SIRT-TV 3D image reconstruction for a simulated muon tomography of the QinShiHuang tomb

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Abstract

Cosmic-ray muons are suitable for non-destructive imaging of large-scale objects. However, due to the low statistics of cosmic-ray muons and the complicated surroundings of transmission muography experiments, transmission muography often faces challenges such as noise and incomplete data, posing certain difficulties for 3D image reconstruction. This paper applies the SIRT-TV algorithm to muon tomography, conducting a 3D image reconstruction simulation study based on a QinShiHuang tomb phantom. The results indicate that compared to the conventional SIRT algorithm, the SIRT-TV algorithm can effectively suppress artifacts in the reconstructed image, allowing for a more accurate reconstruction of the underground palace. The geometric similarity of the walls reconstructed by SIRT-TV is nearly tripled compared to that of SIRT. This study also discusses the impact of TV-minimization algorithm parameters, and the measurement configurations and durations on the quality of the reconstructed image, providing a reference point for future experiments.

Full Text

Preamble

SIRT-TV 3D Image Reconstruction for Simulated Muon Tomography of the QinShiHuang Tomb

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Cosmic-ray muons are well-suited for non-destructive imaging of large-scale objects. However, due to the low flux of cosmic-ray muons and the complex environments of transmission muography experiments, transmission muography often faces challenges such as noise and incomplete data, which pose significant difficulties for 3D image reconstruction. This paper applies the SIRT-TV algorithm to muon tomography, conducting a simulation study of 3D image reconstruction based on a QinShiHuang tomb phantom. The results demonstrate that compared to the conventional SIRT algorithm, the SIRT-TV algorithm can effectively suppress artifacts in reconstructed images, enabling more accurate reconstruction of the underground palace structure. The geometric similarity of the walls reconstructed by SIRT-TV is nearly tripled compared to that of SIRT. This study also examines the impact of TV-minimization algorithm parameters, measurement configurations, and measurement durations on reconstruction quality, providing valuable guidance for future experiments.

Keywords: Muography, Monte Carlo simulation, Image reconstruction

Introduction

Transmission muography utilizes naturally occurring cosmic-ray muons to image the internal structures of objects penetrated by these particles. With an average energy of approximately 4 GeV at sea level [1], cosmic-ray muons possess strong penetrating power and follow nearly straight-line trajectories while being ubiquitous around the Earth. These characteristics make cosmic-ray muons particularly suitable for non-destructive imaging of large-scale objects, including volcanoes [2-4], faults [5-7], mineral deposits [8-11], cultural heritage sites [12-16], and nuclear reactors [17-19].

The attenuation of muon flux due to energy loss during interactions with matter depends on both the thickness and density of the penetrated material. By analyzing cosmic-ray muon flux detected from a single viewpoint and incorporating prior topographic information, the average density of the object along the line of sight can be determined, enabling identification of density anomalies. This 2D transmission muography technique is known as “muon radiography.” However, muon radiography has a fundamental limitation: the resulting density map is only a 2D projection, which introduces ambiguities when imaging complex internal structures where density anomalies may overlap along the measurement direction. In such cases, overlapping structures in muon radiography images become degenerate. To alleviate this ambiguity, 3D information about the object’s internal structure is essential. At least two different viewing angles of cosmic-ray muon flux measurements are required to locate the 3D coordinates of density anomalies. For example, in the 2017 ScanPyramids experiment of the Khufu

Pyramid [12], three independent muon measurement sets were conducted, with each set taken from two different positions to locate hidden chambers through triangulation analysis. The consistent positions identified by the three independent analyses confirmed the discovery with high confidence. With muon flux data obtained from multiple viewpoints, the 3D density distribution of an object can be reconstructed, a technique known as "muon tomography."

Muon tomography faces two major challenges stemming from the characteristics of cosmic-ray muons. First, the relatively low flux of cosmic-ray muons—approximately 1 $cm^{-2} \cdot min^{-1}$ at sea level [1]—limits the number of measurement views due to both experimental time constraints and detector availability. Furthermore, detector placement is often constrained because transmission muography typically investigates large-scale field structures with complex surroundings. Consequently, the image reconstruction problem in muon tomography is frequently under-constrained, potentially leading to ambiguous results. Second, fluctuations in cosmic-ray muon flux data can disturb image reconstruction, including statistical fluctuations caused by low muon statistics and directional fluctuations resulting from multiple Coulomb scattering of muons penetrating objects [20].

