metry energy, is essential in both nuclear physics and astro-physics [1-6]. In nuclear physics, the symmetry energy sig-nificantly affects the structure of finite nuclei, including theit influences the key properties of neutron stars, such as theirmasses and radi [3], as well as their cooling processes [8].Consequently, constraining the symmetry energy is an impor-tant challenge in nuclear physics.Many experimental attempts have been conducted to con-strain the symmetry energy around the saturation densityPo [9-22]. These include measurements of the dipole po-larizabilities of 208pb [13, 14], giant dipole resonance ener-gies [12], isospin diffusion in heavy-ion collisions [10], iso-baric analog states [18], and neutron skin thickness [11, 23].Because of the extensive research on determining the symme-try energy,the constraint on the symmetry energy at saturation density is relativelyprecise,with a commonly acceptedvalue of J =30$±$4MeV [9,10,12-21].In contrast, the symmetry energy at suprasaturation re-mains unclear.Many terrestrial experiments have beenperformed to extract information on symmetry energy atsuprasaturation. In studies on heavy-ion collisions, the con-straints on the suprasaturation density dependence of the sym-metry energy have been obtained from analyses of the π—/π+ratio [24-30] and n/p elliptic flows ratio [31, 32]. However,the symmetry energy at supurasaturation densities exhibit alarge model dependence. This is caused by the diffculties insolving the transport models and extrapolating the finite ex-The mass measurement of the pulsar PSR J0740+6620from the Neutron Star Interior Composition Explorer [34] re-vealedaasW190814from theLIGO/Virgo collaboration[35]observeda 2.5 - 2.67 Mo compact star because of the neutron s-tars impose tight constraints on the EoS [36-39]. For theobservations of the mass around the 1.4 Mo neutron star,the analysis of the tidal deformability fromGW170817byLIGO/Virgo[40.41] along with radius R1.4= 13.02123 k-1.06 m and mass-radius posterior distributions from millisecond-pulsar PSR J0030+0451 via the (NICER) mission[42]havesignificantly enhanced the ability

to constrain the symmetryenergy of nuclear matter [43, 44]. Moreover, constraints onthe symmetry energy have been derived from multiple ob-servations of neutron stars in combination with nuclear the-ory [9, 43-46]. Among these studies, Skyme interaction-s [44, 47, 48], chiral effective field theory [9] and empiri-cal local density functional metamodels[49] were developedunder the non-relativistic framework.Relativistic mean-fieldmodels have been widely used to describe the properties of in-finite nuclear matter, finite nuclei, and stellar matter [5053].A systematically analysis of the influence of all relativisticmean-field (RMF) sets on the properties of neutron stars. Inthis paper, we utilize RMF models to describe the propertiesof neutron stars with a focus on exploring the constraints im-posed by these models on the EoS and symmetry energy atboth saturation and suprasaturation densities.The remainder of this paper is organized as follows. InSect. II, we review the theoretical aspects of RMF models, theEoS of a neutron star, and the Tolman-Oppenheimer-Volkov(TOV) equation. In Sect. III, we present the results of the con-

Full Text

Preamble

Constraint on the symmetry energy at high densities from neutron star observations using relativistic mean-field models

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In this paper, we constrain the symmetry energy at high densities in nuclear matter using recent observations of neutron stars based on calculations from relativistic mean-field (RMF) models. From these neutron star observations, we obtain the following constraints on the symmetry energy at high densities: $S(2\rho_0) = 40.54 \pm 12.47$ MeV and $S(3\rho_0) = 44.12 \pm 29.38$ MeV.

Keywords: Equation of state, Symmetry energy, Neutron star

Introduction

The nuclear equation of state (EoS), particularly the symmetry energy, is essential in both nuclear physics and astrophysics [**?**]. In nuclear physics, the symmetry energy significantly affects the structure of finite nuclei, including the neutron-skin thickness of heavy nuclei [**?**]. In astrophysics, it influences key properties of neutron stars, such as their masses and radii [**?**], as well as their cooling processes [**?**]. Consequently, constraining the symmetry energy represents an important challenge in nuclear physics.

Numerous experimental attempts have been conducted to constrain the symmetry energy around the saturation density ρ_0 [?]. These include measurements of the dipole polarizabilities of ^{208}Pb [?, ?], giant dipole resonance energies [?], isospin diffusion in heavy-ion collisions [?], isobaric analog states [?], and neutron skin thickness [?, ?]. Due to extensive research on determining the symmetry energy, the constraint at saturation density is relatively precise, with a commonly accepted value of $J = 30 \pm 4$ MeV [?, ?, ?].

In contrast, the symmetry energy at suprasaturation densities remains unclear. Many terrestrial experiments have been performed to extract information on the symmetry energy at suprasaturation. In studies of heavy-ion collisions, constraints on the suprasaturation density dependence of the symmetry energy have been obtained from analyses of the π^-/π^+ ratio [?] and the n/p elliptic flow ratio [?, ?]. However, the symmetry energy at suprasaturation densities exhibits large model dependence, arising from difficulties in solving transport models and extrapolating the finite excited nuclear system to infinite nuclear matter at zero temperature. Consequently, further progress in understanding and constraining the equations of state for nuclear matter at high densities is expected from analyses that incorporate the properties of neutron stars.

