Current Distribution in a Parallel Superconducting Circuit

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Abstract

In this research a measurement device is designed capable of measuring the distribution of superconducting currents in parallel circuit. Hall effect sensors are used to measure the magnetic field excited by the current to determine the intensity of the current. The measurement results indicate that the distribution of superconducting current in parallel circuits is inversely proportional to the self-inductance of the branch circuits.

Full Text

Preamble

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ABSTRACT

Superconductors exhibit zero electrical resistance, which raises a fundamental question: how is current distributed in a parallel superconducting circuit? In this study, we derive and experimentally verify the governing rule for current distribution in parallel superconducting circuits based on the law of conservation of magnetic flux within a superconducting loop circuit. A measurement system was designed to accurately assess the current distribution in parallel superconducting circuits. Hall effect detectors were employed to measure current intensity without damaging the zero-resistivity property of the circuits. The experimental results confirm that the current distribution in parallel superconducting circuits is inversely proportional to their inductance.

Keywords: superconductivity, hall sensor, parallel circuit, cuprate superconductors

1. INTRODUCTION

Since Onnes discovered superconductivity in 1911 [1], research in this field has become a major focus in physics. Significant progress has been made in the search for new superconductors, including copper oxide [2], iron-based [3], and nickel-based superconductors [4]. Superconducting currents exhibit many novel properties that differ fundamentally from those in the normal state. In 1933, Meissner and Ochsenfeld discovered perfect diamagnetism in superconductors [5]. In 1956, Cooper proposed that superconducting carriers are electrons bound as Cooper pairs [6]. These Cooper pairs behave as bosons, and many such pairs form a Bose-Einstein condensation at low temperatures, resulting in collective motion behavior; consequently, the superconducting current represents a macroscopic quantum phenomenon. According to the Ginzburg-Landau equation [7], superconducting current is distributed on the surface of a superconductor. These phenomena demonstrate significant differences in transmission properties between superconducting and normal-state currents.

In this study, we designed an experimental system to measure current distribution in parallel superconducting circuits. Furthermore, we discuss and experimentally verify the governing principles that dictate current distribution within these parallel superconducting circuits.

2.1 Experimental Principle

The current distribution in a parallel circuit follows the relationship $I = I_L + I_R$, where I splits into I_L and I_R (with subscripts L and R denoting the left and right circuits, respectively). Figure 1 [Figure 1: see original paper] illustrates a parallel circuit formed by a superconductor, with the inductances of the two paths denoted as L_L and L_R. A current I flows through these parallel circuits, while I_L and I_R denote the currents in the left and right paths. The variation in magnetic flux generated by currents I_L and I_R within the closed loop is given by $\Delta\Phi = L_L \cdot I_L - L_R \cdot I_R$. According to the second London equation [9], which governs the electrodynamics of superconductivity, the magnetic flux within a closed superconducting circuit remains conserved. Thus, $\Delta\Phi = 0$, leading to the relationship $I_L/I_R = L_R/L_L$ [8]. Consequently, the current distribution in parallel superconducting circuits is inversely proportional to their respective inductances.

In the normal state, the current distribution in a parallel circuit is inversely proportional to its resistance, expressed as $I_L/I_R = R_R/R_L$. To verify the relationship $I_L/I_R = L_R/L_L$ in the superconducting state, it is necessary to construct a parallel circuit in which the ratio of resistances differs from that of inductances (i.e., $R_R/R_L \neq L_R/L_L$). Subsequently, we can measure I_L/I_R in both the normal and superconducting states and compare these values with R_R/R_L and L_R/L_L.

2.2 Preparation of Samples

In accordance with the composition of $YBa_2Cu_3O_{7-\delta}$, 40 g of Y_2O_3, $BaCO_3$, and CuO raw materials were weighed, fully mixed, calcined at 1133 K for 20 h, removed, and ground into powder. Two discs, each with a mass of 15 g, were pressed, sintered at 1203 K for 20 h, and processed into two samples of different dimensions, as shown in Figure 2 [Figure 2: see original paper] (units in mm). THK denotes thickness.

For both samples, the thickness of the left semicircle was 1.36 mm, whereas that of the right semicircle was 2.34 mm. Sample 1 features a loop width of 4.38 mm; in contrast, for Sample 2, the loop width is 2.82 mm on the left semicircle and 4.38 mm on the right semicircle.

2.3 Installing the Samples

The installation setup for the samples is illustrated in Figure 3 [Figure 3: see original paper], which includes six electrodes and two Hall detectors evenly distributed around the loop circuit. For both samples, the thicknesses remain consistent: 1.36 mm for the left semicircle and 2.34 mm for the right semicircle.

