A Classical Interpretation of the Nonrelativistic Quark Potential Model: Color Charge Definition and the Meson Mass-Radius Relationship

Authors: Tan, Dr. Zhiguang, Guo, Dr. Youneng, Wang, Dr. Shengjie, ZHENG, Dr. HUA, ZHENG, Dr. HUA

Date: 2025-03-14T00:00:00+00:00

Abstract

Quantum Chromodynamics (QCD) is the fundamental theory describing quark interactions, and various quark models based on QCD have been widely used to study the properties of hadrons, including their structures and mass spectra. However, unlike Quantum Electrodynamics (QED) and the Bohr model of the hydrogen atom, there is no direct classical analogy for hadronic structures. This paper presents a classical interpretation of the nonrelativistic quark potential model, providing a more intuitive and visualizable description of strong interactions through the quantitative formulation of color charge and color flux. Furthermore, we establish the relationship between meson mass and its radius in the nonrelativistic framework and estimate the key parameters of our model using available data from several heavy mesons. We then extend this relationship to a broader range of excited meson states, obtaining structural radii that show good agreement with the root mean square (RMS) radius or charge radius predicted by QCD calculations.

Full Text

Preamble

Quantum Chromodynamics (QCD) is the fundamental theory describing quark interactions, and various quark models based on QCD have been widely used to study the properties of hadrons, including their structures and mass spectra. However, unlike Quantum Electrodynamics (QED) and the Bohr model of the hydrogen atom, there is no direct classical analogy for hadronic structures. This paper presents a classical interpretation of the nonrelativistic quark potential model, providing a more intuitive and visualizable description of strong interactions through the quantitative formulation of color charge and color flux.

Furthermore, we establish the relationship between meson mass and its radius in the nonrelativistic framework and estimate the key parameters of our model using available data from several heavy mesons. We then extend this relationship to a broader range of excited meson states, obtaining structural radii that show good agreement with the root mean square (RMS) radius or charge radius predicted by QCD calculations.

Keywords: Nonrelativistic Quark Potential Model; Color Charge; Color flux; Meson Structure; Mass spectra and radius

Introduction

When solving the Schrödinger equation $\nabla^2\Psi + \frac{2\mu}{\hbar^2}[E - V(r)]\Psi = 0$ to obtain the eigenenergies and mass spectra of a meson system composed of a quark-antiquark pair, the potential function $V(r)$ is pivotal—this is the essence of quark potential models [?, ?, ?]. Among these, the Cornell potential [?], proposed in the 1980s, has proven particularly effective and serves as the foundation for most current potential models, which incorporate various improvements or extensions [?, ?]. The Cornell potential is written as [?]:

$$V(r) = -\frac{a}{r} + br$$

where a and b are two positive parameters. The first term represents a Coulomb-like potential, while the second term accounts for quark confinement, making it extremely difficult to separate a pair of interacting quarks. Solving the Schrödinger equation with this potential to obtain mass eigenstates and quantum properties constitutes a powerful approach for studying hadron structure [?, ?]. To achieve results consistent with experimental measurements, researchers can adjust parameters or extend the potential function further. In Refs. [?, ?], the potential has been generalized to:

$$V(r) = ar^2 + br - \frac{c}{r^2} + \frac{d}{r} + e$$

However, beyond the two terms present in Eq. (2), there are no physically reasonable explanations for the origins of the additional terms in Eq. (3).

According to a classical interpretation, the quantum numbers that describe hadron properties [?] are inherently linked to their internal structure. Therefore, the interaction between quarks should be determined by their color charge values, relative positions, motion states, and spin orientations. This study aims to identify the physical origins of each term in the potential model of Eq. (3) and to provide a classical description of quark interactions.

