SU(3) analysis for B(E2) anomaly

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Abstract

Since the B(E2) anomaly was experimentally found, understanding its mechanism has become an important problem in the field of nuclear structure. Theoretical studies have found that the SU3-IBM and other extended IBM theories can give explanations, but different mechanisms were found. In this paper, we propose the concept of SU(3) analysis for B(E2) anomaly, and some new conclusions can be obtained. This technique can help us to better understand the realistic reason for the B(E2) anomaly.

Full Text

Preamble

SU(3) Analysis for the B(E2) Anomaly

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Since the B(E2) anomaly was discovered experimentally, understanding its underlying mechanism has become a significant challenge in nuclear structure physics. While theoretical studies have shown that the SU3-IBM and other extended IBM theories can provide explanations, different mechanisms have been identified. In this paper, we introduce the concept of SU(3) analysis for the B(E2) anomaly, which yields several new conclusions. This technique can help us better understand the true origin of the B(E2) anomaly.

INTRODUCTION

Fifty years ago, Arima and Iachello proposed the interacting boson model (IBM), an algebraic model for describing collective excitations in nuclear structure [1]. In its simplest formulation, only s and d bosons with angular momenta L = 0 and L = 2 are considered in constructing the Hamiltonian, which possesses SU(6) symmetry. This model exhibits four dynamical symmetry limits: the U(5) limit (spherical shape), the SU(3) limit (prolate shape), the SU(3) limit (oblate shape), and the O(6) limit (γ-soft rotation) [2, 3]. Consequently, shape phase transitions between different shapes can also be studied within this framework [4, 5].

Although the original IBM appears simple and consistent, certain experimental anomalies seem indescribable within this model, such as the B(E2) anomaly [6–9] and the Cd puzzle [10–15]. In the B(E2) anomaly, the ratio E4/2 of the energies of the $4^+{}_1$ state to the $2^+{}_1$ state exceeds 2.0 (a characteristic feature of collective excitations), while the ratio B4/2 of the E2 transition strengths B(E2; $4^+{}_1 \rightarrow 2^+{}_1$) to B(E2; $2^+{}_1 \rightarrow 0^+{}_1$) can be much smaller than 1.0 (a traditional indicator of non-collective behavior). This seemingly contradictory anomaly cannot be explained by previous nuclear structure theories [6–9]. In the Cd puzzle, experimental data failed to confirm the existence of phonon excitations in spherical nuclei [10–14], instead challenging this notion [15]. Experimentally, the B(E2) anomaly and the Cd puzzle may appear in adjacent nuclei, such as $^{72-76}$Zn [16–18], or even within a single nucleus, such as ^{114}Te [19, 20], suggesting they may share a common origin.

Moreover, Otsuka et al. discovered that nuclei previously considered prolate-shaped should actually be described as rigidly triaxial [21–23], making previous IBM descriptions less convenient. In the original IBM, the spectra of prolate (SU(3) limit) and oblate (SU(3) limit) shapes are identical [3], but this symmetry is not observed in real nuclei [24].

Recently, one of the authors (T. Wang) proposed an extension of the interacting boson model with SU(3) higher-order interactions (SU3-IBM) to resolve these various anomalies. In this model, the SU(3) symmetry plays the most important role and dominates all quadrupole deformations. Through detailed investigations, it has been found that this model can indeed better describe the collective behavior of atomic nuclei, exceeding researchers' expectations. It can describe both the B(E2) anomaly and the Cd puzzle [25, 26]. Although the B(E2) anomaly cannot be described by previous nuclear theories [6–9], numerous possibilities exist within SU3-IBM and other extended IBM theories [27–33].