To address these challenges, efforts can be pursued along two avenues: combining cosmic-ray muon data with other geophysical data for joint inversion, and improving image reconstruction algorithms. From the joint inversion perspective, both transmission muography and gravimetry are sensitive to object density, making them naturally complementary. Several studies have reported inversion results combining cosmic-ray muon data with gravity data. For instance, R. Nishiyama et al. combined single-view cosmic-ray muon data with gravity data from 30 stations to reconstruct the 3D density structure of the Showa-Shinzan lava dome, demonstrating through synthetic data inversion that joint inversion using both gravity and cosmic-ray muon data yielded better resolution than using gravity data alone [21]. However, K. Jourde et al. noted that when an object is covered by cosmic-ray muon data from more than two viewpoints, the improvement in image quality from adding gravity data becomes insignificant [22].

From the algorithm enhancement perspective, establishing robust algorithms with strong noise resistance is crucial because 3D image reconstruction in muon tomography involves reconstructing images from noisy and incomplete data. Current algorithms for 3D image reconstruction in muon tomography fall into two main categories. The first is linear inversion methods based on Bayesian principles, widely used in geophysical model inversion. For example, Shogo Nagahara et al. employed linear inversion to reconstruct the 3D density distribution of the Omuroyama scoria cone from cosmic-ray muon data obtained from 10 viewpoints [23]. This method can also be applied to joint inversion of gravity and cosmic-ray muon data, as demonstrated by Anne Barnoud et al., who used linear inversion to reconstruct the 3D density structure of the Puy de Dôme volcano through joint inversion of single-view cosmic-ray muon data and gravimetric data from 650 measurement points [24]. However, this method involves multiple a priori parameters whose selection directly affects inversion results and requires careful tuning. Additionally, the density smoothing constraint used in inversion may blur the boundaries of density anomaly regions [25].

The second category comprises algebraic reconstruction techniques, which are widely used in medical CT image reconstruction and are suitable for image reconstruction from incomplete and noisy data. Many studies have applied algebraic reconstruction techniques to muon tomography. Erlandson et al. simulated cosmic-ray muons passing through the ZED-2 research reactor and performed muon tomography reconstruction using the ART (Algebraic Reconstruction Technique) algorithm [26]. S. Procureur used the SART (Simultaneous ART) algorithm to reconstruct images for simulated muon tomography of a concrete cube with cavities [27]. K. Hartling et al. simulated a 12-view muon tomography of a cylindrical model containing a uranium rod and compared various iterative reconstruction algorithms, concluding that SIRT (Simultaneous Iterative Reconstruction Technique) is particularly suitable for muon tomography reconstruction [28]. Compared to the classic ART algorithm, SIRT better suppresses noise.

Additional information can be incorporated into SIRT to further improve reconstruction quality. One highly effective constraint is total-variation (TV) minimization. Signals or images with transform sparsity can be accurately reconstructed from a small number of samples by minimizing the L1-norm of the transform coefficients [29–31]. For piecewise constant images, their gradient-magnitude images satisfy the transform sparsity condition, and TV represents the L1-norm of the gradient-magnitude image. Therefore, TV-minimization-based image reconstruction algorithms can achieve high-precision reconstruction from limited views [32, 33]. The density distributions of large cultural heritage structures, such as the Khufu Pyramid, are piecewise constant, making them suitable for TV-minimization. Consequently, combining SIRT with TV-minimization could potentially enhance image quality for muon tomography reconstruction of such structures.

In 2022, our group reported preliminary results from a cultural heritage muon radiography simulation study based on archaeological data of the QinShiHuang tomb, demonstrating that muon radiography could identify inner structures of the underground palace, including the tomb chamber, walls, and rammed earth. From muon radiography simulation results obtained from two viewpoints, the length, width, and burial depth of the tomb chamber were estimated [34]. Building upon this previous research, this paper performs 3D image reconstruction for muon tomography of the QinShiHuang tomb. A digital phantom constructed from archaeological data of the QinShiHuang tomb is used for transmission muography simulation in Geant4 and for testing the image reconstruction algorithm. The SIRT-TV algorithm, which combines SIRT and TV-minimization, is applied to image reconstruction of a 12-view muon tomography. This paper is organized as follows: Section II introduces the principles of transmission muography and the SIRT-TV algorithm. Section III describes the simulation setup and presents 2D slices of reconstructed images using both SIRT-TV and SIRT algorithms, with visual comparison of slices obtained under the same number of iterations. Section IV evaluates reconstructed image quality based on geometric similarity indices for underground palace structures, providing a quantitative comparison of SIRT and SIRT-TV. This section also discusses factors influencing reconstruction quality, including TV-minimization algorithm parameters, the number of views in muon tomography, and muon data statistics. The final section concludes the paper.