The paper is organized as follows. In Sec. II, we introduce the concept of unit color charge and discuss the interaction between two stationary color charges in vacuum, providing the rule for the dot product of two color charges along with the basic function corresponding to the Coulomb-like term in Eq. (3). Section III explains the inverse square terms in the potential by introducing the concepts of color flow and color magnetic field. Section IV addresses the harmonic oscillator potential arising from spin, corresponding to the remaining three terms in Eq. (3). In Sec. V, we estimate the relevant model parameters and present numerical results based on our classical description of meson structures, comparing them with data from potential model calculations in the literature. Finally, Sec. VI provides a brief summary and discussion.

II. The Interaction Between a Pair of Stationary Quarks in Vacuum

To provide a classical description of the interaction between a pair of quarks, we first define three fundamental color charges c_r, c_g, c_b and their corresponding anti-color charges $\bar{c}_r$, $\bar{c}_g$, $\bar{c}_b$ as follows:

$$c_r \equiv e^{\theta i}, \quad c_g \equiv e^{(\theta+\frac{2\pi}{3})i}, \quad c_b \equiv e^{(\theta-\frac{2\pi}{3})i}$$

$$\bar{c}_r \equiv e^{(\theta+\pi)i} = -c_r; \quad \bar{c}_g \equiv e^{(\theta-\frac{\pi}{3})i} = -c_g; \quad \bar{c}_b \equiv e^{(\theta+\frac{\pi}{3})i} = -c_b$$

with $0 \leq \theta < \pi$ in the complex plane. In fact, there is only relative meaning between r, g, and b. The modulus of each color charge is 1, so it is called the unit color charge. We can also represent them as unit vectors in the unit circle, as shown in [Figure 1: see original paper].

The color charges satisfy $c_i + \bar{c}_i = 0$ and $c_r + c_g + c_b = 0$ (with $i = r, g, b$), ensuring color neutrality for mesons and baryons. Color charges are quantized, meaning that all color charges can only be integer multiples of the three unit color charges defined above. The color charges can be added together as:

$$C = \sum_{i=r,g,b} (n_i c_i + \bar{n}_i \bar{c}_i)$$

For example, a diquark composed of a red color charge and a blue color charge results in an anti-green color charge, allowing the existence of particles with color charges other than unit color charges.

The interaction between a pair of stationary color charges in vacuum is Coulomb-like and defined as:

$$F_{C_1 C^2} = \frac{Z C_1 \cdot C_2}{r^2}$$

Here, Z is called the vacuum color gravitational constant with units of $\mathrm{N} \cdot \mathrm{m}^2$. The dot product between two unit color charges satisfies:

$$c_i \cdot (\pm c_j) = \begin{cases} \pm 1, & i = j \\ \mp \frac{1}{2}, & i \neq j \end{cases}$$

As shown in [Figure 2: see original paper], one can verify that the total interaction between a blue quark (or antiquark) and a three-quark cluster (with total color charge zero) is zero: $\sum_{i=r,g,b} \pm c_b \cdot c_i = 0$.

III. The Color Magnetic Field from the Motion of Color Charge

The collective motion of color charges generates color flow. Analogous to electrical current intensity, the current intensity of color flow is defined as the amount of color charge flowing through a cross-section per unit time:

$$I_c = \frac{|C|}{t} = \frac{|\sum_{i=r}^{b} (n_i - \bar{n}_i) c_i|}{t}$$

We assume that color flow can generate a color magnetic field, modeled after the Biot-Savart law:

$$\mathbf{B}_c = \frac{T}{4\pi} \int \frac{I_c d\mathbf{l} \times \mathbf{r}}{r^3}$$

Here, T is a vacuum parameter whose value must be measured directly or indirectly through experiments.

