The Impact of Different Effective Models for Star Formation on the Properties of Simulated Milky Way-sized Galaxies postprint

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Date: 2025-02-25T00:00:00+00:00

Abstract

Hydrodynamical cosmological simulations of galaxy formation such as IllustrisTNG or Auriga have shown considerable success in approximately matching many galaxy properties, but their treatment of the star-forming interstellar medium (ISM) has relied on heuristic sub-grid models. However, recent high-resolution simulations of the ISM that directly resolve the regulation of star formation suggest different mean relations for the dependences of pressure and star formation rate on the average gas density. In this study, we adopt such a modern, physically grounded parameterization inspired by the TIGRESS small-scale simulations. We dub this model TEQS and use it for a detailed comparative analysis of the formation and evolution of a Milky Way-sized galaxy when compared with the widely used TNG model. By employing high-resolution simulations in tall box setups, we first investigate the structural differences expected for these two models when applied to different self-gravitating gas surface densities. Our results indicate that TEQS produces considerably thinner gaseous layers and can be expected to form stellar distributions with smaller scale-height than TNG, especially at higher surface density. To test whether this induces systematic structural differences in cosmological galaxy formation simulations, we carry out zoom-in simulations of 12 galaxies taken from the set of Milky Way-sized galaxies that have been studied in the Auriga project. Comparing results for these galaxies shows that disk galaxies formed with the TEQS model have on average very similar stellar mass but are more concentrated in their central regions and exhibit smaller stellar radii compared to those formed with the TNG model. The differences in the scale-heights of the formed stellar disks are only marginal, however, suggesting that other factors for setting the thickness of the disk are more important than the applied ISM equation-of-state model. Overall, the predicted galaxy structure is quite similar for TNG and TEQS despite significant differences in the employed star formation law, demonstrating

that feedback processes are more important in regulating the stellar mass than the precise star formation law itself.

Full Text

Preamble

Research in Astronomy and Astrophysics, 25:015003 (19pp), 2025 January https://doi.org/10.1088/1674-4527/ad9254 CSTR: 32081.14.RAA.ad9254 Development of Gravity Theories in the View of TRAPPIST-1e

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Received 2024 April 10; revised 2024 October 31; accepted 2024 November 08; published 2025 January 2

Abstract

Contrary to the solar system, most exoplanet systems detected hitherto are close-in and compact. One typical system is TRAPPIST-1, which has seven nearly co-planar terrestrial planets all within the orbit of Mercury, including three in the habitable zone. To evaluate the differences in developing sophisticated gravity theories from the solar system, we use N-body integrations to simulate ephemeris and reproduce some important astronomy phenomena observed on the potentially habitable planet TRAPPIST-1e. Retrograde motions of other planets last 1–2 orders of magnitude shorter than in the solar system, but occur much more frequently. Transit events of all inner planets can be observed steadily. Except for Kepler’ s first law, which is hard to notice for low eccentricities of planets, the other two laws can then be precisely verified in 102 days, because the areas swept by planets vary by 0.01% and the observed semimajor axes and periods result in constants with theoretical and observation accuracies both 2%. However, the mean motion correlation implies that the Great Inequality is not always apparent between one pair of planets like Jupiter and Saturn. Furthermore, general relativity can hardly be discovered because it gives rise to perihelion precession of inner planets only ~0.1% of gravity precession, dozens of times smaller than Mercury. Our results support the possibility of developing part of gravity theories by potential exo-civilizations in compact systems like TRAPPIST-1.

Key words: planets and satellites: terrestrial planets –extraterrestrial intelligence –ephemerides –gravitation

1. Introduction

Astronomy plays an important role in understanding the universe, discovering physical laws, and validating these laws at extreme scales. To understand the solar system, in the 3rd century BC Eratosthenes calculated the Earth’ s circumference using the difference in solar radiation angles from two different sites. In the 15th century, the retrograde motion of Mars inspired Copernicus’ heliocentric theory [?], which explained the phenomenon well without the need for the complex epicycles of Ptolemy’ s geocentric theory.