II. Methods

A. Principles of Muon Radiography

As muons pass through an object, they interact with matter and lose energy through ionization, bremsstrahlung, pair production, and photonuclear reactions. The energy loss behavior can generally be described as:

$$\frac{dE}{dX} = a + bE,$$

where a represents the ionization term and b is the sum of the other three radiation loss terms. Both a and b depend on the muon' s energy and the material composition, with values obtainable from tables provided by Groom et al. [35]. $X = \int \rho dl$ is the integral of the object' s density along the muon' s path, referred to as the object' s density length. If a muon with energy $E_{\min}$ loses all its energy after penetrating an object with density length X_1, then X_1 can be expressed as:

$$X_1 = \int_0^{E_{\min}} \frac{dE}{a + bE}.$$

Combining Eqs. (1) and (2), the value of X_1 can be numerically calculated. Alternatively, Monte Carlo simulations can be used to determine the maximum density length that muons of different energies can penetrate. By fitting the simulation results, a fitted formula describing the relationship between X_1 and $E_{\min}$ can be obtained. $E_{\min}$ can be determined from the measured cosmic-ray muon flux I, which can be expressed as the integral of the differential energy spectrum $\phi(E)$:

$$I = \int_{E_{\min}}^{+\infty} \phi(E) dE.$$

Unlike traditional imaging techniques that use artificial radiation sources, transmission muography employs naturally occurring radiation sources whose energy spectrum $\phi(E)$ cannot be manually modulated. Moreover, since targets in transmission muography experiments are typically large, it is impractical to use detectors to measure the cosmic-ray muon energy spectrum $\phi(E)$ in real-time before muons penetrate the target. Nonetheless, theoretical studies and experimental measurements show that the sea-level cosmic-ray muon energy spectrum is relatively stable and can be reliably modeled. The angular distribution of sea-level cosmic-ray muons is roughly $\propto \cos^2\theta$, where θ is the zenith angle of the cosmic-ray muons. Their energy spectrum nearly follows a power law with a negative exponent. Therefore, the transmitted muon flux mainly consists of muons with energy near $E_{\min}$. Classic formulas such as Gaisser/Tang and Reyna are commonly used to describe the cosmic-ray muon energy spectrum at sea level, as they provide reliable descriptions in the energy range most relevant to transmission muography [36].

Combining Eqs. (1)–(3) with the sea-level cosmic-ray muon energy spectrum model, the density length of the object in different directions can be obtained, resulting in a 2D projection map of the object' s density length X.

B. 3D Image Reconstruction of Muon Tomography

Based on the 2D projection map of X obtained in Section II A, the 3D density distribution can be solved. Let m denote the number of directions. Dividing the 3D region of interest (ROI) uniformly into n voxels, the integral $X = \int \rho dl$ can be discretized as:

$$x_i = \sum_{j=1}^{n} l_{i,j}\rho_j, \quad i = 1, \cdots, m,$$

where x_i represents the density length along the i-th ray, ρ_j represents the density of the j-th voxel, and $l_{i,j}$ represents the intersection length of the i-th ray within the j-th voxel. The relationship between density lengths along different directions and the density distribution can be expressed as a system of linear equations:

$$\mathbf{X} = \mathbf{L}\rho,$$

where $\mathbf{X} = (x_1, x_2, \cdots, x_m)^T$ represents the density lengths of the object along different directions, $\mathbf{L} = \{l_{i,j}\}$ represents the intersection lengths of rays within voxels along different directions, and $\rho = (\rho_1, \rho_2, \cdots, \rho_n)^T$ represents the densities of the voxels.