The mass measurement of the pulsar PSR J0740+6620 from the Neutron Star Interior Composition Explorer [?] revealed a neutron star with a mass of $2.08^{+0.07}_{-0.07} M_\odot$, and GW190814 from the LIGO/Virgo collaboration [?] observed a $2.5 - 2.67 M_\odot$ compact star. Because neutron stars impose tight constraints on the EoS [?], observations of masses around $1.4 M_\odot$ neutron stars, such as the tidal deformability from GW170817 [?, ?] along with radius $R_{1.4} = 13.02^{+1.24}_{-1.06}$ km and mass-radius posterior distributions from millisecond pulsar PSR J0030+0451 via the NICER mission [?], have significantly enhanced the ability to constrain the symmetry energy of nuclear matter [?, ?]. Moreover, constraints on the symmetry energy have been derived from multiple observations of neutron stars in combination with nuclear theory [?, ?]. Among these studies, Skyrme interactions [?, ?, ?], chiral effective field theory [?], and empirical local density functional metamodels [?] were developed under the non-relativistic framework. Relativistic mean-field models have been widely used to describe the properties of infinite nuclear matter, finite nuclei, and stellar matter [?]. A systematic analysis of the influence of all relativistic mean-field (RMF) sets on the properties of neutron stars is needed. In this paper, we utilize RMF models to describe the properties of neutron stars with a focus on exploring the constraints imposed by these models on the EoS and symmetry energy at both saturation and suprasaturation densities.

The remainder of this paper is organized as follows. In Section II, we review the theoretical aspects of RMF models, the EoS of a neutron star, and the Tolman–Oppenheimer–Volkov (TOV) equation. In Section III, we present the results of the constraint on the symmetry energy obtained from neutron star observations. Finally, the summary is presented in Section IV.

II. Model

A. Relativistic mean field

As an example, we use the nonlinear RMF model to present the expressions of relevant quantities [?]. In the RMF model, meson fields can be replaced with their expectation values. In this paper, we employ the framework of RMF models to extract information on the symmetry energy in symmetric nuclear matter (SNM) systems. RMF models provide a comprehensive description of both nuclear matter and finite nuclei and are widely used to study neutron star properties. To better categorize the parametrizations associated with RMF models, Ref. [?] defined three distinct types: (i) nonlinear, (ii) density-dependent, and (iii) point-coupling models.

The Lagrangians for the different RMF models are:

1. Nonlinear (NL) model

$$\mathcal{L}_{NL} = \bar{\Psi}[i\gamma^\mu\partial_\mu - m_N]\Psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - U(\sigma) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{\zeta}{4}(\omega_\mu\omega^\mu)^2 - g_{\sigma NN}\bar{\Psi}\Psi\sigma - g_{\omega NN}\bar{\Psi}\gamma^\mu$$

2. Density-dependence (DD) model

$$\sigma \to \langle\sigma\rangle = \bar{\sigma}, \quad \omega_\mu \to \langle\omega_\mu\rangle = \bar{\omega}_0\delta_{\mu 3}, \quad \delta_\mu \to \langle\delta_\mu\rangle = \bar{\delta}_0\delta_{\mu 3}, \quad \rho_\mu \to \langle\rho_\mu\rangle = \bar{\rho}_0\delta_{\mu 3}$$

The equations of motion (EOMs) for nucleons and mesons were derived from the Lagrangian density:

$$[\gamma^\mu i\partial_\mu - g_\omega\bar{\omega}_0 - g_\rho\bar{\rho}_0\tau_3 - (m_N - g_\sigma\bar{\sigma} - g_\delta\bar{\delta}_3\tau_3)]\Psi = 0$$

$$\omega(\bar{\omega}_0)^2(\alpha_1 + \alpha_1' g_\sigma\bar{\sigma})\bar{\sigma} = g_\omega\rho - \zeta g_\omega^4(\bar{\omega}_0)^3 - g_\sigma g_\omega^2\bar{\sigma}(\bar{\omega}_0)^2(2\alpha_1 + \alpha_1' g_\sigma\bar{\sigma})$$

$$\sigma\bar{\sigma} = g_\sigma\rho_s - g_2\bar{\sigma}^2 - g_3\bar{\sigma}^3 + g_\sigma g_\omega^2(\bar{\omega}_0)^2(\alpha_1 + \alpha_1' g_\sigma\bar{\sigma}) + g_\sigma g_\rho^2(\bar{\rho}_0^3)^2(\alpha_2 + \alpha_2' g_\sigma\bar{\sigma})$$

$$\rho\bar{\rho}_0^3 = g_\rho\rho_3 - g_\sigma g_\rho^2\bar{\sigma}(\bar{\rho}_0^3)^2(2\alpha_2 + \alpha_2' g_\sigma\bar{\sigma}) - \alpha'(\bar{\rho}_0^3)^2(\bar{\omega}_0)^2$$

$$\delta\bar{\delta}_3 = g_\delta\rho_{s3}$$

where the different densities are defined as:

$$\rho_s = \langle\bar{\Psi}\Psi\rangle = \rho_s^n + \rho_s^p, \quad \rho = \langle\bar{\Psi}\gamma^0\Psi\rangle = \rho_n + \rho_p,$$