After the medieval period, scientists like Galileo began using telescopes to observe the night sky with high precision, ushering astronomy into a modern era. Gravitational theory was subsequently extensively developed through understanding celestial motion over the following centuries. Based on accurate solar system motion data from Tycho Brahe, Kepler derived three well-known planetary motion laws in *Astronomia nova* [?] and *Harmonices mundi* [?]. These became the cornerstones of gravitational theories, i.e., Newtonian mechanics [?], as well as celestial mechanics. After Herschel & Watson’ s discovery of Uranus [?], Adams and Le Verrier predicted Neptune’ s motion from O-C deviations in Uranus’ orbit [?, ?, ?], representing a remarkable example of the successful application of Newtonian mechanics.

By the 20th century, with advances in physics and high-precision astronomical observations, accounting for the theory of relativity became increasingly important in astronomy, as opposed to pure Newtonian mechanics. For instance, general relativity [?] precisely predicts Mercury’ s perihelion precession [?] and provides a theoretical basis for studying black holes, dark matter, and many other astronomical objects under extreme conditions.

It is evident that observations of the major planets in the solar system have played a vital role in the history of understanding gravity theory. According to statistics from the NASA Exoplanet Archive, more than 5,000 exoplanets had been detected by December 2022, with over 5,000 candidates awaiting confirmation. Space telescopes such as Kepler [?] and TESS [?] have discovered over 1,000 planets with radii below two Earth radii, including dozens of temperate terrestrial planets [?]. Recently, the Closeby Habitable Exoplanet Survey (CHES) mission has been proposed to discover habitable-zone Earth-like planets around nearby solar-type stars [?]. The possibility of civilizations on these planets has long been an attractive topic for humanity. If intelligent creatures exist on other planets, could they discover gravity theory from observations in their own “solar system,” as humans have done?

Until now, exoplanet systems have shown many differences compared to the solar system. Due to observational selection bias, most planets in the habitable zone (hereafter HZ) are detected around M-type stars, since these stars have a much closer HZ than the solar system. These exoplanets are usually in compact systems, and most of them are rocky planets of similar sizes [?, ?]. Multiple-planet systems around M dwarfs are common, and the gravitational perturbations in such systems are usually stronger than those in the solar system because of the compact architectures and small mass of M-type dwarfs. How these stronger perturbations influence planetary motion, and whether the ephemerides of other planets deviate from Kepler' s laws, are crucial for civilizations on habitable planets to discover and understand gravity theory.

Our motivation is to examine the difficulty of discovering gravity theory for civilizations around M-type dwarfs via the same methods as humans and test the precision compared with historical data on Earth.

As an extreme example, the TRAPPIST-1 system has seven terrestrial planets, including three in the HZ. The star, TRAPPIST-1, is a cool red dwarf discovered by the Two Micron All-Sky Survey (2MASS) in 1999 [?]. In 2017, the Transiting Planets and Planetesimals Small Telescope (TRAPPIST, [?]) discovered seven planets (b–h) orbiting TRAPPIST-1 [?]. The longest period of the outermost planet h is about 19 days [?], which is even shorter than Mercury' s period. The masses of the seven Earth-like planets are not well determined, and the planetary system may be dynamically unstable in some cases [?]. Thus, the TRAPPIST-1 system is more compact and dynamically perturbed compared with the solar system. The formation of such a compact and flat system has been studied extensively in previous works [?, ?, ?, ?]. It is speculated that planets e, f, and g are located in the habitable zone, hence with the probability of harboring civilization [?].

In this paper, we focus on the TRAPPIST-1 system and choose planet e as a potentially habitable world. We simulate the planet ephemeris on the sky of planet e based on an N-body numerical model. To understand planetary motion in the context of heliocentric theory in the TRAPPIST-1 system, we calculate the retrograde motions for other planets and the transit events of inner planets. We also test Kepler' s Laws and general relativity via the planetary ephemeris. Comparing with ephemeris on Earth, we assess how well gravity theory can be tested in the TRAPPIST-1 system.

This paper is arranged as follows. In Section 2, we build up a numerical model and introduce the initial setups for the TRAPPIST-1 system. In Section 3, we calculate the retrograde motions and transit events in the sky of TRAPPIST-1e. We also use the ephemeris to verify Kepler' s Laws and evaluate the Great Inequality effect in Section 4. General relativity is tested via the orbital precession of inner planets in Section 5. Finally, we conclude our results and discuss the validation in Section 6.

