The technique for extracting initial parameters of longitudinal phase space of freshly injected bunches in storage rings and its applications

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Abstract

This paper presents a technique for extracting the initial parameters of the longitudinal phase space of freshly injected bunches in an electron storage ring. This technique combines the development of a single-bunch injection phase space simulation software with the establishment of a bunch-by-bunch data acquisition and processing system, enabling high-precision acquisition of the initial parameters of injected bunches during the injection process into the electron storage ring (including initial phase, initial bunch length, initial energy offset, initial energy spread, and initial energy chirp). The experiment utilizes a high-speed oscilloscope to capture the beam injection signals, which are then processed by a data processing script to calculate and extract the phase and bunch length evolution information of the injected bunches. The data acquisition length covers several thousand turns per capture, with a phase measurement accuracy of 0.2 ps and a bunch length measurement accuracy of 1 ps. Additionally, a single-bunch simulation software based on the mbtrack2 and PyQt5 packages has been developed. This software can simulate the phase space evolution of bunches under different initial conditions after injection. By employing a genetic algorithm and integrating experimental data with simulation data, it can obtain the optimized initial parameters of the injected bunches.

Full Text

Preamble

A Technique for Extracting Initial Parameters of the Longitudinal Phase Space of Freshly Injected Bunches in Storage Rings and Its Applications

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This paper presents a novel technique for extracting the initial parameters of the longitudinal phase space of freshly injected bunches in electron storage rings. The method integrates the development of single-bunch injection phase space simulation software with a bunch-by-bunch data acquisition and processing system, enabling high-precision measurement of injected bunch parameters—including initial phase, initial bunch length, initial energy offset, initial energy spread, and initial energy chirp—during the injection process. The experimental system employs a high-speed oscilloscope to capture beam injection signals, which are then processed by a dedicated data processing script to extract phase and bunch length evolution information. Each acquisition covers several thousand turns, achieving a phase measurement accuracy of 0.2 ps and a bunch length measurement accuracy of 1 ps. Additionally, we have developed single-bunch simulation software based on the mbtrack2 and PyQt5 packages, which can simulate phase space evolution under various initial conditions. By employing a genetic algorithm to integrate experimental and simulation data, the technique successfully obtains optimized initial parameters for injected bunches.

Keywords: injection transient, phase measurement, bunch length measurement, genetic algorithm

Introduction

In advanced synchrotron light sources, the injection process in electron storage rings represents a critical factor influencing beam quality and stability. Optimizing this process can significantly reduce beam loss, enhance injection efficiency, and minimize experimental interference \cite{1,2,4?–12}. Consequently, in-depth research and systematic optimization of electron storage ring injection processes hold substantial theoretical and practical importance.

Transverse injection technology has matured considerably in recent years. For instance, researchers at the University of Guilan proposed a multi-turn transverse injection scheme for a 3.5 GeV synchrotron storage ring [**?**], utilizing short-pulse nonlinear kickers to achieve efficient beam injection. Their work focused on designing injection systems (including septa and pulsed sextupole magnets) to enable on-axis injection, thereby improving storage ring stability and reducing interference with the stored beam. Through simulations and theoretical analysis, this approach demonstrated advantages over traditional off-axis injection, including reduced beam emittance growth and improved injection efficiency. Similarly, PLS-II (Pohang Light Source-II) employed the particle tracking software ELEGANT to simulate and analyze transient behaviors in both transverse and longitudinal phase spaces during beam injection [?]. By studying the distribution of injected beams in these phase spaces, they characterized post-injection beam behavior and assessed how system errors affect capture efficiency, thereby determining optimal injection conditions and guiding practical system commissioning.

Despite these advances, current diagnostic methods face significant limitations. The arrival time and bunch length of freshly injected bunches are typically measured using streak cameras [?], which can capture longitudinal bunch size changes but cannot simultaneously provide high temporal resolution and large dynamic range. Moreover, existing tools struggle to directly and accurately measure critical parameters such as central energy, energy spread, and energy chirp [?, ?]. These parameters are essential for injection optimization and light source performance enhancement, yet effective experimental methods for their precise characterization remain lacking [?].

Addressing these challenges, this paper proposes a novel technique for extracting the initial longitudinal phase space parameters of freshly injected bunches in electron storage rings. The method employs a high-speed oscilloscope to capture beam signals during injection, combined with advanced data processing algorithms to obtain phase and bunch length evolution within a single injection cycle. By integrating this approach with newly developed single-bunch tracking simulation software based on the mbtrack2 and PyQt5 packages, the technique achieves precise measurement of initial energy offset, energy spread, and energy chirp while enabling predictive simulation of beam behavior in the storage ring. This advancement provides a powerful tool for comprehensive injection process optimization.