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$$\rho_{s3} = \langle \bar{\Psi}\tau_3\Psi \rangle = \rho_s^p - \rho_s^n, \quad \rho_3 = \langle \bar{\Psi}\gamma^0\tau_3\Psi \rangle = \rho_p - \rho_n,$$

$$\mathcal{L}_{DD} = \bar{\Psi}[i\gamma^\mu\partial_\mu - m_N]\Psi + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}(\partial_\mu\delta\partial^\mu\delta - m_\delta^2\delta^2) + \Gamma_\sigma(\rho)\bar{\Psi}\Psi\sigma -$$

3. Point-coupling (PC) model

$$\mathcal{L}_{PC} = \bar{\Psi}[i\gamma^\mu\partial_\mu - m_N]\Psi - \alpha_S(\bar{\Psi}\Psi)^2 - \alpha_V(\bar{\Psi}\gamma^\mu\Psi)(\bar{\Psi}\gamma_\mu\Psi) - \alpha_{TS}(\bar{\Psi}\tau\Psi)\cdot(\bar{\Psi}\tau\Psi) - \alpha_{TV}(\bar{\Psi}\gamma^\mu\tau\Psi)\cdot(\bar{\Psi}\gamma_\mu\tau\Psi) - \alpha_3(\bar{\Psi}\Psi)^3$$

Filling up to the Fermi momenta $k_{F,i}$ for $i = n$ or p in nuclear matter, the neutron (n) and proton (p) scalar and baryon densities are given by:

$$\rho_{si} = \frac{C}{(2\pi)^3}\int_{k<k_F^i} d^3k = \frac{C}{2\pi^2}[k_F^i E_F^{*i} - m^{*2}\ln\frac{k_F^i + E_F^{*i}}{m^*}],$$

$$\rho_i = \frac{C}{(2\pi)^3}\int_{k<k_F^i} d^3k = \frac{C}{6\pi^2}(k_F^i)^3,$$

where the degeneracy factor $C(i = n, p) = 2$, and $E_F^{*i} = \sqrt{(k_F^i)^2 + m^{*2}}$ is the Fermi energy of neutrons and protons. Here, m^* is the Dirac effective mass of nucleons, given by the relation:

$$m_n^* = m_N - g_\sigma\bar{\sigma} + g_\delta\bar{\delta}_3,$$
$$m_p^* = m_N - g_\sigma\bar{\sigma} - g_\delta\bar{\delta}_3.$$

The eigenvalues of the neutrons and protons from the Dirac equation are:

$$\epsilon_n = g_\omega\bar{\omega}_0 - g_\rho\bar{\rho}_0 + \sqrt{k^{*2} + m_n^{*2}},$$
$$\epsilon_p = g_\omega\bar{\omega}_0 + g_\rho\bar{\rho}_0 + \sqrt{k^{*2} + m_p^{*2}},$$

where $\omega_{\mu\nu}$ and $\rho_{\mu\nu}$ are defined by $\partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ and $\partial_\mu\rho_\nu - \partial_\nu\rho_\mu$, respectively. In Eq. (1), $U(\sigma) = \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4$ is the nonlinear potential of the σ field.

The expressions for the energy density and pressure are obtained from the given Lagrangian using the energy-momentum tensor relation:

$$T_{\mu\nu} = \sum_i \frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi_i)}\partial_\nu\phi_i - g_{\mu\nu}\mathcal{L},$$

where ϕ_i runs over all possible fields. The energy density ϵ and pressure P can be obtained from the energy-momentum tensor:

$$\epsilon_{NL} = \langle T^{00}\rangle = \frac{1}{2}m_\sigma^2\bar{\sigma}^2+\frac{g_2}{3}\bar{\sigma}^3+\frac{g_3}{4}\bar{\sigma}^4+\frac{1}{2}m_\omega^2\bar{\omega}_0^2+\frac{\zeta}{4}g_\omega^4(\bar{\omega}_0)^4+g_\omega\bar{\omega}_0\rho-g_\sigma g_\omega^2\bar{\sigma}(\bar{\omega}_0)^2(\alpha_1+\alpha_1' g_\sigma\bar{\sigma})+\frac{1}{2}m_\rho^2(\bar{\rho}_0^3)^2+g_\rho\bar{\rho}_0^3\rho_3$$

$$P_{NL} = \frac{1}{3}\sum_i\langle T^{ii}\rangle = -\frac{1}{2}m_\sigma^2\bar{\sigma}^2-\frac{g_2}{3}\bar{\sigma}^3-\frac{g_3}{4}\bar{\sigma}^4+\frac{1}{2}m_\omega^2\bar{\omega}_0^2+\frac{\zeta}{4}g_\omega^4(\bar{\omega}_0)^4+g_\sigma g_\omega^2\bar{\sigma}(\bar{\omega}_0)^2(\alpha_1+\alpha_1' g_\sigma\bar{\sigma})+\frac{1}{2}m_\rho^2(\bar{\rho}_0^3)^2-g_\rho\bar{\rho}_0^3\rho_3+$$

The same calculations for the density-dependence and point-coupling models are given in Refs. [**?**].