2. Numerical Simulations of the TRAPPIST-1 System

In this section, we introduce how we integrate the planetary ephemeris in the TRAPPIST-1 system and the initial setups.

We choose the REBOUND package for our N-body simulations to calculate ephemeris [?]. A 15th order Gauss-Radau integrator, IAS15 [?], is used. We only consider the classic N-body Newtonian gravity to calculate the positions and motions of the seven planets.

The initial conditions for each planet, which include semimajor axis a, eccentricity e, inclination i, argument of pericenter ω, true anomaly f, mass M_p, and radius R_p, are adopted from [?] at epoch BJD = 2457257.931, as shown in Tables 1 and 2. The mass of the host star M_* is 8.98×10^{-2} solar masses [?]. All longitudes of ascending nodes are set as 0°.

We then obtained the evolution of the orbital elements over the next 5,000 Earth years (i.e., 3×10^5 orbital periods of TRAPPIST-1e) and recorded outputs every 0.5 Earth days. Table 1 shows the orbital elements of all planets after 5 kyr. Given the positions and velocities of the seven planets, we transform them into the horizontal coordinates of TRAPPIST-1e (Section 3). Based on the horizontal coordinates, we can study the ephemeris observed from TRAPPIST-1e. In addition, the sidereal period T is calculated according to the evolution of the orbital elements and is used to test Kepler' s laws and the Great Inequality (Section 4).

Even though 5 kyr is already comparable to the time humans developed gravity theories, there is a concern that the initial conditions may lead to long-term instability after the early stable evolutions analyzed in this work, because of the system' s compactness and active resonances. To quantify the compactness of the TRAPPIST-1 system, we check the separation of planet pairs using the Hill radius defined by [?] and the separation factor $K = |a_1 - a_2|/R_H$, where a_1 and a_2 are the semimajor axes of two planets, and M_1 and M_2 represent the masses of the two planets. The smaller the separation scaled by R_H, the more compact these two planets are. Here we use the semimajor axes of the planets in the TRAPPIST-1 system from Table 1. As shown in Table 3, the average separation in the TRAPPIST-1 system is $ 10.3R_H, whilethemostcompactplanetpairisfandg, withaseparationof6.6R_H$.

Compared to the inner solar system, the TRAPPIST-1 system is 3–6 times more compact. According to the empirical relationship between K and stability [?], TRAPPIST-1 should be marginally stable. In addition, the resonance ratio of the seven planets forms a long resonance chain: 24:15:9:6:4:3 [?]. The diversity of these extra-terrestrial resonances shows the complexity of compact planet systems. However, if convergent migration occurs, the TRAPPIST-1 system can still be stable on a timescale of $ 10^7$ yr [?].

Our early evolutions are shown in Figure 1. The oscillating patterns of eccentricity, inclination, and pericenter of each planet are illustrated. The seven planets are on a common plane inclined at about 89.8° with a maximum deviation of no more than 0.3°. Thus, compared with the solar system, the TRAPPIST-1 system is more co-planar. The eccentricity of planet e, oscillating between 0 and 0.03, is similar to that of Earth. Pericenter precession can be seen on each planet in the system, which will be discussed further in Section 5. We further lengthen the simulation time and compare the evolutions after 1 Myr with the early evolutions to clarify the long-term stability. Figure 2 shows that even under gravitational perturbations, the semimajor axes of the seven planets nearly remain the same as the initial values without irregular jumps.

The eccentricities still oscillate, but with magnitudes close to the early evolutions, and the oscillation of planet e is 0.017, even smaller than the early value. Therefore, we point out that some values of eccentricities 5 kyr later in Table 1 seem to increase significantly during the simulation, which could misleadingly suggest system instability with excited orbits. However, those values at merely one instant are contributed by oscillations that are no stronger even after 1 Myr than in the early evolution, though the orbital elements' evolutions contain periodic signals (Figure 1). Briefly, the stability of the TRAPPIST-1 system over the timescale of 5 kyr is certain.