3.1 Measurement of Superconducting Transition

Sample 1 was installed according to the specifications shown in Figure 3, with electrodes 2 and 6 connected to a constant-current source set at 20 mA. The voltmeter was linked to electrodes 3 and 5 to establish a four-probe measurement system. As the temperature gradually decreased, the variations in voltage between electrodes 3 and 5 were recorded, and the results are presented in Figure 4 [Figure 4: see original paper].

As shown in Figure 4, the superconducting transition occurs at a critical temperature of approximately 102 K and is completed when cooled to 98 K, where zero resistance is achieved.

3.2 Calibration of Two Hall Detectors

Subsequently, the sample temperature was increased to 108 K, placing it in the normal state. Electrodes 2 and 6 were supplied with a constant current of 180 mA. Under these conditions, both Hall detectors measured identical currents; thus, their respective Hall voltages could be utilized for calibration purposes to eliminate inconsistencies caused by factors such as mounting position and installation angle.

3.3 Measurements and Analysis

A current of 180 mA was subsequently applied between electrodes 1 and each of the other electrodes in turn. The variation in Hall voltage was measured by altering the direction of the constant current. The recorded Hall voltages were adjusted based on calibration data. The ratio of the Hall voltage in the left path (V_L) to that in the right path (V_R) was equivalent to the ratio of the current I_L to I_R. This calibration and measurement process was repeated at a temperature of 90.5 K with a constant current of 100 mA. As illustrated in Figure 4, the sample exhibited superconducting properties at this temperature.

The measurement results for Sample 1 at both 108 K and 90.5 K are presented in Table 1 and Figure 5 [Figure 5: see original paper]. R_R/R_L represents the ratio of resistance in the right path to that in the left path, whereas L_R/L_L denotes the ratio of inductance in the right path to that in the left path. Both R_R/R_L and L_R/L_L were calculated based on the electrode positions in the normal state (the methods for calculating R_R/R_L and L_R/L_L are provided in the appendix).

Table 1. The measurement data I_L/I_R for Sample 1 at temperatures of 108 K and 90.5 K, respectively.

Electrodes	I_L/I_R (108 K)	R_R/R_L	I_L/I_R (90.5 K)	L_R/L_L
1-2				
1-3				
1-4				
1-5				
1-6				

From Table 1, it is evident that the calculated values of R_R/R_L differ from those of L_R/L_L across all electrode connection modes, thereby fulfilling the requirements outlined in Section 2.1 of this article.

Figure 5 [Figure 5: see original paper] illustrates the relationships between I_L/I_R (dashed lines, measured at temperatures of 108 K and 90.5 K, respectively), R_R/R_L (solid lines), and L_R/L_L (solid lines) as functions of electrode position. Sample 1 is in the normal state at 108 K and transitions to the superconducting state at 90.5 K. Figure 5 shows that the line I_L/I_R (at 108 K) differs from the line I_L/I_R (at 90.5 K), indicating that current distribution in parallel circuits varies between the normal and superconducting states.

Figure 5 demonstrates that at 108 K, I_L/I_R aligns with R_R/R_L, which precisely corresponds to the expected current distribution in parallel circuits under normal conditions. At a temperature of 90.5 K, I_L/I_R coincides with L_R/L_L, that is, $I_L/I_R = L_R/L_L$, which is consistent with the theoretical results presented in Section 2.1 of this article.

The measurement results for Sample 2 are presented in Table 2 and Figure 6 [Figure 6: see original paper].

Table 2. The measurement data I_L/I_R for Sample 2 at temperatures of 109 K and 84 K, respectively.

Electrodes	I_L/I_R (109 K)	R_R/R_L	I_L/I_R (84 K)	L_R/L_L
1-2				
1-3				
1-4				
1-5				
1-6				

Sample 2 exhibited distinct shapes and sizes compared with Sample 1. Comparing R_R/R_L and L_R/L_L in Tables 1 and 2, it is clear that the calculated R_R/R_L and L_R/L_L ratios for Sample 1 differ from the corresponding values for Sample 2.

Figure 6 [Figure 6: see original paper] shows the relationships between I_L/I_R (dashed lines, measured at temperatures of 109 K and 84 K, respectively), R_R/R_L (solid line), and L_R/L_L (solid line) as functions of electrode position. The line I_L/I_R at 109 K deviates from the line I_L/I_R at 84 K, indicating that current distribution in parallel circuits differs between the normal and superconducting states. In the normal state at 109 K, I_L/I_R aligns with R_R/R_L, whereas in the superconducting state at 84 K, I_L/I_R corresponds to L_R/L_L. The measurement results for Sample 2 were consistent with those for Sample 1.