The binding energy per particle in asymmetric nuclear matter can be expressed as:

$$E(\rho,\alpha) = \frac{\epsilon}{\rho} - m_N = E_0(\rho) + S(\rho)\alpha^2 + \mathcal{O}(\alpha^4),$$

where the isoscalar term $E_0(\rho) = E(\rho, \alpha = 0)$ is the binding energy per nucleon in SNM, and the isovector term $S(\rho)$ is the symmetry energy. Here, $\rho = \rho_n + \rho_p$ is the nuclear matter density, and $\alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is the isospin asymmetry. The nuclear symmetry energy $S(\rho)$ is defined as:

$$S(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\alpha)}{\partial\alpha^2}\bigg|_{\alpha=0}.$$

For SNM, $m_n^* = m_p^* = m_N^*$ because $\bar{\delta}_3$ vanishes. The symmetry energies of the RMF models are expressed as follows:

$$S(\rho)_{NL} = \frac{k_F^2}{6E_F^*}[1 + g_\rho^2 A(\rho, m_N^*)],$$

$$S(\rho)_{DD} = \frac{k_F^2}{6E_F^*}[1 + \Gamma_\rho^2 A(\rho, m_N^*)],$$

$$S(\rho)_{PC} = \frac{k_F^2}{6E_F^*}[1 - \alpha_{TV} S A(\rho, m_N^*)],$$

where $m_\rho^{*2} = m_\rho^2 + g_\sigma g_\rho^2\bar{\sigma}(2\alpha_2 + \alpha_2' g_\sigma\bar{\sigma}) + \alpha'(\bar{\omega}_0)^2$, and

$$A(\rho, m_N^*) = \frac{3}{\pi^2}\left(\frac{\rho_s}{m_N^*}\right).$$

The expressions for the slope of symmetry energy (L) of the various RMF models are:

$$L_{NL} = \frac{k_F^2}{6E_F^*} \times \left\{ 3 - \frac{m_N^{*2}}{E_F^{*2}} + 6\left(1 - \frac{m_N^*}{E_F^*}\right) + \frac{g_\rho^2}{m_\rho^{*2}} \left[1 + \frac{g_\rho^2}{m_\rho^{*2}} A(\rho, m_N^*) \right] \right\},$$

$$L_{DD} = \frac{k_F^2}{6E_F^*} \times \left\{ 3 + 6\frac{\Gamma_\rho^2}{m_\rho^{*2}} A(\rho, m_N^*) \left(1 + \frac{\Gamma_\rho^2}{m_\rho^{*2}} A(\rho, m_N^*)\right) + \frac{6}{1 + \Gamma_\rho^2 A(\rho, m_N^*)/m_\rho^{*2}} \right\},$$

$$L_{PC} = \frac{k_F^2}{6E_F^*} \times \left\{ 3 - \frac{m_N^{*2}}{E_F^{*2}} + 6\left(1 - \frac{m_N^*}{E_F^*}\right) + \frac{3\alpha_{TV} S}{1 - \alpha_{TV} S A(\rho, m_N^*)} \right\}.$$

The symmetry energy is expanded in terms of $(\rho - \rho_0)/3\rho_0$:

$$S(\rho) = J + L\frac{\rho - \rho_0}{3\rho_0} + \cdots,$$

where $J = S(\rho_0)$ is the symmetry energy, and $L = 3\rho_0 \frac{\partial S}{\partial \rho}|_{\rho=\rho_0}$ is the slope of the symmetry energy at saturation density.

In this paper, we utilize 180 interaction parameter sets selected from those used in RMF models [?]. These parameter sets satisfy the incompressibility at saturation densities K_0 within the range of 200 to 300 MeV by computing the distribution of isoscalar monopole strength in ^{208}Pb with relativistic models [?]. The interaction parameter sets are listed below, and detailed information is available in Ref. [?]:

(i) 165 nonlinear RMF models (E [?], ER [?], NL1 [?], NL3 [?], NL3-II [?], NL3* [?], NL4 [?], NLC [?], NLB1 [?], NLB2 [?], NLRA1 [?], NLS [?], P-067 [?], P-070 [?], P-075 [?], P-080 [?], GL1 [?], GL2 [?], GL3 [?], GL4 [?], GL5 [?], GL6 [?], GL7 [?], GL8 [?], GL82 [?], GL9 [?], GM1 [?], GM2 [?], GM3 [?], GPSa [?], GPSb [?], NLρA [?], NLρB [?], RMF301 [?], RMF302 [?], RMF303 [?], RMF304 [?], RMF305 [?], RMF306 [?], RMF307 [?], RMF308 [?], RMF309 [?], RMF310 [?], RMF311 [?], RMF312 [?], RMF313 [?], RMF314 [?], RMF315 [?], RMF316 [?], RMF317 [?], RMF401 [?], RMF402 [?], RMF403 [?], RMF404 [?], RMF405 [?], RMF406 [?], RMF407 [?], RMF408 [?], RMF409 [?], RMF410 [?], RMF411 [?], RMF412 [?], RMF413 [?], RMF414 [?], RMF415 [?], RMF416 [?], RMF417 [?], RMF418 [?], RMF419 [?], RMF420 [?], RMF421 [?], RMF422 [?], RMF423 [?], RMF424 [?], RMF425 [?], RMF426 [?], RMF427 [?], RMF428 [?], RMF429 [?], RMF430 [?], RMF431 [?], RMF432 [?], RMF433 [?], RMF434 [?], Q1 [?], G1 [?], G2 [?], SMFT2 [?], DJM [?], S271 [?], Z271 [?], SRK3M5 [?], HD