Consider the color magnetic field generated by a circular color flow. As shown in [Figure 3: see original paper], assuming the radius of the circular color current is a and the color current intensity is I_c, the following formulas can be derived by analogy with the magnetic field generated by a circular current [**?**]:

$$B_{cx} = T I_c a r \cos\theta \int_0^{2\pi} \frac{\cos\phi \, d\phi}{(r^2 + a^2 - 2ra \sin\theta \cos\phi)^{3/2}}$$

$$B_{cy} = 0$$

$$B_{cz} = T I_c \int_0^{2\pi} \frac{a^2 - ar \sin\theta \cos\phi \, d\phi}{(r^2 + a^2 - 2ra \sin\theta \cos\phi)^{3/2}}$$

For points on the color flow plane, $\theta = \pi/2$, $\sin\theta = 1$, and $\cos\theta = 0$, therefore:

$$B_{cx} = B_{cy} = 0$$

$$B_{cz} = TI_c \int_0^{2\pi} \frac{a^2 - ar\cos\phi}{(r^2 + a^2 - 2ra\cos\phi)^{3/2}}\, d\phi$$

When taking infinity as the zero point of color potential energy, the color potential energy between two color charges can be calculated by:

$$E_p = -\int_\infty^r \frac{F_{C_1 C^2}}{r^2}\, dr = -\frac{ZC_1 \cdot C_2}{r}$$

When $C_1 = -C_2$ and $|C_1| = 1$, this result corresponds to the third term in Eq. (3)—the Coulomb-like term.

The integral for B_{cz} can be expressed using elliptic functions:

$$B_{cz} = \frac{2TI_c}{a-r}\left[\frac{1}{r}E(k) + \frac{1}{a+r}K(k)\right] = 2TI_c X(a,r)$$

where $E(k)$ and $K(k)$ are the elliptic functions ellipticE and ellipticK, respectively, with $k = \frac{2\sqrt{ar}}{a+r}$. The color magnetic field diverges on the color flow circular line, as shown in [Figure 4: see original paper].

Through segmented fitting for different radii of circular currents, we obtain a simple explanatory formula for the inner and outer color magnetic fields:

For $a = 0.5$: $X_i = 1.2215 + 9.9248(a-r)$, $X_o = -0.8433 + 5.0440(r-a)$

For $a = 0.6$: $X_i = 1.2001 + 7.0576(a-r)$, $X_o = -0.8593 + 3.6412(r-a)$

For $a = 0.7$: $X_i = 1.1833 + 5.2816(a-r)$, $X_o = -0.8717 + 2.7549(r-a)$

For $a = 0.8$: $X_i = 1.1696 + 4.1042(a-r)$, $X_o = -0.8817 + 2.1587(r-a)$

For $a = 0.9$: $X_i = 1.1583 + 3.2828(a-r)$, $X_o = -0.8900 + 1.7381(r-a)$

For $a = 1.0$: $X_i = 1.1487 + 2.6866(a-r)$, $X_o = -0.8970 + 1.4301(r-a)$

Here, X_i and X_o represent the values of $B_{cz}/2TI_c$ inside and outside the circle, respectively.

The color magnetic field energy stored in a color flow ring can be calculated as:

$$E_{Bc} = \int B_c \cdot dS = TI_c^2 \int_0^a X_i^2 2\pi r\, dr$$

Since the value of T has not been determined yet, [Figure 5: see original paper] shows the calculated values and fitting relationship of E_{Bc}/TI_c^2 versus a, yielding:

$$\frac{E_{Bc}}{TI_c^2} = 22.97a$$

We can now apply this scenario to mesons. When one quark orbits another in circular motion with radius r, its equivalent color flow intensity is $I_c = |c|/t$. Considering the centripetal force provided by the color charge force:

$$\frac{mv^2}{r} = \frac{Z}{r^2}$$

The color magnetic energy stored in the meson is:

$$E_{Bc} = 22.97TI_c^2 r = 0.5818\frac{Tm}{r^2}$$

This represents the fourth term in the potential energy Eq. (3), which is inversely proportional to the square of the distance.