Since density length data is obtained from cosmic-ray muon flux data, which is usually noisy and incomplete, the linear equations in Eq. (5) face issues such as being under-determined, inconsistent, and having a large sparse system matrix. Algebraic reconstruction techniques are commonly used to solve such problems. The most basic algorithm in algebraic reconstruction techniques is the ART algorithm [37], which is based on the Kaczmarz optimization algorithm. The principle of ART can be understood as gradually projecting the trial solution onto hyperplanes determined by each equation to obtain a new solution. If the system of equations has a unique solution, the trial solution will iteratively converge to that solution. In every iteration of the ART algorithm, each equation of Eq. (5) updates ρ once. This equation-by-equation update method is sensitive to noise and can even cause divergence if the noise is severe. To suppress noise, the SIRT algorithm was developed based on the basic ART algorithm [38]. In one iteration of the SIRT algorithm, the contribution of each equation to ρ is first calculated without updating ρ. After processing all equations, the contributions from all equations are averaged and then used to update ρ. The update equation of SIRT is:

$$\rho_j^k = \rho_j^{k-1} + \lambda \frac{\sum_{i=1}^m L_{i,j} \frac{X_i - \sum_{j=1}^n L_{i,j}\rho_j^{k-1}}{\sum_{j=1}^n L_{i,j}}}{\sum_{i=1}^m L_{i,j}},$$

where k represents the number of iterations and λ controls the iteration step size, usually set to 1.

Both ART and SIRT solve the reconstruction problem solely based on projection data $\mathbf{X}$, without introducing additional constraints. However, muon tomography often lacks sufficient views, resulting in ill-posed image reconstruction problems. To further improve reconstruction quality, additional constraints must be introduced. We observe that the density distribution of targets in muon tomography typically exhibits piecewise constancy, meaning its gradient-magnitude image is sparse, making it suitable for TV-minimization as a constraint.

Let the voxel density at coordinates (r, s, t) be $\rho_{r,s,t}$. Then, the total variation of the density image is:

$$\|\rho\|_{TV} = \sum_{r,s,t} |\nabla \rho_{r,s,t}| = \sum_{r,s,t} \sqrt{(\rho_{r,s,t} - \rho_{r-1,s,t})^2 + (\rho_{r,s,t} - \rho_{r,s-1,t})^2 + (\rho_{r,s,t} - \rho_{r,s,t-1})^2}.$$

Using the gradient descent algorithm to solve the TV-minimization problem, let $\rho_{TV}^0 = \rho_{SIRT}^k$, $d\rho = \|\rho_{SIRT}^k - \rho_{SIRT}^{k-1}\|_2$, $\vec{v}_0 = \nabla_\rho \|\rho_{TV}^0\|_{TV}$, and $\hat{v}_0 = \vec{v}_0 / |\vec{v}_0|$. For $k' = 1, 2, \cdots, N_{TV}$, perform TV gradient descent:

$$\rho_{TV}^{k'} = \rho_{TV}^{k'-1} - \alpha \cdot d\rho \cdot \hat{v}_{k'-1}.$$

where α controls the TV gradient descent step size and N_{TV} controls the total number of gradient descent iterations.

In Ref. [32], TV-minimization is combined with ART. However, considering that data noise is more significant in muon tomography, this paper opts to combine

SIRT with TV-minimization for image reconstruction. The SIRT-TV procedure terminates when the number of iterations reaches N_{max}.

The overall process of muon tomography image reconstruction in this work is shown in Fig. 1 [Figure 1: see original paper]. Note that after the SIRT step in each iteration k, a non-negative constraint is applied to ρ^k_{SIRT}. Any additional prior information about ρ can also be incorporated at this step.

III. Simulation and Reconstruction

A. Muography Simulation

The simulation is performed using Geant4, a Monte Carlo toolkit developed by CERN based on C++ [39]. Geant4 can simulate particle transport and interactions in matter and is a commonly used and reliable simulation platform for transmission muography. The simulation model consists of three parts: the QinShiHuang tomb phantom, the muon track detector, and the cosmic-ray muon source.

The QinShiHuang tomb has been studied using 20 geophysical approaches as part of the national 863 Hi-tech project. Archaeological results confirmed the depth of the underground palace, as well as the size and location of the tomb chamber and surrounding walls [40]. Based on this archaeological data, we constructed a phantom model of the QinShiHuang tomb in our simulation, which includes the mound, rammed earth, loam wall, stone wall, burial chamber, and surrounding land, as shown in Fig. 2 [Figure 2: see original paper].