4 CONCLUSIONS

In this experiment, a system was designed to measure the current distribution in parallel superconducting circuits. The current distribution in a parallel circuit under normal conditions was measured, and the results agreed with $I_L/I_R = R_R/R_L$, which represents the current distribution law under normal conditions. This confirms that the designed measurement system is accurate and reliable.

Measurements of two samples with different shapes and sizes in the superconducting state show that the current distribution in parallel superconducting circuits is inversely proportional to their inductance, that is, $I_L/I_R = L_R/L_L$.

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APPENDIX

(i) Calculation of Inductance Ratio L_R/L_L

L_R/L_L can be calculated based on the electrode positions and circuit geometry. The sample was installed as shown in Figure 3. The left and right semicircles were uniform. Let the inductance per unit length in the left semicircle be L_s and that in the right semicircle be R_s. Because the six electrodes and the two Hall detectors are evenly distributed on the loop circuit, the arc length between neighboring electrodes 2, 3, and 4 can be set as L_k, the arc length between neighboring electrodes 4, 5, and 6 can be set as R_k, the arc length between electrodes 1 and 2 can be set to $2L_s$, and that between electrodes 1 and 6 can be set to $2R_s$. Taking the current flowing through electrodes 1 and 2 as examples, see $L_{LL} = L_k + 2L_s$ and $L_{RR} = R_k$. If the current flows through electrodes 1 and 5, $L_{LL} = L_k + 2L_s$ and $L_{RR} = R_k + 2R_s$. For connections between any two electrodes, L_R/L_L can be calculated in the same manner.

Because the cross-section of the circuit for both samples is a geometrically symmetrical rectangle, when calculating the magnetic field excited by the current through it, it can be assumed that the current passes through the center of the cross-section, such as a straight wire passing through the center. According to the formula for the magnetic induction intensity of an infinitely long straight wire, the magnetic induction intensity B at point A in Figure A1 is inversely proportional to the width of the cross-section, i.e., $B \propto 1/a$, so the inductance per unit length is inversely proportional to the width of the circle loops.

For Sample 1, because the widths of the left and right semicircles are the same (4.38 mm), it is reasonable to assume $L_s = R_s$. For Sample 2, the widths of the left and right semicircles are 2.82 mm and 4.38 mm, respectively, so $L_s/R_s = 4.38/2.82 = 1.55$.

The L_R/L_L values of samples 1 and 2 were calculated for various connection modes between electrodes 1 and 6. Table A1 presents the results.

Table A1. L_R/L_L values calculated for Sample 1 and Sample 2.

Electrodes	L_R/L_L Formula	L_R/L_L Sample 1	L_R/L_L Sample 2
1-2			
1-3			
1-4			
1-5			
1-6			

(ii) Calculation of Resistance Ratio R_R/R_L

Let the electrical resistance per unit length in the left semicircle be L_r and that in the right semicircle be R_r. Because the six electrodes and the two Hall detectors are evenly distributed on the loop circuit, the arc length between neighboring electrodes 2, 3, and 4 can be set as L_s, the arc length between neighboring electrodes 4, 5, and 6 can be set as R_s, the arc length between electrodes 1 and 2 can be set to $2L_s$, and that between electrodes 1 and 6 can be set to $2R_s$. Taking the current flowing through electrodes 1 and 2 as examples, see $R_{LL} = L_s + 2L_r$ and $R_{RR} = R_s$. If the current flows through electrodes 1 and 5, then $R_{LL} = L_s + 2L_r$ and $R_{RR} = R_s + 2R_r$. For connections between any two electrodes, R_R/R_L can be calculated in the same manner.

For Sample 1, $L_r/R_r = 2.34/1.36 = 1.72$. For Sample 2, $L_r/R_r = (2.34 \times 2.82)/(1.36 \times 4.38) = 1.67$.

The R_R/R_L values of samples 1 and 2 were calculated for various connection modes between electrodes 1 and 6. Table A2 presents the results.

Table A2. R_R/R_L values calculated for Sample 1 and Sample 2.

Electrodes	R_R/R_L Formula	R_R/R_L Sample 1	R_R/R_L Sample 2
1-2			
1-3			
1-4			
1-5			
1-6			

Note: Figure translations are in progress. See original paper for figures.

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