[?], HC [?], MS1 [?], MS3 [?], XS [?], NLSV1 [?], NLSV2 [?], TM1 [?], PK1 [?], hybrid [?], Z271* [?], G2* [?], BKA20 [?], BKA22 [?], BKA24 [?], FSUGOLD [?], FSUGOLD4 [?], FSUGOLD5 [?], FSUGZ00 [?], FSUGZ03 [?], FSUGZ06 [?], IU-FSU [?], NL3V1 [?], NL3V2 [?], NL3V3 [?], NL3V4 [?], NL3V5 [?], NL3V6 [?], S271V1 [?], S271V2 [?], S271V3 [?], S271V4 [?], S271V5 [?], S271V6 [?], Z271S1 [?], Z271S2 [?], Z271S3 [?], Z271S4 [?], Z271S5 [?], Z271S6 [?], Z271V1 [?], Z271V2 [?], Z271V3 [?], Z271V4 [?], Z271V5 [?], Z271V6 [?], TM1* [?], BSR1 [?], BSR2 [?], BSR3 [?], BSR4 [?], BSR5 [?], BSR6 [?], BSR7 [?], BSR8 [?], BSR9 [?], BSR10 [?], BSR11 [?], BSR12 [?], BSR13 [?], BSR14 [?], BSR15 [?], BSR16 [?], BSR17 [?], BSR18 [?], BSR19 [?], BSR20 [?], BSR21 [?], SVI-1 [?], SVI-2 [?], SIG-OM [?], NL$\rho\delta$A [?], NL$\rho\delta$B [?]);

(ii) Nine density-dependent RMF models (TW99 [?], DD-ME1 [?], PKDD [?], DD-ME2 [?], DD [?], DD-F [?], DD2 [?], DDMEδ [?], DDRH$\rho\delta$ [?]);

(iii) Six point-coupling RMF models (FA3 [?], FA4 [?], FZ3 [?], VZ3 [?], PC-F1 [?], PC-F3 [?]).

B. Equation of state of a neutron star

For different density regions of a neutron star, different forms of the EoS are employed in this paper as follows:

(i) For the outer crust of a neutron star, the EoS provided by Baym, Pethick, and Sutherland (BPS) [?] is adopted for densities in the range $\rho_{\rm min} \leq \rho \leq \rho_{\rm outer}$. Here, $\rho_{\rm min}$ is the minimum density ($\rho_{\rm min} = 4.73 \times 10^{-15}$ fm^{-3}), with the corresponding energy density $\epsilon_{\rm min} = 4.38 \times 10^{-12}$ MeV fm^{-3} and pressure $P_{\rm min} = 6.3 \times 10^{-25}$ MeV fm^{-3}. The outer crust density is $\rho_{\rm outer} = 2.57 \times 10^{-4}$ fm^{-3} with an energy density $\epsilon_{\rm outer} = 0.24$ MeV fm^{-3} and pressure $P_{\rm outer} = 4.86 \times 10^{-4}$ MeV fm^{-3}. In this region, because of the heavy nuclei, primarily around the iron mass number, a Coulomb lattice coexists in β-equilibrium (i.e., equilibrium with respect to weak interaction processes) with an electron gas in the neutron star outer crust [?].

(ii) For the inner crust, $\rho_{\rm outer} < \rho \leq \rho_T$, the EoS takes the form $P = A + B\epsilon^{4/3}$ from Refs. [?, ?], where two constants A and B are adjusted to match the outer crust EoS to that of a liquid core at the crust-core transition density ρ_T ($\rho_T = 0.5\rho_0$ [?]).

(iii) In the liquid core region, also referred to as the outer core (OC) region, the density range is $\rho_T < \rho \leq \rho_{OC}$ ($\rho_{OC} = 3\rho_0$), where the EOS is obtained using the RMF. This region requires the system to be in β-equilibrium and is composed of protons, neutrons, electrons, and muons. For β-equilibrium to be attained in nuclear matter, the following electroweak processes must occur: $n \to p + e^- + \bar{\nu}_e$, $p + e^- \to n + \nu_e$, $e^- \to \mu^- + \nu_e + \bar{\nu}_\mu$, $p + \mu^- \to n + \nu_\mu$, and $n \to p + \mu^- + \bar{\nu}_\mu$. The β-equilibrated matter must satisfy the following conditions for the chemical potentials: $\mu_n - \mu_p = \mu_e$, $\mu_e = \mu_\mu$, where $\mu_{\nu_i} = 0$, $i = e, \mu$. At zero temperature, the neutron and proton chemical potentials are $\mu_n = \epsilon_n$ and $\mu_p = \epsilon_p$, respectively. For relativistic degenerate electrons and muons:

$$\mu_e = \sqrt{m_e^2 + (3\pi^2 x_e \rho)^{2/3}}, \quad \mu_\mu = \sqrt{m_\mu^2 + (3\pi^2 x_\mu \rho)^{2/3}},$$

where $m_e = 0.511$ MeV and $m_\mu = 0.105$ GeV. With the particle fraction $x_i = \rho_i/\rho$ $(i = n, p, e, \mu)$, $x_p = x_e + x_\mu$ satisfies charge neutrality.