In this paper, building upon previous results [27–33], we continue to investigate the mechanism underlying the B(E2) anomaly. Since many possible explanations for the B(E2) anomaly now exist, more fundamental discussions are needed to clarify key aspects. We introduce the concept of SU(3) analysis, which re-examines the results obtained in these studies within the SU(3) limit. As the coefficient of the SU(3) third-order interaction $[L \times Q \times L]^{(0)}$ evolves, we discuss the partial low-lying levels and the values of B(E2; $2^{+}{}_{1} \rightarrow 0^{+}{}_{1}$), B(E2; $4^{+}{}_{1} \rightarrow 2^{+}{}_{1}$), and B(E2; $6^{+}{}_{1} \rightarrow 4^{+}{}_{1}$). We find three new results: (1) The interaction $[L \times Q \times L]^{(0)}$ is critical for the SU(3) anomaly; (2) Level-crossing within the SU(3) limit is crucial for the SU(3) anomaly; (3) Not only the value of B(E2; $4^{+}{}_{1} \rightarrow 2^{+}{}_{1}$) but also those of B(E2; $6^{+}{}_{1} \rightarrow 4^{+}{}_{1}$) and B(E2; $2^{+}{}_{1} \rightarrow 0^{+}{}_{1}$) can be anomalous. Thus, SU(3) analysis is a useful tool for identifying the true cause, and we expect it to be employed in future discussions of the B(E2) anomaly.

II. SU3-IBM HAMILTONIAN

In the SU3-IBM, only the U(5) limit and the SU(3) limit are included. In the SU(3) limit, the SU(3) second-order Casimir operator $-\hat{C}_2[SU(3)]$ can describe the prolate shape, while the SU(3) third-order Casimir operator $\hat{C}_3[SU(3)]$ can describe the oblate shape, which fundamentally distinguishes this model from previous IBM versions [37]. The rigid triaxial shape can be obtained through combinations of the square of the SU(3) second-order Casimir operator $\hat{C}_2{}^2[SU(3)]$ with $-\hat{C}_2[SU(3)]$ and $\hat{C}_3[SU(3)]$. Moreover, the SU(3) invariants $[L \times Q \times L]^{(0)}$ and $[(L \times Q)^{(1)} \times (L \times Q)^{(1)}]^{(0)}$ are necessary.

The Hamiltonian is as follows:

$$\hat{H} = \alpha \hat{n}_d + \beta \hat{C}_2[SU(3)] + \gamma \hat{C}_3[SU(3)] + \delta \hat{C}_2^2[SU(3)] + \eta[\hat{L}\times\hat{Q}\times\hat{L}]^{(0)} + \zeta[(\hat{L}\times\hat{Q})^{(1)}\times(\hat{L}\times\hat{Q})^{(1)}]^{(0)} + \xi \hat{L}^2$$

where α, β, γ, δ, , , and are seven fitting parameters. $\hat{Q} = [d^{\dagger} \times \tilde{s} + s^{\dagger} \times \tilde{d}]^{(2)} - \frac{\sqrt{7}}{2}[d^{\dagger} \times \tilde{d}]^{(2)}$ is the SU(3) quadrupole operator. The first four interactions determine the quadrupole shapes of the nuclear ground state and the positions of the 0^{+} states in the excited levels. The latter three are dynamical interactions that can modify the features of non-0^{+} states.

To understand the B(E2) anomaly, the B(E2) values are essential. The E2 operator is defined as $\hat{T}(E2) = q\hat{Q}$, where q is the boson effective charge. The evolutions of B(E2; $2^{+}{}_{1} \rightarrow 0^{+}{}_{1}$), B(E2; $4^{+}{}_{1} \rightarrow 2^{+}{}_{1}$), and B(E2; $6^{+}{}_{1} \rightarrow 4^{+}{}_{1}$) values are discussed with various parameters deduced from [27-33].

III. SU(3) ANALYSIS

Although the B(E2) anomaly cannot be explained by previous nuclear structure theories, it can be described in many ways within the extended IBM with higher-order interactions [27-33]. A key question is whether these descriptions are related to SU(3) symmetry. Obviously, this requires considering the SU(3) limit in these explanations. For explaining the B(E2) anomaly, the boson number operator $\hat{n}_d$ is needed in some descriptions, or the SU(3) quadrupole operator $\hat{Q}$ is replaced by the generalized quadrupole operator $\hat{Q}_\chi = [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]^{(2)} + \chi[d^\dagger \times \tilde{d}]^{(2)}$. Thus, for SU(3) analysis, the $\hat{n}_d$ interaction should be removed (here $\alpha = 0$), and $\hat{Q}_\chi$ is replaced by $\hat{Q}$ again (here = -√7/2). If these operations are feasible, the SU(3) analysis exists, which is important for understanding the emergence of the B(E2) anomaly. This analysis has been ignored in previous studies, but as we will demonstrate, it is very important.