IV. The Harmonic Oscillator Potential Originating from Spin

In quantum mechanics, spin is an intrinsic property of particles. However, in classical terms, we propose that particle spin is an external manifestation of its internal structure. We assume that the quark color charge undergoes circular motion around its own central axis, equivalent to a circular color flow ring, thereby possessing a color magnetic moment. For a meson system composed of a quark-antiquark pair, as shown in [Figure 6: see original paper], the color magnetic moments of the two quarks must be coplanar due to the effect of chromomagnetic torque. Consequently, their spin orientations can only have two states: parallel or antiparallel. During their respective rotations, when the directions of the two color flows are parallel, the spin interaction is attractive; when antiparallel, the interaction is repulsive. If the color flow directions are perpendicular, no force acts between them.

Therefore, spin-induced interactions can be described by a harmonic oscillator, whose dynamic equilibrium position lies along the circumference of the quark’ s orbital motion:

$$F_{S_{1S^2}} = -k(r' - r) = -m\omega^2(r' - r)$$

Here, k is the elastic coefficient and ω is the angular frequency. The total energy of the oscillator is:

$$E_s = \frac{1}{2}kA^2 = \frac{1}{2}k(r_m - r)^2$$

Expanding the right-hand side yields the remaining three terms of Eq. (3). Due to the extremely small size of $r_m - r$ (comparable to the quark radius), the vibrational energy is expressed using the quantum mechanical harmonic oscillator energy formula:

$$E_s = \left(L + \frac{1}{2}\right)\hbar\omega_{nL}, \quad (L = 0, 1, 2, \cdots, n-1)$$

The top row in [Figure 7: see original paper] represents the state where two quarks have parallel spins, while the bottom row shows the antiparallel state ($S_1 + S_2 = 0, 1$). For a meson system to be stable, the period of quark circular motion $T_\theta = 2\pi r/v$ must be an odd multiple (for $S = 0$) or even multiple (for $S = 1$) of the vibration half-period:

$$\frac{2\pi r_{nL}}{v_{nL}} = \begin{cases} (2n-1)\frac{\pi}{\omega_{nL}} & (S=0) \\ 2n\frac{\pi}{\omega_{nL}} & (S=1) \end{cases}, \quad n = 1, 2, \cdots$$

According to Eq. (20), one obtains:

$$\omega_{nL} = \begin{cases} \frac{(2n-1)Z}{2r_{nL}^2}\sqrt{\frac{M_Q M_{\bar{q}}}{M_Q + M_{\bar{q}}}} & (S=0) \\ \frac{nZ}{r_{nL}^2}\sqrt{\frac{M_Q M_{\bar{q}}}{M_Q + M_{\bar{q}}}} & (S=1) \end{cases}$$

Note that ω_{nL} here does not represent the angular velocity of orbital motion, i.e., $\omega_{nL} \neq v_{nL}/r_{nL}$.

V. The Values of Z and T and Results

Due to color confinement, it is impossible to directly measure the values of Z and T by measuring forces between free quarks or color currents. Instead, we can estimate their values using experimentally measured masses and radii of certain hadrons.

In the center-of-mass frame of a meson, the mass can be calculated as [**?**, **?**]:

$$M_{nL} = M_Q + M_{\bar{q}} + E_{nL}$$

Here, E_{nL} includes the previously mentioned E_p, E_{Bc}, E_s, and the kinetic energy $E_k = \frac{1}{2}\mu v^2 = \frac{Z}{2r}$, where the reduced mass is $\mu = \frac{M_Q M_{\bar{q}}}{M_Q + M_{\bar{q}}}$. Therefore, for a meson in a state with quantum numbers n and L, the classical calculation of its mass can be expressed as:

$$M_{nL} = M_Q + M_{\bar{q}} - \frac{Z}{r_{nL}} + \frac{0.5818T}{r_{nL}^2} + \left(L + \frac{1}{2}\right)\hbar\omega_{nL} + \frac{Z}{2r_{nL}}$$

According to Refs. [?, ?], the root mean square radius of the $\Upsilon(1S)$ state is approximately 0.2671 fm. Ref. [?], through a comprehensive contact interaction analysis, determined that the ground state charge radius of the pseudoscalar meson η_b is about 0.07 fm. We use data from these two mesons to determine the parameters Z and T. The meson masses are obtained from the Particle Data Group (PDG), while the constituent b quark mass is taken as $m_b = 4.95$ GeV, consistent with Refs. [?, ?]. These data are listed in .