Based on the time series of the seven planets, we mimic virtual observations from TRAPPIST-1e and produce the celestial ephemeris that would be seen from the planet. In the next section, we will investigate the retrograde motions and transits of the inner planets in the TRAPPIST-1 system.

3. The Observation of Retrograde Motion and Transit

On Earth, retrograde motions of Mercury, Venus, and Mars have been observed. In fact, Martian retrograde motion is what caused Copernicus to become suspicious of geocentric theory and establish heliocentric theory, starting a new era of astronomical exploration. In addition, transits of the inner planets—an important phenomenon that once helped humans recognize the scale of the solar system—are also studied in this section to characterize the TRAPPIST-1 system. Hereafter, we inspect the synchronous status and habitability of TRAPPIST-1e, and then search for and study the occurrence of these two important phenomena in the sky of TRAPPIST-1e.

3.1. Spin Evolution and Insolation Distribution of TRAPPIST-1e

To derive virtual observations from TRAPPIST-1e, the astrocentric coordinates of the system need to be transformed into the horizontal coordinates of TRAPPIST-1e, so its spin evolution must be known. According to simulations by [**?**], an Earth-mass planet orbiting a 0.08 $M_\odot$ host evolves toward synchronization in less than 2×10^3 yr, indicating a similar damping timescale for TRAPPIST-1e. We confirm this by comparing four simulations with the same initial conditions (Table 1), except that planet e is synchronous with obliquity $\epsilon = 0°$ in the first case but has various spin rates and obliquities (half, equal, and twice the values of Earth) in the other cases. To track the spin evolution of the planet, we use the tides_{spin} implementation of equilibrium tide theory [**?**, **?**] included in the REBOUNDx package of add-ons [**?**].

Figure 3 shows that the spin rate and obliquity damp quickly to synchronization even when planet e is initially non-synchronous, so it is reasonable to assume that planet e has long been synchronous at the initial date of the simulation.

The first case, where planet e has reached synchronization from the beginning, is then used to derive virtual observations and subsequent analyses.

Starting from the initial synchronous status, planet e remains synchronous throughout the following 5 kyr (Figure 3). Still, to obtain precise projected positions of other planets, we consider the precession and nutation of the spin axis of TRAPPIST-1e due to perturbations from other planets. According to the simulation results shown in Figure 3, the precession period of planet e is about 278 yr, and its nutation period is about 227 yr with a variation of ϵ no more than 0.8°. Both periods are much longer than the timescale of the phenomena discussed in Section 3, so the precession and nutation of TRAPPIST-1e should not affect observations from the planet. The precession effects due to tides and general relativity are taken into account in Section 5 to evaluate the possibility of discovering general relativity theory.

We further investigate the insolation distribution of TRAPPIST-1e given its spin evolution to assess its habitability. The mean insolation over one revolution can be estimated analytically using the elliptic integral method adopted from [?]. The insolation W depends on the planet' s semimajor axis, eccentricity, obliquity, and the host star' s luminosity L_*:

$$W = \frac{L_*}{4\pi a^2}\frac{1}{\pi}\int_0^{2\pi}\frac{(1-e^2)^{3/2}}{(1+e\cos f)^2}\frac{1}{\sqrt{1-\sin^2\epsilon\sin^2(\omega+f)}}df$$

where E, K, Π are the first, second, and third complete elliptic integrals, respectively. Using the simulated parameter values of TRAPPIST-1e and $L_* = 0.000553 L_\odot$ [?], the insolation distribution is obtained as shown in Figure 4. The insolation of TRAPPIST-1e has a similar latitudinal trend to Earth, i.e., a convex shape with one peak at the equator and two equal minima at the poles, because of its small obliquity. The insolation value of TRAPPIST-1e is smaller than that of Earth at each latitude but comparable overall. Latitudes lower than 52° on TRAPPIST-1e overlap with latitudes higher than 51° on Earth, where the insolation ranges between 173 and 281 W m^{-2}. As the orbit and obliquity of TRAPPIST-1e evolve, its insolation varies by only 0.7% over 5 kyr. Given that the insolation is suitable and stable, it should also be easy to satisfy the heat transport efficiency needed to maintain the global atmosphere of tidally locked planets [?]. Thus, there should be no obstacles to the habitability of TRAPPIST-1e in terms of its insolation and synchronous status.