(iv) For the neutron star inner core (IC) region at high density ($\rho > \rho_{OC}$), studies have incorporated exotic particles such as hyperons, Δs, quarks, and dark matter into the core of massive neutron stars at densities exceeding $3\rho_0$ [?]. However, owing to the significant uncertainties in the interactions between nucleons and exotic particles (e.g., hyperons, Δs, and quarks), and the unclear composition of the neutron star core, a polytropic EoS is employed instead of explicitly modeling all possible exotic particles. In this paper, a piecewise polytropic EoS of the form $P_i(\rho) = \kappa_i \rho^{\gamma_i}$ [?] is used to smoothly extend the EoS to the density region of the IC of the neutron star ($\rho > \rho_{OC}$). The polytropic EoS is constructed as follows:

$$P_i = \kappa_i \rho^{\gamma_i}, \quad \text{with} \quad \kappa_i \approx \kappa_{i-1} \frac{\ln(P_i/P_{i-1})}{\ln(\rho_i/\rho_{i-1})},$$

$$\epsilon_i = \epsilon_{i-1} - \frac{P_i}{\gamma_i - 1} + \frac{P_{i-1}}{\gamma_i - 1} \left(\frac{\rho_i}{\rho_{i-1}}\right)^{\gamma_i},$$

where the set of dividing densities $\rho_{OC} = \rho_1 < \rho_2 < \cdots$, and $\rho_{i+1} - \rho_i = \Delta\rho = 0.05\rho_0$.

In summary, the EoS of neutron star matter is:

$$P(\epsilon) = \begin{cases} P_{BPS}(\epsilon) & \text{for } \rho_{\min} \le \rho \le \rho_{\text{outer}} \\ A + B\epsilon^{4/3} & \text{for } \rho_{\text{outer}} < \rho \le \rho_T \\ P_{RMF}(\epsilon) & \text{for } \rho_T < \rho \le \rho_{OC} \\ P_{IC}(\epsilon) & \text{for } \rho > \rho_{OC} \end{cases}$$

where $P = P_N + P_e + P_\mu$, $\epsilon = \epsilon_N + \epsilon_e + \epsilon_\mu$, and the total pressure and energy density should include the leptons. The thermodynamically stable EoS must satisfy $\partial P/\partial\epsilon \ge 0$. The adiabatic speed of sound can be expressed as:

$$v_s^2 = \frac{\partial P}{\partial \epsilon}.$$

C. Tolman–Oppenheimer–Volkov equation

The structure of a neutron star is obtained by solving the TOV equation derived from General Relativity [?]. The TOV equations are:

$$\frac{dP}{dr} = -\frac{G\epsilon}{r^2}\frac{(1+P/\epsilon)(1+4\pi r^3 P/M)}{1-2GM/r},$$

$$\frac{dM}{dr} = 4\pi r^2 \epsilon,$$

where $G = 6.707 \times 10^{-45}$ MeV^{-2} is the gravitational constant, r is the distance from the core of the star, $P = P(r)$ is the pressure, and $M = M(r)$ is the mass enclosed within radius r.

The in-spiral phase of two merging neutron stars creates strong tidal gravitational fields, resulting in the deformation of the multipolar structure of the star. The deforming effects are quantified through the tidal deformability parameter Λ, which relates the induced mass quadrupole moment Q_{ij} to the time-independent external tidal field $\mathcal{E}_{ij}$ through the relation [?]:

$$Q_{ij} = -k_2 \frac{2R^5}{3G}\mathcal{E}_{ij}.$$

Here, k_2 is the Love number, which can be obtained from the solution of the first-order differential equation [?]:

$$\frac{dy}{dr} = -\frac{y^2}{r} - \frac{y-6}{r-2GM} - 4\pi G r^2 Q,$$

where

$$Q = \frac{G(5\epsilon + 9p + (\epsilon+p)/c_s^2) - 6\pi r^2}{1-2GM/r} - \left[\frac{2G(M+4\pi p r^3)}{r(r-2GM)}\right]^2,$$

and

$$k_2 = \frac{8\beta^5}{5}(1-2\beta)^2[2-y_R+2\beta(y_R-1)]\times\{2\beta[6-3y_R+3\beta(5y_R-8)]+4\beta^3[13-11y_R+\beta(3y_R-2)+2\beta^2(1+y_R)]+3(1-2$$

where $y_R = y(R)$ and $\beta = GM/R$. Equations (40), (41), and (43) are solved using the boundary conditions at the center of the star: $M(0) = 0$, $P(0) = P_c$, and $y(0) = 2$, where P_c is the central pressure. Varying P_c yields all possible stars for a given EoS. The condition $P(R) = 0$ (vacuum pressure being set to zero) defines the radius of the star R, and the total gravitational mass of the star is $M(R)$, which is simply denoted by M in the following.