The obtained values are:

$$Z = 9.46, \quad T = 0.615$$

Converting to International System of Units:

$$Z = 2.98 \times 10^{-25}\,\mathrm{N \cdot m^2}, \quad T = 1.13 \times 10^{-43}\,\mathrm{N \cdot s^2}$$

For the ground state radius r_{10} and mass M_{10}:

$$M_{10} = M - \frac{Z}{r_{10}} + \frac{0.5818T}{r_{10}^2} + \frac{1}{4}\frac{Z}{r_{10}}$$

Comparing the magnitudes of forces between a quark-antiquark pair separated by 1 fm (with $m_b = m_{\bar{b}} = 4.95$ GeV, $q_b = -q_{\bar{b}} = -\frac{1}{3}e$):

$$F_m = G\frac{m_b m_{\bar{b}}}{r^2} = 5.16 \times 10^{-33}\,\mathrm{N}$$

$$F_e = k\frac{q_b q_{\bar{b}}}{r^2} = 2.56 \times 10\,\mathrm{N}$$

$$F_c = Z\frac{c_i \bar{c}_i}{r^2} = 8.17 \times 10^4\,\mathrm{N}$$

where the natural unit $\hbar = 0.197\,\mathrm{GeV \cdot fm} = 1$ has been used. The strong interaction based on color charge is much greater than the other two, consistent with the known hierarchy of force magnitudes.

The values of Z and T are universal, allowing us to use a small amount of experimental data to deduce them and then calculate results for other mesons to test our model. It is important to note that our discussion has not considered relativistic effects, which may require corrections for light meson systems. For heavy mesons, relativistic effects are likely less significant, so we use heavy meson data to determine Z and T.

Unfortunately, due to current experimental limitations, directly measuring meson radii is very challenging. However, model calculations of meson sizes have garnered significant scientific interest. Currently, two important radii are used to describe meson systems: the root mean square (r.m.s.) radius $\langle r^2\rangle_{\rm rms}$ [?, ?] and the charge radius $\langle r^2\rangle_E$ [?], defined as:

$$\langle r^2\rangle_{\rm rms} = \int_0^\infty r^2[\psi(r)]^2 dr$$

$$\langle r^2\rangle_E = -6\frac{dF(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

where $\psi(r)$ is the radial wave function and $F(Q^2)$ is the meson form factor [?, ?].

Using meson masses from the Particle Data Group (PDG) [?] as inputs and applying Eq. (29), we calculate the corresponding meson radii and compare them with results from other models, as shown in . The table demonstrates that some of our results are in good agreement with literature values. It must be noted that existing literature on meson radius calculations employs various models [?, ?, ?, ?, ?], each with its own parameters, and most focus only on the lowest few states. Our model also relies on data from a few mesons, which is necessary due to the current inability to experimentally measure Z and T. Once these values are scientifically determined, a predictable relationship between meson mass and structural radius can be established. As shown in [Figure 8: see original paper], we present the mass-radius relationship for several quantum states of $b\bar{b}$ mesons using Eq. (29), which is of significant importance for understanding hadron structure.