The cosmic-ray muon source is sampled using the Reyna formula [41]:

$$\phi(p,\theta) = c_1 \cos^3\theta (p^*)^{-[c_2 + c_3 \log_{10}(p^*) + c_4 \log^2_{10}(p^*) + c_5 \log^3_{10}(p^*)]},$$

where θ is the zenith angle of cosmic-ray muons, p is the momentum of cosmic-ray muons in GeV/c, $p^* = p\cos\theta$, $c_1 = 0.00253$, $c_2 = 0.2455$, $c_3 = 1.288$, $c_4 = -0.2555$, and $c_5 = 0.0209$. This formula is recommended for a momentum range of 1 GeV/c $< p <$ 2000/cosθ GeV/c and a zenith angle range of $0° \leq \theta \leq 90°$.

The coordinate system is defined as shown in Fig. 2(b-d). The muon track detector is configured as a 1 m × 1 m ideal detector placed horizontally, recording the direction of cosmic-ray muons passing through it. To avoid interfering with the underground palace structure, detector positions are selected 5 m away from the boundaries of the underground palace. Twelve detector locations are chosen, evenly spaced at 30° intervals surrounding the center of the underground palace and numbered 1 to 12, as shown in Fig. 2(b). Since incident cosmic-ray muons always travel downward, detectors must be positioned lower than the underground palace to ensure their field of view fully covers the ROI.

Selecting an appropriate detector depth is important because the angular size of the ROI, the thickness of overburden above the detector, and the zenith angle of detected muons traversing the ROI must all be considered. We set the detector burial depth to 90 m, as shown in Fig. 2(c). At this depth, the maximum zenith angle of detected muons crossing the ROI does not exceed 75°. To reduce simulation time, we adopted the method from Reference [42] to determine the minimum energy of muons that can penetrate the phantom and reach the detector. Only muons with energy exceeding this minimum energy are sampled. The simulated cosmic-ray muon statistics for a single view are equivalent to 180 days of physical measurement.

B. Image Reconstruction

Since the tomb phantom is much larger than the detectors, the detectors are treated as points for calculating muon flux and density length maps. The simulated cosmic-ray muon fluxes after penetrating the QinShiHuang tomb phantom, as detected by each detector, are divided into 75 × 180 directions according to their zenith and azimuth angles, as shown in Fig. 3 [Figure 3: see original paper]. The direction of each pixel in Fig. 3 is represented by its center coordinates. The minimum energy E_{min} of cosmic-ray muons penetrating the phantom can be calculated using Eq. (3). For conversion to density length, this paper uses Geant4 to simulate the average penetration range of muons with different energies in loess, with simulation results shown in Fig. 4 [Figure 4: see original paper]. A second-order polynomial is fitted to the curve in Fig. 4.

Since we are primarily interested in the underground palace structure, we define the ROI as shown in Fig. 2(c) and 2(d) and perform 3D image reconstruction only for the density distribution within the ROI. The density length data obtained from different directions in the simulation include contributions from both inside and outside the ROI. For the tomb phantom, the density distribution outside the ROI is known, allowing direct calculation of the density length outside the ROI. By subtracting the density length outside the ROI from the obtained density length data, we obtain the density length only within the ROI, as shown in Fig. 5 [Figure 5: see original paper]. Note that some directions in the figure have negative density length values due to fluctuations in cosmic-ray muon flux. Only non-negative density length values are used in the image reconstruction process.

In 3D image reconstruction, the ROI is divided into 60 × 50 × 20 voxels, resulting in a voxel size of 3 m × 3.2 m × 4 m. This voxelization ensures that the number of voxels is not too large for solving the linear equations in Eq. (5) while keeping the voxel size small enough to reconstruct fine structures in the underground palace. Both SIRT and SIRT-TV are used to reconstruct images from the same density length data to compare their results. Following parameter values from Ref. [32], we set $\lambda = 1$, $\alpha = 0.2$, and $N_{TV} = 20$. The initial density values for iteration are set to $\rho^0 = 1.6$ g/cm^3, matching the loess density in the phantom. Both algorithms are iterated $N_{\text{max}} = 50$ times. An additional constraint is incorporated into each iteration, restricting voxel density outside the burial mound to 0.00129 g/cm^3, matching the air density in the phantom. Slices of the reconstructed images at three representative locations are selected for comparison with phantom slices, as shown in Fig. 6 [Figure 6: see original paper].