Nevertheless, as an M dwarf, the stellar activity of TRAPPIST-1 may be considered disadvantageous to habitability. Simulations by [?] showed that even at the peak of an M dwarf flare, the ultraviolet radiation received by Earth-like planets within HZs would exceed the level received by Earth for only a few minutes. Additionally, although ozone depletion can be induced by protons, it is temporal and recoverable. In other words, a single flare event has no permanent hazard on habitability. Repeated flares can result in direct harm to life and comprehensive destruction of ozone shields [?], but M dwarfs usually cool down after the initial $ $1 Gyr, and recovery of the planetary atmosphere during long-term evolution is achievable. Considering factors including insolation, the host' s stellar activity, and orbital stability [?, ?], it can be assumed that during the simulation, TRAPPIST-1e has a stable and habitable environment for a civilization to make astronomical observations and develop theories.

3.2. Retrograde Motions Inspiring Heliocentric Theory

Retrograde motion is a basic and important phenomenon in the solar system. In Ptolemy' s geocentric model, to satisfy both the motion of the star along its perfect circular trajectory and the retrograde motions of planets, it was necessary to add epicycles to the deferent and introduce a series of complex concepts such as eccentric and equant. The final "wheel-on-wheel" system could include more than 80 epicycles at a scale affecting interplanetary motions, as shown in Figure 5. Because the interpretation of retrograde motion was too complex, Copernicus developed his heliocentric theory in response. After Copernicus introduced the heliocentric model, the epicycles of the planets were reduced to about 30, and their sizes—previously considered perfect circumferences—were reduced to very small extents. From then on, heliocentric theory began to be accepted and recognized by the public at large.

Kepler then assumed that planets moved along elliptical orbits and eventually proposed Kepler's first law based on the 8 uncertainty in the orbital observations of Mars. It can be seen that these two important milestones in astronomy are closely related to observations of retrograde motion, which therefore inspires us to explore it in the TRAPPIST-1 system.

Based on the projected positions of other planets on the TRAPPIST-1e celestial sphere, we can draw their trajectories and observe if and when retrograde motion occurs. While transforming the astrocentric coordinates of the system into the horizontal coordinates of TRAPPIST-1e, the planet' s spin evolution (Section 3.1) is considered. Note that annual parallax is considered automatically in the coordinate conversion. Typical retrograde motions are illustrated in Figure 6. We can obtain some features about this phenomenon observed in the TRAPPIST-1 system and compare it with that observed on Earth. Since the durations and trajectories of retrograde motions vary from time to time, we have measured quantitative characteristics of retrograde motions in the TRAPPIST-1 system over 500 days ($ $82 orbital periods of planet e), with results shown in Table 4 and Figure 7.

As we can see, in the solar system, planets have longer durations and lower frequencies of retrograde motions, whereas the frequencies in the TRAPPIST-1 system are significantly larger. The typical duration and interval in the latter system are both 1–2 orders of magnitude shorter than in the former. These phenomena can be explained by the compactness of the TRAPPIST-1 system: a smaller orbital period leads to a shorter duration of retrograde motion, and a closer distance leads to a higher frequency. However, compared with Mercury and Mars, the ratios of total durations of all retrograde motions in a given time to that time length for planets in TRAPPIST-1 are comparable and no more than double. Additionally, the arcs of retrograde motions in the two systems are quite close, and the displacements in the sky are almost equally obvious. Hence, there should also be a good chance to notice this phenomenon in the TRAPPIST-1 system.

3.3. Transit Events Allowing Distance Measurements

Transits are important phenomena for understanding the absolute scale of the solar system. The first known attempt to determine the Sun–Earth distance geometrically was made by Aristarchus [?]. Shortly after, Hipparchus realized that determining the Sun–Earth distance in units of Earth' s radius was equivalent to determining the solar parallax (the angle between the line through the centers of Earth and the Sun and the tangent line touching Earth' s surface through the center of the Sun) [?]. In the 17th century, benefiting from telescopic observations, Kepler limited the solar parallax to 1 , corresponding to a Sun–Earth distance of 3,469 Earth radii [?]. In the 18th century, Halley proposed determining the solar parallax via observations of the transit of Venus [?]. In this method, the difference in transit duration observed at different locations on Earth is compared to the expected difference calculated from an estimate of the solar parallax, so the estimate can be effectively corrected [?]. After international efforts to observe the 1761 and 1769 transits, the mean parallax was precisely determined to be 8.78 , leading to an eight-tenths of a percent difference in the Sun–Earth distance [?].