The B(E2) anomaly implies that B4/2 < 1.0 while E4/2 ≥ 2.0. These two quantities involve only the $0^+{}_1$, $2^+{}_1$, and $4^+{}_1$ states, so they are important for the success of any nuclear structure theory. This point is similar to magic numbers in atomic nuclei, which are only related to the $0^+{}_1$ states. It should be noted that this B(E2) anomaly was first observed theoretically by Y. Zhang et al. in [44], but it is not consistent with experimental data in [6–9], where the B4/2 values are much smaller than 1.0. To date, we have realized that the B(E2) anomaly is just as important a phenomenon as magic numbers. If a theory cannot explain the B(E2) anomaly, it is insufficient.

Through careful calculations, we find that the SU(3) cubic interaction $[L \times Q \times L]^{(0)}$ plays an important role in explaining the B(E2) anomaly. (Whether this relationship is unique remains unclear.) Thus, when discussing this problem, we vary the coefficient of this interaction to observe the characteristics of the B(E2) anomaly.

IV. ANALYSIS RESULTS

We now discuss the first example of SU(3) analysis. Fig. 1(a) shows the evolution of partial low-lying levels as a function of in the SU(3) limit from Ref. [27]. This represents the first successful explanation of the B(E2) anomaly in the realistic nucleus ^{170}Os. The boson number is N = 9. In [27], the parameters are α = 302.4 keV, β = -30.09 keV, γ = 3.79 keV, δ = 0.0 keV, = -10.38 keV, = 0.0 keV, and = 18.66 keV. The ground state has a prolate shape with SU(3) irrep (18,0). In that paper, the two SU(3) third-order interactions were considered, but the two fourth-order interactions were not.

From Fig. 1(a), we observe that as varies from 0 to -20.76 keV (with the midpoint being the parameter used in [27]), the line of the $4^+{}_1$ state intersects with another 4^+ state at = -6.45 keV. In [27], it was established that the 4^+ state in the SU(3) irrep (10,4) becomes lower than the 4^+ state in the SU(3) irrep (18,0). However, Fig. 1(a) also reveals new features not observed in previous studies. The $6^+{}_1$ state can intersect with another 6^+ state at = -4.25 keV, and importantly, the $2^+{}_1$ state can intersect with another 2^+ state at = -12.97 keV. Thus, new results can be identified.

In the SU(3) limit, if two states belong to different SU(3) irreps, the E2 transitions between them must be zero. Fig. 1(b) shows the evolution of B(E2; $2^+{}_1$

$\rightarrow 0^+{}_1$), B(E2; $4^+{}_1 \rightarrow 2^+{}_1$), and B(E2; $6^+{}_1 \rightarrow 4^+{}_1$) as functions of . We see that the B(E2; $4^+{}_1 \rightarrow 2^+{}_1$) anomaly (where anomaly means the E2 transition is zero) can appear when $\leq$ -6.45 keV. We also note that the B(E2; $6^+{}_1 \rightarrow 4^+{}_1$) anomaly can occur when $\leq$ -4.25 keV, and B(E2; $2^+{}_1 \rightarrow 0^+{}_1$) can occur when $\leq$ -12.97 keV. These anomalies were not mentioned in previous studies, and importantly, we find they actually exist in realistic nuclei, such as the B(E2; $6^+{}_1 \rightarrow 4^+{}_1$) anomaly in ^{72}Zn [16] and the B(E2; $2^+{}_1 \rightarrow 0^+{}_1$) anomaly in ^{166}Os [43]. We believe that the B(E2; $2^+{}_1 \rightarrow 0^+{}_1$) anomaly in ^{166}Os is critical for understanding the origin of the B(E2) anomaly in realistic nuclei.

Clearly, B(E2) analysis is a powerful technique for understanding the B(E2) anomaly. Within the SU(3) symmetry limit, level crossing phenomena can occur, induced by the $[L \times Q \times L]^{(0)}$ interaction. If two levels belong to different SU(3) irreps, the E2 transition must be zero. This case differs from the SU(3) counterpart of the rigid triaxial description found in [44].