Comparing the reconstructed image slices from both algorithms with the phantom slices reveals that the shapes and positions of the tomb chamber and walls in the SIRT-TV reconstruction match the phantom well. In contrast, slices obtained using the SIRT algorithm exhibit significant salt-and-pepper artifacts, particularly noticeable at the bottom of the reconstructed image, around the tomb chamber, and at the top of the burial mound. These artifacts could potentially lead to misinterpretation of the underground palace structure. The SIRT-TV reconstruction does not show such significant artifacts, preliminarily verifying that TV-minimization effectively suppresses artifacts.

IV. Discussion

A. Reconstructed Image Quality Assessment

Fig. 6 presents a visual comparison of reconstructed images using SIRT and SIRT-TV. However, visual comparison lacks quantitative information and is non-comprehensive since the reconstructed images are 3D. To quantitatively compare the reconstruction results, appropriate image quality assessment metrics are necessary. Classic metrics include mean squared error (MSE), peak signal-to-noise ratio (PSNR), and structural similarity (SSIM) [43], which evaluate overall similarity between reconstructed images and the phantom. However, this study focuses on whether underground palace structures–specifically the tomb chamber and walls–can be properly reconstructed. Referring to image quality assessment metrics used in Ref. [28], we employ the Jaccard similarity index J to assess geometric similarity of reconstructed images to the phantom for the tomb chamber and walls. The definition of J is:

$$J(P,R) = \frac{|P \cap R|}{|P \cup R|},$$

where P represents the set of voxels belonging to the tomb chamber or walls in the phantom, and R represents the set of voxels belonging to the same structure in the reconstructed image. The term $|P \cap R|$ denotes the number of elements in the intersection of P and R, while $|P \cup R|$ denotes the number of elements in the union of P and R. A larger $J(P,R)$ indicates higher geometric similarity, with $J(P,R) = 1$ signifying identical geometric structures between the phantom and reconstructed image.

When calculating $J(P,R)$ for reconstructed images in Section III, we face the issue of inconsistent voxel segmentation between the phantom and reconstructed images, as the phantom established in Geant4 is not voxelized. To address this, we modify Eq. (14). We define a vector $\mathbf{p}$ representing the proportion of overlap between voxels defined in reconstructed images and the corresponding structure in the phantom. Consequently, the element p_i corresponding to the i-th voxel can take three values: 0 for voxels outside the structure, 1 for voxels inside the structure, or q for voxels partially overlapping with the structure with proportion q.

We define a vector $\mathbf{r}$ to represent whether each voxel belongs to the structure in the reconstructed image. The element r_i corresponding to the i-th voxel can take two values:

$$r_i = \begin{cases} 0 & \text{voxel outside the corresponding structure,} \\ 1 & \text{voxel inside the corresponding structure.} \end{cases}$$

Therefore, Eq. (14) can be redefined as:

$$J(\mathbf{p}, \mathbf{r}) = \frac{\sum_{i=1}^{n} p_i r_i}{\sum_{i=1}^{n} p_i + \sum_{i=1}^{n} r_i - \sum_{i=1}^{n} p_i r_i}.$$

When calculating $\mathbf{r}$, image segmentation is necessary to extract the tomb chamber and walls from reconstructed images. Since the density of the tomb chamber and walls correspond to the minimum and maximum densities in the phantom, respectively, image segmentation can be performed using density thresholds. Structures are extracted based on whether voxel density values fall within specified density ranges. For the tomb chamber:

$$r_i^{\text{chamber}} = 1, \quad \text{if } \rho_i < \text{threshold}_{\text{chamber}} \text{ and } z < 0.$$

And for the walls:

$$r_i^{\text{wall}} = 1, \quad \text{if } \rho_i > \text{threshold}_{\text{wall}}.$$

Ideally, thresholds should be set according to density values in the phantom. However, due to differences between reconstructed image densities and phantom densities–especially since tomb chamber densities in reconstructed images are much higher than air density in the phantom–an optimization approach is used to find the most suitable segmentation threshold. Considering that chamber density is lower than loess density while wall density is higher, the tomb chamber threshold is searched within (0, 1.6 g/cm^3] and the wall threshold within (1.6 g/cm^3, 2.7 g/cm^3], with a step size of 0.1 g/cm^3. For each threshold, J is calculated, and the highest J value is taken as the geometric similarity of the reconstructed image, denoted as J_{chamber} for the tomb chamber and J_{wall} for the walls.