The dimensionless deformability Λ is defined as:

$$\Lambda = \frac{2k_2}{3}\left(\frac{R}{GM}\right)^5 = \frac{2k_2}{3\beta^5}.$$

III. Results and Discussions

Generally, most RMF models are adjusted to describe nuclei and nuclear matter in the density region from near sub-saturation density $\rho \approx 2/3\rho_0$ (average between central and surface densities [?, ?, ?]) to saturation density. Moreover, the symmetry energy shows large variations at high densities when obtained using different parameters of RMF models. The symmetry energy as a function of density obtained by RMF models with 180 parameter sets is shown in Fig. 1 [Figure 1: see original paper].

We observe that various RMF models predict very different density behaviors of the symmetry energy, particularly at suprasaturation densities $\rho > \rho_0$. For example, the magnitude of the symmetry energy varies from 29.5 to 114.8 MeV at $2\rho_0$ and from about 14.7 to 188.7 MeV at a density of $3\rho_0$. In Fig. 1, the symmetry energy behaviors are classified into three types: hard, linear, and soft, represented by the green, gray, and orange lines, respectively. We observe from the figure that most of the symmetry energy curves in RMF models occur near the linear type.

In this paper, we use observations of neutron stars to constrain the nuclear EoS at high densities $\rho \geq \rho_0$ because neutron star properties are strongly correlated with the nuclear EoS, as mentioned in Refs. [?, ?, ?]. This constraint can be obtained from measurements of neutron stars with a mass of $M = 1.4M_\odot$, such as the tidal deformability from the analysis of gravitational wave data, $\Lambda_{1.4} \approx 190^{+390}_{-120}$ from GW170817 [?], and the radius-mass relation from both GW170817 and PSR J0030+0451 [?, ?]. For the maximum masses of neutron stars, we use the constraint $M_{\max} \geq 2M_\odot$ from Refs. [?, ?], because a compact star with a mass of $2.59^{+0.08}_{-0.09}M_\odot$ may be the lightest black hole [?].

Our results for tidal deformability with the empirical values from GW170817 (red point) [?] are plotted in panel (a) of Fig. 2 [Figure 2: see original paper]. The mass-radius relationship of neutron stars is shown in panel (b) of Fig. 2, where the pink shaded areas represent the region of the posterior distributions at 90% confidence from the analysis of PSR J0030+0451 [?], the blue shaded region is the posterior distribution at 95.4% confidence from PSR J0740+6620 [?], and the purple and green shaded areas denote the region of the posterior distributions at 90% confidence for GW170817's lighter and heavier neutron stars [?], respectively. In Fig. 2, the gray lines represent the results for all the selected 180 RMF parameter sets, which are models without the constraint of neutron star observations.

With the constraint from observables of neutron stars such as tidal deformability [?] and mass-radius relation around $1.4M_\odot$ [?, ?] combined with the maximum masses of neutron stars above $2M_\odot$, the results of restricted RMF models are indicated by black lines in Fig. 2. Because of the neutron star multi-observables, the analysis of empirical values excludes most RMF parameter sets, and only nine sets remain: HC, FSUGZ03, IU-FSU, G2*, BSR8, BSR9, DD-F, FA3, and FZ3, which can simultaneously describe both the tidal deformability [?] and the overlap of the two mass-radius relation regions [?, ?]. The symmetry energy for RMF models HC, FSUGZ03, IU-FSU, G2*, BSR8, BSR9 is described by Eq. (26), that for DD-F by Eq. (27), and that for FA3, FZ3 by Eq. (28).

Fig. 3 [Figure 3: see original paper] shows a comparison of the pressure-density relations between the empirical values from measurements of neutron stars and the RMF model calculations. The green shaded regions enclose the empirical pressure given by the "spectral" EoS inferred from the Bayesian analysis of the GW170817 data at a 90% confidence level, maintaining the lower limit of the maximum neutron star mass at $2M_\odot$. Our results of the pressure-density relation (black lines) for neutron-rich matter with β stability are mostly under the limit of the empirical region at the OC ($0.5\rho_0 < \rho \leq 3\rho_0$), where the EoS is described by RMF models. For the IC area, $\rho > 3\rho_0$, the order-by-order polytropes EoS in Eqs. (36)-(38) occur primarily in the region of neutron star measurement (except the EoS from the HC model).

Fig. 4 [Figure 4: see original paper] depicts the constraints on the $J - L$ relation, which were compiled in [?, ?, ?]. In this paper, the symmetry energy $J-L$ constraint from neutron star observables based on the RMF models is shown as scattering points (cyan stars) with the range of symmetry energy at saturation $J = 30.66 \pm 0.96$ MeV and slope $L = 49.47 \pm 20.39$ MeV. Fig. 4 includes the $J - L$ constraints obtained in the analysis of finite nuclei (neutron skin thicknesses of Sn isotopes and ^{209}Pb, isobaric analog states and isovector skins, and the dipole polarizability of ^{209}Pb) and nuclear matter (heavy-ion collisions and chiral effective field theory calculations of nuclear matter). The enclosed overlap region [?] from constraints obtained from experimental measurements of the neutron skin thicknesses of Sn, dipole polarizabilities, giant dipole resonances, heavy-ion collisions, and nuclear mass fitting corresponds to J of approximately 29.0-32.7 MeV, and L is approximately 40.5-61.9 MeV [?, ?]. In this paper, the constraints for the symmetry energy and its slope at saturation, which are obtained from neutron star observables, are almost in this overlap region, as shown in Fig. 4. Our results can reproduce the properties of neutron stars but cannot reproduce the PREX-II neutron skin using the RMF model. Compared with the research by Chen et al. [?], where the slope parameter $L = 80 \pm 25$ MeV was obtained from a study of 23 RMF parameter sets, the result in this paper, $L = 49.47 \pm 20.39$ MeV, is relatively soft.