The same method can be used in the TRAPPIST-1 system, based on the transit timings of the inner planets, which would allow possible extrasolar civilizations to calculate the distance from TRAPPIST-1e to their host star. From the perspective of a civilization on TRAPPIST-1e, we can model the trajectory of each inner planet and TRAPPIST-1 itself on the sky. The radius of TRAPPIST-1 is 0.1192 solar radii [?], leading to an apparent radius of about 1.1°. Transits are spotted when the angular distance between an inner planet and the host is smaller than the host' s apparent radius. Some typical transits are shown in Figure 8. The transits of all three inner planets are obvious. In particular, simultaneous transits of two planets are not difficult to find. This is because the TRAPPIST-1 system is both more co-planar and compact than the solar system, and the apparent radius of TRAPPIST-1 is much larger than that of the Sun as seen from Earth (16). These facts ensure that observed transits are not rare on TRAPPIST-1e.

It should be noted that since the eccentricities of TRAPPIST-1b and TRAPPIST-1d can increase to greater than that of Venus, we need to estimate the speed during transit to ensure the speed can be considered constant for calculating the distance between the host star and the planet. Our calculation confirms that the speed variation during transit for planets b, c, and d can all be neglected (Figure 9). During those transits, the speeds of the planets vary by only 0.1%. Thus, it can be assumed that the speeds of inner planets are constant during transits, and distance calculation via trigonometric parallax is valid, assuming the transit duration is determined by the contact parameters. For comparison, the transit speed of Venus seen from Earth is about 8.3×10^{-3} au day^{-1} and differs by less than 1%.

Since we have simulated transit events in the TRAPPIST-1 system, the distance of TRAPPIST-1e to the host when a transit is observed can be calculated as $R \sin \alpha$. To determine the horizontal stellar parallax of the host star α, Short' s method [?, ?] is implemented, as it was for the solar parallax with data collected from observations of the 1761 transit of Venus. An initial estimate is set to be $\alpha_i = 0.070°$, smaller than the accurate value 0.077° by an error of 9%, which is comparable to that of the 8.5 hypothesis of solar parallax [?]. The virtual observation of a transit of the closest planet, TRAPPIST-1d, is taken as an example. As seen from an observer at latitude 60°N on TRAPPIST-1e, this transit duration is observed to be $t_o = 0.3837$ hr (the mid-transit occurs at BJD = 2457738.602). Additionally, the theoretical duration for this observer calculated from α_i is $t_{th} = 0.3852$ hr, and the theoretical duration as viewed from the center of TRAPPIST-1e is $t_c = 0.4063$ hr. (This calculation process is detailed by [?].) Thus, the proportion yields $\alpha = 0.075°$, equivalent to a distance from TRAPPIST-1e to the host of 0.02998 au, with a deviation from the accurate distance of 3%. Therefore, the absolute scale of the TRAPPIST-1 system can be determined via observations of transits.

4. The Test of Kepler' s Laws of Planetary Motion

Both the phenomena of retrograde motions and transits in the TRAPPIST-1 system will help develop an understanding of heliocentric theory and assist in measuring the semimajor axes of the planets. In this section, we test the precision of Kepler' s laws according to the simulated planetary ephemeris on TRAPPIST-1e.

4.1. Kepler' s First Law

Kepler's first law states that planets'orbits are ellipses. Kepler first verified Mars' elliptical orbit with his ingenious method of triangulation in *Astronomia nova* [?]. Before implementing this method, Kepler knew that Mars' orbital period was 687 days, which could be determined from the recorded time between its successive oppositions (synodic period) as well as Earth' s orbital period of 365 days. Kepler then collected from Tycho' s logbooks the apparent positions of Mars 687 days apart, when Mars was assumed to be at the same position in its orbit relative to the Sun but Earth was not (Figure 10). The directions to Mars indicated by at least two of those observations from Earth at different positions along its assumed circular orbit led to one common intersection point, which was taken as one orbital position of Mars. To locate another orbital position of Mars, a series of observations every 687 days apart with another start date was used. Given access to Martian data covering a couple of decades, Kepler triangulated several orbital positions of Mars. Thus, the motion trajectory of Mars was traced out and well fitted by an ellipse (with error reduced to 2 in the *Rudolphine Tables* [**?**]).