VI. Summary

In this article, we propose a classical interpretation based on color charge interactions for quark pairs. Through the design of superposition and dot product rules for quark color charges, we predict the existence of non-unit color charge elementary particles. Analogous to electromagnetic fields, we introduce the concepts of color flow, color field, and color magnetic field. Model parameters Z and T are estimated using known data from a few heavy mesons. The effectiveness of our model has been validated by comparing our results with literature data, which show good agreement. However, precise calculation of the energy spectrum structure of hadrons relies on Quantum Chromodynamics. We expect that the classical approach presented in this article provides a simple and visualizable method to study the interior of microscopic particles, analogous to the Bohr model of the hydrogen atom in quantum mechanics.

Acknowledgments

This work was supported in part by the Department of Education (grant No. 21A0541) and Natural Science Foundation (grant No. 2025JJ50382) of Hunan province, China.

References

[1] C. Semay, B. Silvestre-Brac, Potential models meson https://doi.org/10.1016/S0375-9474(97)00060-2 Phys. A 618, spectra. Nucl. (1997).

[2] K. Watanabe, Quark-diquark potential and diquark mass from lattice QCD. Phys. Rev. D 105, 074510 (2022). https://doi.org/10.1103/PhysRevD.105.074510

[3] Z. Zhao, K. Xu, A. Limphirat et al., Mass spectrum of 1—— heavy quarkonium. Phys. Rev. D 109, 016012 (2024). https://doi.org/10.1103/PhysRevD.109.016012

[4] E. Eichten, K. Gottfried, T. Kinoshita et al., Charmonium: comparison with experiment. Phys. Rev. D 21, 203 (1980). https://doi.org/10.1103/PhysRevD.21.203

[5] A.I. Ahmadov, K.H. Abasova, M.S. Orucova, Bound state solution schr¨odinger equation for extended cornell potential at finite temperature. Advances in High Energy Physics 2021, 1861946 (2021). https://doi.org/10.1155/2021/1861946

[6] L.P. Fulcher, Z. Chen, K.C. Yeong, Energies of quark- antiquark systems, the Cornell potential, and the spin- less Salpeter equation. Phys. Rev. D 47, 4122 (1993). https://doi.org/10.1103/PhysRevD.47.4122

[7] F.X. Liu, R.H. Ni, X.H. Zhong et decays al., Charmed- strange poten- their tial quark model. Phys. Rev. D 107, 096020 (2023). https://doi.org/10.1103/PhysRevD.107.096020 tetraquarks

[8] T.Y. Li, L. Tang, Z.Y. Fang et al., Higher states of the Bc meson family. Phys. Rev. D 108, 034019 (2023). https://doi.org/10.1103/PhysRevD.108.034019

[9] V. Kumar, R.M. Singh, S.B. Bhardwaj et al., Analytical so- lutions to the Schr¨odinger equation for a generalized Cor- nell potential and its applications to diatomic molecules and heavy mesons. Mod. Phys. Lett. A 37, 2250010 (2022). https://doi.org/10.1142/S0217732322500109

[10] M. Abu-Shady, E.M. Khokha, Bound state solutions of the generalized Cornell poten- Int. J. Mod. Phys. A 36, 2150195 (2021). the Dirac equation for tial model. https://doi.org/10.1142/S0217751X21501955

[11] Z.G. Tan, S.J. Wang, Y.N. Guo et al., A novel encoding mech- anism for particle physics. Nucl. Sci. Tech. 35, 144 (2024). https://doi.org/10.1007/s41365-024-01537-8

[12] J.M. Griffith, G.W. Pan, Time harmonic fields produced by circular current loops. IEEE transactions on magnetics 47, 2029 (2011). https://doi.org/10.1109/TMAG.2011.2132731

[13] M. Abu-Shady, T.A. Abdel-Karim, S. Y. Ezz-Alarab, Masses and thermodynamic properties of heavy mesons in the non-relativistic quark model using the Nikiforov- Uvarov method. J. Egyptian Math. Soc. 27, 14 (2019). https://doi.org/10.1186/s42787-019-0014-0

[14] R. Rani, S.B. Bhardwaj, F. Chand, Mass spectra of heavy and light mesons using asymptotic iteration method. Commun. Theor. Phys. 70, 179 (2018). https://doi.org/10.1088/0253-6102/70/2/179