Y. Zhang et al. continued the rigid triaxial description in [28]. Here we apply SU(3) analysis to ^{168}Os. The SU(3) counterpart of the rigid triaxial description was found in [45–47] and was subsequently used in the IBM to remove the degeneracy of the β and γ bands [48] and to describe rigid triaxial spectra [49]. A key step was Fortunato et al.' s discussion of the extended cubic Q-consistent Hamiltonian, which revealed a new evolutionary path from prolate to oblate shapes [50]. This asymmetric shape evolution was also studied analytically by Y. Zhang et al. [51]. In [44], Y. Zhang et al. first discovered the B(E2) anomaly theoretically while studying the rigid triaxial rotor, though they did not realize that realistic nuclei could actually exhibit such features. The characteristic in [44] is that as the angular momentum L in the ground band increases, the E2 transition values B(E2; $L^+{}_1 \rightarrow (L\text{-}2)^+{}_1$) decrease, but initially slowly.

For ^{168}Os, the boson number is N = 8. The parameters in Fig. 2, deduced from [28], are α = 0.022 MeV, β = 0.096 MeV, γ = 0.0027 MeV, δ = 0.0003 MeV, = 0.053 MeV, = -0.0045 MeV, and = 0.094 MeV. For SU(3) analysis, we set $\alpha = 0$. Fig. 2(a) shows the evolution of partial low-lying levels as a function of from 0 to 0.106 MeV, with the midpoint being the parameter from [28]. Clearly, the $4^+{}_1$ state cannot intersect with other 4^+ states, but we see that the $6^+{}_1$ state intersects with another 6^+ state. As shown in Fig. 2(b), the B(E2; $4^+{}_1 \rightarrow 2^+{}_1$) value can be lower than the B(E2; $2^+{}_1 \rightarrow 0^+{}_1$) value, so the B(E2) anomaly exists and results from the rigid triaxial rotor effect. However, this ratio B4/2 is 0.6, larger than the experimental value of 0.34. Whether the rigid triaxial rotor effect can produce such a small B4/2 value remains an open question for future study. Clearly, the B(E2; $6^+{}_1 \rightarrow 4^+{}_1$) anomaly results from the level crossing effect. Here the SU(3) irrep of the ground state is (4,6), so the B(E2) values are smaller than those in Fig. 1(b) when they exist.

In [28], another parameter set for ^{168}Os was presented: α = 0.099 MeV, β = 0.0504 MeV, γ = 0.0035 MeV, δ = 0.00055 MeV, = 0.043 MeV, = -0.0097 MeV, and = 0.026 MeV. For SU(3) analysis, we set $\alpha = 0$. Fig. 3(a) shows the evolution of partial low-lying levels as a function of from 0 to 0.086 MeV, with the midpoint being the parameter from [28]. Clearly, the $4^+{}_1$ state intersects with another 4^+ state at = 0.023 MeV, and the $6^+{}_1$ state intersects with another 6^+ state at = 0.0301 MeV, which is larger than the crossing point of the 4^+ states. Thus, this B(E2) anomaly results from the level crossing effect. However, these crossing effects may be very complex. Confirming the actual crossing effects in experimental results requires more experimental data. If confirmed, the emergence of the B(E2) anomaly may be an accidental effect. In Fig. 3(b), the B(E2; $4^+{}_1 \rightarrow 2^+{}_1$) anomaly and the B(E2; $6^+{}_1 \rightarrow 4^+{}_1$) anomaly can occur simultaneously. Here the SU(3) irrep of the ground state is (2,4), so the B(E2) values become smaller when they exist.

Similar results were mentioned in a previous work [5]. In the original IBM, O(5) symmetry exists when evolving from the U(5) limit to the O(6) limit, resulting in the crossing of the $0^+{}_2$ and $0^+{}_3$ states. When parameters deviate slightly, significant energy level repulsion can occur. Undoubtedly, confirming such phenomena is important. Recently, a similar result was found in the SU3-IBM [40], representing a level anti-crossing phenomenon.