II. Longitudinal Beam Dynamics in Storage Rings

In particle storage rings, charged particles circulate as bunches that fill different buckets. The longitudinal motion of a bunch is described by position L (or phase ϕ) and momentum p. For non-equilibrium particles with coordinates L and p, relative to equilibrium particle coordinates L_0 and p_0, the following relationship holds:

$$\begin{pmatrix} \Delta L \\ \Delta p \end{pmatrix} = \begin{pmatrix} \eta \\ \rho \end{pmatrix} \frac{\Delta p}{p_0}$$

where α_c is the momentum compaction factor, ΔL is the position difference from the equilibrium particle, Δp is the momentum difference, η is the dispersion function, and ρ is the curvature radius.

As particles circulate, they gain energy from the RF cavity's electric field and lose energy through synchrotron radiation in bending magnets, establishing dynamic equilibrium. The total energy change is:

$$\Delta E = qV(\psi) - U(E)$$

where q is particle charge, V is RF cavity voltage, ψ is accelerating phase, and $U(E)$ is energy-dependent synchrotron radiation loss.

In most practical applications, velocity and energy changes during acceleration are slower than phase changes, allowing them to be treated as constants. Under the small-deviation approximation, the particle phase motion equation in the RF field becomes:

$$\ddot{\phi} + 2\alpha_z\dot{\phi} + \Omega^2\phi = 0$$

Solving this yields the phase evolution:

$$\phi = \hat{\phi}e^{-\alpha_z t}\cos(\Omega t + \psi)$$

Similarly, solving the energy motion equation gives the energy oscillation:

$$\delta = -\frac{h\omega_0\eta_c}{\Omega}\hat{\phi}e^{-\alpha_z t}\sin(\Omega t + \psi)$$

where δ is particle energy offset. A typical particle trajectory in longitudinal phase space is shown in Fig. 1 [Figure 1: see original paper], demonstrating exponentially damped harmonic oscillations in both phase and energy.

The damping factor is $\alpha_z = -\frac{1}{2}\frac{dU}{dE}$, and the synchrotron oscillation frequency is [**?**]:

$$\Omega = \sqrt{\frac{h\eta_c e\hat{V}_0\cos\psi_s}{2\pi\beta c p_0 T_0}}$$

where T_0 is revolution period, ω_{rev} is revolution frequency, h is harmonic number, and ψ_s is synchronous phase. In circular accelerators, particles periodically encounter a locally synchronized accelerating field, oscillating around the synchronous phase at frequency Ω. Multiple RF cavities provide the necessary field, with $V_0\cos\psi_s$ representing the total accelerating voltage per revolution. The RF frequency is an integer multiple of the revolution frequency, requiring strict synchronization between RF system design and particle motion to ensure efficient energy gain. Precise control of RF cavity voltage and phase maximizes acceleration efficiency while regulating bunch phase stability.

III. Simulation Development and Single-Bunch Tracking

The preceding analysis shows that individual particles undergo exponentially damped harmonic oscillations in phase space during storage ring injection, with similar behavior for particle bunches. However, since each particle within a bunch has different initial phase and energy, analyzing the overall bunch motion becomes highly complex. Generally, the bunch distribution in phase space can be approximated as a two-dimensional Gaussian, as shown in Fig. 2 [Figure 2: see original paper], which simplifies analysis of collective motion.

Theoretically, for a Gaussian-distributed bunch, measuring the phase oscillation and bunch length evolution curves after injection enables reconstruction of the initial two-dimensional Gaussian distribution at injection. This includes critical parameters: initial phase, initial bunch length, initial energy offset, initial energy spread, and initial energy chirp—all essential for injection system optimization.

Based on this theoretical foundation, we developed a single-bunch simulation tool using the mbtrack2 and PyQt5 packages \cite{20-22} to investigate longitudinal phase space evolution during injection. The software interface, shown in Fig. 3 [Figure 3: see original paper], is organized into distinct functional areas: the left visualization area displays longitudinal phase space distribution (top-left), phase evolution (middle-left), and bunch length evolution (bottom-left) for specified turn numbers; the right parameter input area (top-right) controls initial bunch length, energy spread, phase, energy offset, and energy chirp; the bottom-right operation area manages turn selection and displays current values; and the bottom section contains controls for data/image saving and simulation execution. The software supports external machine parameter files, enabling theoretical adaptation to any accelerator.