Thresholds are searched for reconstructed images obtained in Section III. Voxel sets belonging to the tomb chamber and walls are extracted according to the thresholds yielding the highest J, and voxel positions from image segmentation are shown in Fig. 7 [Figure 7: see original paper]. Both wall and tomb chamber geometries in the SIRT reconstruction show noticeable overflow compared to the phantom. In contrast, their counterparts extracted from the SIRT-TV reconstruction align better with the phantom. Table 1 provides J calculation results for structures in images obtained by both algorithms. Both J_{chamber} and J_{wall} in the SIRT-TV result are superior to those from SIRT, with J_{wall} being 2.8 times that of SIRT.

TABLE 1. J_{chamber} and J_{wall} of reconstructed images obtained using SIRT and SIRT-TV.

Algorithm	J_{chamber}	J_{wall}
SIRT		
SIRT-TV		

B. Iterative Convergence

Convergence speed is an important metric for evaluating iterative image reconstruction algorithms. In Section IV A, J_{chamber} and J_{wall} of reconstruction results are compared after the same number of iterations for both algorithms, but the number of iterations required for convergence may differ. To compare convergence speeds, both algorithms were iterated 150 times, and curves showing changes in J_{chamber} and J_{wall} with iteration number are plotted in Fig. 8 [Figure 8: see original paper].

J_{chamber} and J_{wall} for SIRT-TV generally increase with iteration number until convergence, whereas those for SIRT initially increase but then decrease with additional iterations before convergence. To quantitatively evaluate convergence speed, we define a convergence metric:

$$\frac{\Delta J}{J} = \frac{|J^{k+1} - J^k|}{J^k},$$

where k represents the iteration number and ϵ is the convergence threshold. The $\Delta J/J$ values for both algorithms as functions of iteration number are plotted in Fig. 8. These values fluctuate with increasing iteration numbers while converging, due to the discreteness of structure thresholds and voxelization. With ϵ set to 0.002, J_{chamber} and J_{wall} for SIRT-TV converge after 36 and 24 iterations, respectively, whereas those for SIRT have not yet converged. This indicates that SIRT-TV converges faster than SIRT. The faster convergence of SIRT-TV is due to the effective constraint of TV-minimization on the reconstruction procedure. For the SIRT algorithm, convergence speeds of J_{chamber} and J_{wall} differ because of structural differences: the tomb chamber is a relatively large cubic volume, while the walls are more complex and thinner.

C. Impact of TV Parameters on Reconstruction

Parameter values in the TV-minimization algorithm may affect reconstruction quality. To assess this impact, we vary the maximum iteration number N_{TV} and TV gradient descent step size α. J_{chamber} and J_{wall} as functions of iterations for different parameter values are shown in Fig. 9 [Figure 9: see original paper].

For most (N_{TV}, α) combinations, convergence is achieved within 50 iterations, indicating that choosing $N_{\max} = 50$ as the termination criterion for SIRT-TV is reasonable. For J_{wall}, when $N_{TV} = 10$ and $\alpha = 0.1$, the general trend with increasing iterations is similar to SIRT, with a downward trend still present after 150 iterations. This indicates that TV-minimization with these parameter values does not effectively improve reconstruction quality. With different α values, J_{wall} is always lowest when $N_{TV} = 10$. Unlike J_{wall}, for different α values (except $\alpha = 0.1$), J_{chamber} is always highest when $N_{TV} = 10$, remaining above 0.5 regardless of parameter values. This suggests that compared to walls, tomb chamber reconstruction does not require stringent TV constraint.

Overall, different parameter values in Fig. 9 (except $N_{TV} = 10$ or $\alpha = 0.1$) do not significantly affect reconstruction results, indicating that the SIRT-TV algorithm can reliably produce high-quality images.

D. Impact of Measurement Configuration and Duration

As mentioned in Section I, the main challenges in 3D muon tomography reconstruction are noise and incomplete data. Section III simulated relatively ideal conditions to test algorithm feasibility. This section discusses the impact of less ideal measurement configurations and durations—specifically fewer views and lower muon statistics—on SIRT-TV reconstruction.

1. Measurement Configuration Using the same simulation data from Section III, the number of views is reduced to 4, and three different detector configurations are selected for reconstruction. Image reconstruction algorithm parameters remain the same as in Section III. Fig. 10 [Figure 10: see original paper] shows slices of the phantom and reconstructed images for the 12-view configuration and three 4-view configurations.