The symmetry energy at $\rho_0 \leq \rho \leq 3\rho_0$ constrained using the neutron star multi-observables is displayed as a cyan shaded region in Fig. 5 [Figure 5: see original paper]. Analysis of doubly magic nuclei and masses of neutron-rich nuclei [?] (black square), isobaric analog states (IAS) [?] (red region), isospin diffusion in heavy-ion collisions [?] (olive region), the electric dipole polarizability in ^{209}Pb [?] (red point), the multi-observables (isospin diffusion, neutron skin and neutron star) [?] (blue dash line), χEFT (magenta hatched) based on Gaussian Process–BUQEYE (χEFT-GPB) Collaboration [?, ?, ?], the constraint from neutron star observations and nuclear matter experiments (gray hatched shaded area) [?], and neutron skin thicknesses of ^{209}Pb by PREX-II [?] (up triangle), are also included for comparison. To obtain full information on the symmetry energy, we also present the range of symmetry energies at suprasaturation: $S(2\rho_0) = 28.07 - 53.00$ MeV and $S(3\rho_0) = 14.74 - 73.49$ MeV in Fig. 5. Our result for the symmetry energy constraint is similar to that from the χEFT-GPB Collaboration, which is a microscopic calculation. The symmetry energy at high density in this paper is softer than the result from Tsang et al. [?] (gray hatched area), which used parametric priors based on an expansion that is widely used in nuclear physics. This discrepancy suggests that further constraints on the symmetry energy should be achieved by reducing the uncertainties in the HIC experiments. Additionally, the pressure of SNM is presented in Appendix A.

IV. Summary

We have extracted information on the symmetry energy at suprasaturation densities from astronomical observations using relativistic mean-field models. In this paper, we have employed 180 RMF parameter sets with incompressibility at saturation density $K_0 = 200 - 300$ MeV, which are suitable for describing the isoscalar monopole distribution strength in ^{208}Pb. By combining the measurements of 1.4 solar-mass neutron stars, such as tidal deformability ($\Lambda_{1.4} \approx 190^{+390}_{-120}$), the mass-radius relation [?, ?], and the maximum mass of at least $2M_\odot$ neutron stars, we derive constraints on the symmetry energy in the density region $\rho_0 - 3\rho_0$. At saturation density, the symmetry energy is $J = 30.66 \pm 0.96$ MeV and the slope is $L = 49.47 \pm 20.39$ MeV, which are consistent with the overlap region of $J - L$ constraints from some terrestrial experiments. The symmetry energy constraints at $2\rho_0$ and $3\rho_0$ are $S(2\rho_0) = 40.54 \pm 12.47$ MeV and $S(3\rho_0) = 44.12 \pm 29.38$ MeV, as shown in Fig. 5.

In the next step, we will explore the entire parameter space of relativistic mean-field models using Bayesian inference with neutron star observational data, which can refine the parameter constraints and provide quantitative constraints on the EoS. Furthermore, the combination of constraints on the EoS from heavy-ion collision analyses (e.g., K^-/K^+ data), neutron star cooling properties such as luminosity data, and measurements of neutron skin thickness in finite nuclei (^{208}Pb and ^{48}Ca) could also reduce the uncertainties of the constraints in future research.

A. Appendix

Pressure is plotted as a function of density (ρ/ρ_0) in Fig. 6 [Figure 6: see original paper], which depicts several constraints for the pressure: pressure constrained by neutron star observables in this paper (blue shaded region), experimental flow (orange area) [**?**], kaon production (dark purple region) [**?**], Giant Monopole Resonance (red dashed line) [**?**], χEFT (magenta hatches) based on the Gaussian Process–BUQEYE Collaboration [**?**, **?**, **?**], and the constraint from astronomical observations and nuclear experiments (gray hatched shaded area) [**?**]. We can observe from Fig. 6 that the pressure is within the regions of the kaon production and Giant Monopole Resonance at $\rho_0 \leq \rho \leq 2.2\rho_0$. In addition, the pressure is almost entirely in the regions of the experimental flow at densities $\rho > 2.2\rho_0$. Compared with the constraint by experimental data, we support all regions of the pressure-density region from ρ_0 to $3\rho_0$, which can complement the constraint on the EoS.

AUTHOR CONTRIBUTIONS

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Ying Cui, Yuan Tian, Cheng-Jun Xia, Ying-Xun Zhang, and Zhu-Xia Li. The first draft of the manuscript was written by Ying Cui and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are openly available in Science Data Bank at http://cstr.cn/31253.11.sciencedb.j00186.00663 and http://www.doi.org/10.57760/sciencedb.j00186.00663.

CONFLICT OF INTEREST

The authors declare that they have no competing interests.

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Note: Figure translations are in progress. See original paper for figures.