All simulations employ a single-bunch model with 10,000 particles tracked for 3,000 turns (adjustable parameters). In mbtrack2, a particle is described by six-dimensional phase space coordinates $(x, x', y, y', \tau, \delta)$, where x and y are horizontal and vertical positions, x' and y' are transverse momenta, τ is time difference relative to the reference particle, and $\delta = (E - E_0)/E_0$ is relative energy offset. For $\tau > 0$, the particle is delayed relative to the reference. The longitudinal motion iteration for τ and δ under RF cavity and synchrotron radiation damping effects follows:

$$\tau_{n+1} = \tau_n + \eta T_0 \delta_n$$

$$\delta_{n+1} = \delta_n - \frac{U_0}{E_0}\left(1 - \frac{2T_0}{\tau_\delta}\right)\delta_n + \frac{2\sigma_\delta}{\sqrt{T_0}}\cos(h\omega_{rev}\tau + \phi_s)\epsilon$$

where T_0 is a time constant, τ_δ is a δ-related time parameter, σ_δ is the standard deviation of δ, and ϵ is a normally distributed random number simulating synchrotron radiation stochastic processes. This formulation captures energy dissipation and noise effects on particle motion.

Fig. 4 [Figure 4: see original paper] illustrates longitudinal phase space evolution under specific initial conditions, with subfigures showing distributions at different turns. The X-axis represents phase time offset τ and Y-axis represents energy offset δ. Early evolution shows gradual bunch bending and noticeable tail filamentation, indicating nonlinear effects. Intermediate stages exhibit further bending and stretching into complex structures from combined synchrotron oscillation and radiation damping effects. Later stages show stabilization into compact distributions as the system reaches equilibrium with weakened nonlinearities and balanced internal energy exchange. This progression from initial perturbation to stabilization reflects both nonlinear dynamics and self-stabilizing capacity in electron storage rings.

Additional simulations explore typical variations in initial distributions. Table 1 lists the initial phase space parameters, while Fig. 5 [Figure 5: see original paper] shows initial distributions and reconstructed bunch length-phase two-dimensional distributions. In these distributions, the X-axis represents time, Y-axis represents bunch charge density (color-coded from blue to yellow for low to high density), reflecting bunch center position and length—similar to streak camera measurements. Energy chirp represents intra-bunch energy variation over time. Zero chirp yields a regular elliptical phase space distribution with equal head/tail energies, while non-zero chirp produces tilted distributions. We define this tilt angle with values between -1 and 1 to quantify chirp.

Fig. 5.d shows a realistic injection scenario with non-zero initial phase, energy offset, and energy chirp, exhibiting a "winged" shape near phase oscillation extrema where bunch length reaches maxima. These results demonstrate that different initial distributions produce distinct post-injection evolution patterns, enabling the simulation tool to model various injection scenarios through parameter adjustment.

IV. Storage Ring Bunch-by-Bunch Phase and Length Measurement System

The simulation tool establishes the relationship between injected bunch initial parameters and post-injection phase space evolution. By analyzing experimental bunch length and phase evolution data, we can determine these initial parameters. While streak cameras are the primary method for measuring phase and bunch length in storage rings, they cannot simultaneously provide high temporal resolution and large dynamic range, limiting data acquisition to the first few tens of post-injection turns—insufficient for comprehensive analysis.

To address this limitation, we developed a diagnostic system for individual bunches that computes three-dimensional position, charge, bunch length, and other parameters from raw BPM data [**?**, **?**]. Each acquisition covers several thousand turns with phase measurement accuracy of 0.2 ps and bunch length accuracy of 1 ps. The complete system comprises data acquisition and offline processing components, as shown in the block diagram in Fig. 6 [Figure 6: see original paper].

The data acquisition component uses a high-speed oscilloscope (6 GHz bandwidth, 16 GHz sampling rate, 16 bits) to collect button electrode coupling signals. Each acquisition lasts several milliseconds, covering thousands of revolution periods. For the Shanghai Synchrotron Radiation Source (SSRF), the RF frequency is 499.5 MHz with approximately 2 ns bunch signal length, allowing 32 sampling points per bunch for accurate waveform reconstruction. Offline scripts then process the collected data.

The phase extraction algorithm is illustrated in Fig. 7 [Figure 7: see original paper]. Accounting for bunch length and charge effects, we construct a response function for each bunch (red signal in Fig. 7.a shows a single-turn BPM signal; blue shows the constructed response function). A lookup table is established based on this function, traversing all phase values in 0.1 ps steps. The correlation function method then identifies the table element with highest correlation to measured data [**?**, **?**], with the bunch signal zero-crossing defining that turn' s phase.

The bunch length calculation process is shown in Fig. 8. Due to mismatches between system sampling rate and storage ring RF frequency, we first determine single-bunch signal length T from measured BPM signals. The original one-dimensional BPM signal array is then sliced and restructured into a three-dimensional array (bunch signal slice × total turns × harmonic number) based on T. Fig. 8.a shows a particular bunch' s signal waveform in a specific turn. Harmonic analysis of the sliced waveform yields the signal spectrum (Fig. 8.b), which is multiplied by the system transfer impedance (Fig. 8.c) to produce a Gaussian-distributed spectrum for fitting. The transfer impedance is calibrated using a streak camera with a fixed coefficient for consistent acquisition systems. Gaussian fitting is applied to the calibrated spectrum, where the reciprocal of the fitted frequency-domain distribution σ represents the time-domain bunch length for that bunch in that turn [**?**]. Table 2 lists SSRF' s main machine parameters.