Some important new results have been obtained recently. In [31], the SU(3) quadrupole operator $\hat{Q}$ is replaced by the generalized quadrupole operator $\hat{Q}_\chi$. We again take ^{168}Os as an example, with $\alpha = 0$ and other parameters β = -25.0 keV, $\gamma = 0$, $\delta = 0$, = -26.9 keV, = 0, = 43.65 keV, and = -0.39. For SU(3) analysis, we set = -√7/2. Here the SU(3) irrep of the ground state is (16,0) with prolate shape. Fig. 4(a) shows the evolution of partial low-lying levels as a function of from 0 to -53.8 keV, with the midpoint being the parameter from [31]. Intuitively, this evolution is very similar to that in Fig. 1(a). The $4^+{}_1$ state intersects with another 4^+ state at = -44.8 keV, and the $6^+{}_1$ state also intersects with another 6^+ state at = -29.4 keV. This implies that this B(E2) anomaly is also related to SU(3) symmetry. We note that at the midpoint, the B4/2 value appears normal but is near the anomalous region. This is very interesting and provides a new mechanism for the B(E2) anomaly, related to the level anti-crossing phenomenon, which will be discussed in detail in a future paper.

A similar phenomenon to that in [31] can also appear in the SU3-IBM, as found in [33]. We again take ^{168}Os as an example. Fig. 5(a) shows the evolution of partial low-lying levels as a function of from 0 to -53.8 keV, with the midpoint being the parameter from [33]. Here the SU(3) irrep of the ground state is (0,8) with oblate shape. We see that the $2^+{}_1$ and $2^+{}_2$ states intersect at = 12.1 keV, the $4^+{}_1$ and $4^+{}_2$ states intersect at = 4.6 keV, and the $6^+{}_1$ and $6^+{}_2$ states intersect at = 2.5 keV. At the midpoint, the B4/2 value is infinite. This is very interesting. When the d-boson number operator is added, the B(E2) anomaly can also be found. This is also a level anti-crossing phenomenon, which will be discussed in the future. We can confirm that this B(E2) anomaly is related to SU(3) symmetry.

These results broaden our understanding of the B(E2) anomaly. Even when the conditions that (1) B(E2; $2^+{}_1 \to 0^+{}_1$) is zero and (2) B(E2; $4^+{}_1 \to 2^+{}_1$) exists are not satisfied in the SU(3) limit, the B(E2) anomaly can still appear. If an explanation for the B(E2) anomaly can have a SU(3) symmetry limit, SU(3) analysis should be performed. From the above examples [27-33], we obtain many new results. Some phenomena appear unrelated to SU(3) symmetry but are actually connected to it. This may represent a more complicated mechanism that requires further elaboration. In SU(3) analysis, even when no level crossing occurs, the B(E2) anomaly can still happen, suggesting it may arise from the rigid triaxial rotor effect. If a B(E2) anomaly does not exist in SU(3) analysis but appears when the d-boson number operator is added (which is almost impossible) or when $\hat{Q}$ is replaced by $\hat{Q}_\chi$, this would be very important. A general discussion will be provided in the future using the extended Q-consistent Hamiltonian with up to fourth-order interactions. In a previous paper [25], one of the authors (T. Wang) proved that in this extended Hamiltonian, when = 0 (the O(6) symmetry), the B(E2) anomaly cannot occur. Thus, it is important to discuss the evolution region from the SU(3) symmetry limit to the O(6) symmetry limit.

V. CONCLUSION

In this paper, we propose a powerful technique for understanding the B(E2) anomaly: SU(3) analysis. Through discussions of examples from [27-33], many new results are presented, with three being the most important. First, the SU(3) third-order interaction $[L \times Q \times L]^{(0)}$ is critical for the SU(3) anomaly. Whether this relationship is unique requires further investigation. Second, when this interaction is included, various level crossing phenomena can occur. The causes of the B(E2) anomaly in realistic experiments require more experimental research to determine. Searching for more mechanisms for the B(E2) anomaly also seems necessary. Third, not only the value of B(E2; $4^+{}_1 \to 2^+{}_1$) but also B(E2; $6^+{}_1 \to 4^+{}_1$) and B(E2; $2^+{}_1 \to 0^+{}_1$) can be anomalous. It is very important to find more and more B(E2) anomaly results in experiments.

In these SU(3) analyses, new results can be obtained. These new mechanisms are related to SU(3) symmetry but at a more complex level. We find they may be relevant to the level anti-crossing phenomenon, which will be discussed in a subsequent paper.

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