[15] N. Akbar, B. Masud, S. Noor, Wave-function-based charac- teristics of hybrid mesons. Eur. Phys. J. A 47, 124 (2011). https://doi.org/10.1140/epja/i2011-11124-2

[16] A.M. Yasser, G.S. Hassan, T.A. Nahool, A study of some properties of bottomonium. Journal of Modern Physics 5, 1938 (2014). http://dx.doi.org/10.4236/jmp.2014.517188

[17] K.U. Can, G. Erkol, M. Oka et al., Vector and axial- in 2+1 fla- 103 (2013). vector couplings of D and D∗ mesons lattice QCD. Phys. Lett. B 719, https://doi.org/10.1016/j.physletb.2012.12.050

[18] D.P. Stanley, D. Robson, Nonperturbative potential model for light and heavy quark-antiquark systems. Phys. Rev. D 21, 3180 (1980). https://doi.org/10.1103/PhysRevD.21.3180

[19] H.T. Ding, X. Gao, A.D. Hanlon et al., QCD Pre- Form Factors Electromagnetic dictions at High Momenta: Testing Factorization in Exclusive Processes. Phys. Rev. Lett. 133, 181902 (2024). https://doi.org/10.1103/PhysRevLett.133.181902 for Meson

[20] C.W. Hwang. Charge Phys. mesons, https://doi.org/10.1007/s100520200904 radii light heavy (2002).

[21] T. Das, Treatment of N -dimensional Schr¨o dinger equation for anharmonic potential via Laplace transform. arXiv:1408.6139. https://doi.org/10.48550/arXiv.1408.6139

[22] R.J. Hern´andez-Pinto, L.X. Guti´errez-Guerrero, A. Bashir et al., Electromagnetic form factors and charge radii of pseudoscalar and scalar mesons: A comprehensive con- tact interaction analysis. Phys. Rev. D 107, 054002 (2023). https://doi.org/10.1103/PhysRevD.107.054002

[23] D. Ebert, R.N. Faustov, V.O. Galkin, Spectroscopy and Regge trajectories of heavy quarkonia and Bc mesons. Eur. Phys. J. C 71, 1825 (2011). https://doi.org/10.1140/epjc/s10052-011-

[24] L. Adhikari, Y. Li, M. Li et al., Form factors and gen- eralized parton distributions of heavy quarkonia in basis light front quantization. Phys. Rev. C 99, 035208 (2019). https://doi.org/10.1103/PhysRevC.99.035208

[25] T. Das, D.K. Choudhury, K.K. Pathak, RMS and charge radii in a potential model. Indian J. Phys. 90, 1307 (2016). https://doi.org/10.1007/s12648-016-0866-1

[26] T. Das, K.K. Pathak, D.K. Choudhury, Three-Loop Effect in rms and Charge Radii of Heavy Flavored Mesons in a QCD Potential Model. Int. J. Theor. Phys. 63, 172 (2024). https://doi.org/10.1007/s10773-024-05697-6

[27] (Particle Data Group) C. Patrignani, K. Agashe, G. Aielli et al., Review of Particle Physics. Chin. Phys. C 40, 100001 (2016). https://iopscience.iop.org/article/10.1088/1674-1137/40/10/100001

[28] A. H¨oll, A. Krassnigg, P. Maris et al., Electromag- netic properties of ground-state and excited-state pseu- doscalar mesons. Phys. Rev. C 71, 065204 (2005). https://doi.org/10.1103/PhysRevC.71.065204

[29] A.F. Krutov, R.G. Polezhaev, V.E. Troitsky, Radius of the meson determined from its decay constant. Phys. Rev. D 93, 036007 (2016). https://doi.org/10.1103/PhysRevD.93.036007

Note: Figure translations are in progress. See original paper for figures.