The reduction in view number leads to increased artifacts. Among the 4-view configurations, the symmetrical 4-view configuration shows better geometric symmetry in its reconstruction, while the other two configurations exhibit distortions in tomb chamber and wall shapes. J_{chamber} and J_{wall} calculation results in Table 2 also indicate that reconstruction quality is highest for the symmetrical 4-view configuration. These artifacts can be explained by ray densities in voxels. Given identical voxelization for each reconstruction, more views lead to more rays, higher ray densities in voxels, and more information about voxel density. When voxelization and view number are the same for each reconstruction, asymmetrical measurement configurations cause greater variation in ray densities across voxels, meaning some voxels are crossed by only a few rays, making their densities difficult to reconstruct. Symmetrical measurement configurations yield better reconstruction quality.

TABLE 2. J_{chamber} and J_{wall} of images obtained from 12-view and three 4-view reconstructions.

Measurement configuration	J_{chamber}	J_{wall}
12-view		
4-view Configuration 1		
4-view Configuration 2		
4-view Configuration 3		

2. Measurement Duration The number of cosmic-ray muons detected is proportional to measurement duration, affecting statistical fluctuations and consequently reconstruction quality. To investigate reconstruction quality under different durations, we select datasets from the beginning of Section III simulation data, corresponding to 7, 14, 30, 90, and 180 days of equivalent measurement time. Algorithm parameters remain the same as in Section III. Reconstructed image slices are shown in Fig. 11 [Figure 11: see original paper].

In reconstructed image slices corresponding to 7-day and 14-day measurement durations, severe artifacts appear between the chamber and walls, particularly at the bottom of reconstructed images. As measurement duration exceeds 30 days, artifacts are effectively suppressed, and the boundary between the chamber and rammed earth becomes relatively clear. Reconstructed image slices for 90-day and 180-day durations are quite similar, with both J_{chamber} and $J_{\text{wall}} \geq 0.5$, as shown in Table 3 .

TABLE 3. J_{chamber} and J_{wall} of reconstructed images obtained using SIRT-TV under different equivalent measurement durations.

Time (day)	J_{chamber}	J_{wall}
7		
14		
30		
90		
180		

V. Conclusion

This paper conducts simulation research on 3D image reconstruction of muon tomography for a QinShiHuang tomb phantom using the SIRT-TV algorithm.

Reconstruction quality is evaluated based on geometric similarity of structures. The results show that compared to the SIRT algorithm, SIRT-TV effectively suppresses salt-and-pepper artifacts. It improves geometric similarity J_{chamber} by nearly one-third and J_{wall} by nearly twofold, while converging faster. This suggests that the SIRT-TV algorithm is more suitable for 3D image reconstruction of large-scale cultural heritage muon tomography. Using this algorithm with simulated 12-view cosmic-ray muon data, the locations and shapes of the tomb chamber and walls are successfully reconstructed.

This study also investigated the impact of TV-minimization algorithm parameters on reconstruction quality by varying the maximum iteration number N_{TV} and TV gradient descent step size α. The results indicate that although optimal TV parameters differ between wall and tomb chamber reconstruction, selecting parameters within a reasonable range achieves high-quality reconstructions. Considering potential experimental constraints, this paper also discusses the impact of measurement configurations and durations on reconstruction results. Reducing views from 12 to 4 decreases J values. Among three different 4-view configurations, J values are highest when detectors are positioned along ROI symmetry axes. For the 12-view configuration, J values steadily increase with equivalent measurement duration, with both J_{chamber} and $J_{\text{wall}} \geq 0.5$ after 90 days of equivalent measurement. These findings provide guidance for selecting measurement configurations and durations in future experiments.

While SIRT-TV substantially improves reconstruction quality compared to SIRT, room for enhancement remains: the two walls with different densities remain indistinguishable in reconstructed images, and geometric similarity J could be further improved. Future work could enhance image quality through machine learning techniques. By simulating muon data from the QinShiHuang tomb phantom under varying viewpoints and measurement durations, reconstructed images from the SIRT-TV algorithm could serve as training inputs for machine learning models to enable further quality improvements.

Although the QinShiHuang tomb phantom used in this study is simplified, it retains essential structural features of the underground palace. As one of China' s most renowned imperial tombs, the QinShiHuang tomb structure is likely representative. Transmission muography simulation based on this phantom would also provide insights for potential transmission muography experiments on other imperial tombs.

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