We determine the orbit of TRAPPIST-1f using the same method with the simulated ephemeris on TRAPPIST-1e (simulation over 500 days with an integration step of 1/2000 days). The orbit of planet e is assumed to be circular. Using each pair of data $T_f = 9.220$ days apart to triangulate just one position of planet f, ephemeris in two successive periods from BJD = 2457263.960 results in the orbit constructed by connecting every triangulated position, as shown in Figure 11. The reason why only a small portion of the whole orbit can be realistically constructed is the special proportion of orbital periods of planets e and f. Because $T_f/T_e = 1.5$, at dates T_f apart, planet e is always in either one of two roughly opposite directions from the host. In addition, once either opposition or conjunction of planet f occurs (when the host and two planets are lined up), these two events occur in turn in this series of observations. Even without considering the difficulty of observing planet f at conjunction (when the host can hinder the view), its directions seen from planet e at this date and the date T_f apart at its opposition are almost parallel, so their intersection point approaches infinity (gray dots in Figure 11), leading to a failure of triangulation. The method of triangulation becomes even more restricted when applied to planets g and h. Because T_g and T_h are nearly double and triple T_e, respectively, at dates T_g or T_h apart, not only the planet observed but also planet e is in the same position, so no triangle can be formed. This obstacle to constructing the orbits of planets is hardly noticed in the solar system.

Nevertheless, we can still fit most of the orbit of TRAPPIST-1f with an ellipse. The semimajor axis of planet f is fitted to be 1.3160 (scaled to the semimajor axis of planet e, as Kepler scaled the solar system with 1 au), with an error of only 0.02%. However, the eccentricity is fitted to be 0.0109, 34% larger than the accurate value 0.0081 during that duration. The error of eccentricity can be as large as 100% when adopting data at other dates, revealing another obstacle. While the discovery of Kepler's first law is partly owed to the relatively high eccentricity of Mars in the solar system, the planets in the TRAPPIST-1 system all have very low eccentricities (Table 1), resulting in large errors in fitted eccentricity. The high precession rate, which makes the orbit non-elliptical, may also contribute to the error. The small orbital period, allowing a short duration for observation, partly compensates for this. Still, the orbit orientation of planet f changes by \$ \$0.1° during the observation time (based on the orbital evolution shown in Section 2), two orders of magnitude larger than that of Mars [**?**].

Therefore, the low eccentricities, resonance chain, and possibly fast precession in the TRAPPIST-1 system can make Kepler's first law quite difficult to discover by triangulation. Still, the semimajor axes of planets can be precisely determined, and the relative scale of the whole system can be realized, which helps develop Kepler' s second and third laws.

4.2. Kepler' s Second and Third Laws

Kepler' s second and third laws are both based on the determination of planets' distances to the Sun. Although Kepler' s triangulation method can only determine the relative scale of the TRAPPIST-1 system (Section 4.1), transit events allow measurement of the absolute distance of TRAPPIST-1e to the host (Section 3.3). Thus, we consider it realistic to use the planets' absolute distances or semimajor axes to test the following two laws.

Kepler' s second law states that a line connecting any planet and the Sun sweeps out equal areas in equal lengths of time [?]. To validate Kepler' s second law, we obtain both the position and velocity vectors in heliocentric coordinates from numerical integration (simulation over 100 days with an integration step of 1/2000 days). By simulating the orbital motions of the seven planets, we obtain the areas swept every 1 day by each planet, as shown in Figure 12. The relative variations between the swept areas obtained over different times have magnitudes between 10^{-4} and 10^{-5}. This illustrates that in the TRAPPIST-1 system, Kepler' s second law can still be discovered and validated precisely over short timescales, e.g., 100 days.