V. Beam Experiment

We collected multiple datasets under vacuum injection conditions in the SSRF storage ring. When beam injection occurs, the oscilloscope captures a trigger signal and records data at regular intervals. Through our bunch-by-bunch diagnostic system script, we obtained longitudinal phase and bunch length evolution for a single bunch over several thousand turns during injection. A typical result is shown in Fig. 9 [Figure 9: see original paper].

Fig. 9.a shows phase evolution during injection, following a nearly exponentially decaying sinusoidal oscillation that matches theoretical predictions well. The maximum amplitude remains below 100 ps, depending on injector beam energy,

injection timing, and initial phase-space distribution. After 7-8 milliseconds, oscillation amplitude decays to within 20 ps. Fig. 9.b shows bunch length evolution, with the upper envelope also exhibiting exponential decay. The initial oscillation amplitude reaches 80 ps, stabilizing around 20 ps after 3000 turns. However, significant low-frequency noise appears in bunch length results because phase represents mean arrival time while bunch length is the RMS arrival time, making it more susceptible to noise. Future work will optimize the bunch length calculation algorithm.

To visualize beam trajectory within the vacuum chamber, we reconstructed the two-dimensional bunch length-phase distribution. After applying low-pass filtering to reduce noise, we calculated each turn' s longitudinal distribution under Gaussian assumptions, stored results in a matrix, and visualized using the 'imagesc' function. Fig. 9.c shows the reconstructed result, similar to streak camera measurements but covering several thousand turns with higher temporal resolution beyond streak camera dynamic range.

Zooming into different experimental samples reveals evolution details. Fig. 10.a [Figure 10: see original paper] shows maximum bunch length occurring at phase oscillation maxima, with minima near phase center. Fig. 10.b shows the opposite pattern. As previously discussed, combining simulation with experimental data yields injected bunch initial parameters through a genetic algorithm that minimizes variance between simulation and experimental data as the fitness function. The algorithm automatically optimizes simulation parameters until reaching preset iteration limits or target accuracy. Simulation parameter steps are: initial phase 0.5 ps, initial bunch length 1 ps, initial energy offset 0.01%, initial energy spread 0.01%, and initial energy chirp 0.01. The optimal simulation achieves Pearson correlation coefficients of approximately 0.99 for phase (variance 5-10 ps^2) and 0.98 for bunch length (variance 10-20 ps^2). The implementation block diagram is shown in Fig. 11 [Figure 11: see original paper].

Fig. 12 [Figure 12: see original paper] presents typical experimental and corresponding simulated reconstructed two-dimensional distributions from 2023 data. The high consistency between simulation and experimental results validates the simulation software accuracy and reflects the high precision of our bunch-by-bunch diagnostic system, demonstrating the technique's capability to accurately determine experimental initial parameters. The error distribution, calculated by subtracting experimental from simulation data, is shown in Fig. 12.c. Errors gradually increase over 3000 turns, possibly due to inconsistent longitudinal oscillation normalization frequencies or imperfect machine parameter matching, but remain relatively small, with excellent agreement within the first 1000 turns.

Table 3 lists initial parameter calculation results from multi-year injection data. Early data show initial bunch length not starting from maximum oscillation amplitude, indicating non-zero initial energy chirp. Over time, energy chirp approaches zero and initial bunch length stabilizes, while initial phase amplitude and energy offset decrease, indicating continuous SSRF injection system optimization.

VI. Summary and Conclusions

Through our bunch-by-bunch data processing script, we obtain evolution of injected bunch initial phase and bunch length over several thousand turns. However, initial energy offset, energy spread, and energy chirp cannot be directly determined. To address this, we developed single-bunch injection phase space evolution simulation software using mbtrack2 and PyQt5. By providing initial parameters, the software simulates phase space evolution under various injection conditions. A genetic algorithm then optimizes these parameters by aligning simulation with experimental data, yielding the experimental initial parameters. This capability provides a powerful tool for injection system optimization.

Furthermore, the simulation software can generate extensive single-bunch injection phase space evolution data with known initial distributions. This enables machine learning training on large simulation samples to analyze experimental data and determine initial injection parameters. Future development will focus on simulating original signal distributions rather than phase space distributions, using these as training samples to directly analyze raw sampling signals without requiring the bunch-by-bunch diagnostic system, enabling faster and more convenient initial parameter extraction.

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