In Kepler' s book *Harmonices Mundi* [?], he proposed his third law in Chapter 5, after describing the similar pattern of normal phenomena and the harmony of musical notes. The general form of Kepler' s third law can be expressed as:

$$T^2 = Ca^3$$

where T is the orbital period and C is a constant for all planets in the solar system. The original data in the book are shown in Figure 13, which demonstrates that his measurements of the semimajor axes and periods of the planets are only slightly different from modern values (Table 5). After converting the original data into modern units, we list Kepler' s calculation results in Table 5. It can be seen that the main error in Kepler' s calculation is due to uncertainty in the semimajor axis.

Using an expression similar to Kepler' s in the book (Figure 13), we list the semimajor axes and orbital periods of planets in the TRAPPIST-1 system in Table 6. All accurate semimajor axes or orbital periods are averages of simulated values over 100 days. The observed semimajor axis of planet e is set as the same as the accurate value 0.02929 au. The other planets' observed semimajor axes are obtained by Kepler' s triangulation method (Section 4.1) and then scaled to their absolute values. The observed period of planet e is the length of time the host takes to return to the same apparent position in the sky, while the other planets' observed periods are determined by the time between two successive transits.

As can be seen from Table 6, the data observed on TRAPPIST-1e basically support the validation of Kepler' s third law. The observed gravity constants C_o are consistent with theoretical values $C_{G\mu}$ with differences of no more than about 2%. Compared to Kepler’ s data for the solar system, the absolute errors of semimajor axes in the TRAPPIST-1 system are smaller. The constants C_k obtained by Kepler deviate from the theoretical values $C_{G\mu}$ by no more than 2%, the same amount as in the TRAPPIST-1 system, although the deviation is smaller for planets near Earth. In addition, the deviation from two-body motion caused by gravitational perturbations E_{o-a} is smaller than anticipated in such a compact system. Figure 14 further shows that on longer timescales like 1000 yr, the variation of the simulated Kepler constant C_a stays no greater than 0.05%, which is much less than the observational error E_{o-a}. In brief, the Kepler constant deviates from theoretical values due to both gravitational perturbations and observational error. Since the deviation is at a magnitude of 1%, we can still conclude that Kepler’ s third law can be proven in the TRAPPIST-1 system.

4.3. Great Inequality Disobeying Kepler’ s Laws

Extensive research on the Great Inequality is an indispensable chapter in the development of celestial mechanics, which has allowed astronomers to explicitly understand the motion of celestial bodies and the stability of the solar system [?]. Briefly, the Great Inequality refers to the deviation from empirical predictions and even ephemerides derived from Kepler’ s law or Newton’ s universal gravity theory before the 18th century. Extraordinary astronomers like Kepler and Halley observed that Jupiter always accelerates while Saturn decelerates, leading to an alarming prediction that Jupiter would fall into the Sun and Saturn would be driven away. To avoid these disasters and confirm the stability of our solar system, many outstanding scientists tried to reconcile this paradox. Euler and Lagrange introduced trigonometric functions into the calculation of perturbation, which made the analytical solution possible. Eventually, Laplace [?] and Lagrange [?] solved the problem by adding high-order perturbations between the planets and interpreting the Great Inequality as a 5:2 resonance between Saturn and Jupiter. They derived from energy conservation that:

$$\sum_i m_i \sqrt{a_i} \Delta n_i = 0$$

The other planets in the solar system can be ignored in the equation. Given Kepler’ s third law, this implies:

$$\frac{\Delta n_{\mathrm{Sat}}}{\Delta n_{\mathrm{Jup}}} = -\frac{m_{\mathrm{Jup}} a_{\mathrm{Jup}}}{m_{\mathrm{Sat}} a_{\mathrm{Sat}}} = -2.47$$

(calculated with modern values of planet parameters), where Δn is the deviation of mean motion from the unperturbed elliptical motion.

The research process of the Great Inequality is quite an excellent saga. In this work, we consider whether such a problem would impact observers on

TRAPPIST-1e. In the TRAPPIST-1 system, planets b, c, f, and g all have masses slightly more than 1 $M_\oplus$ and are in a long resonance chain. Therefore, it is not as realistic as in the solar system to estimate the ratio of mean motion deviations of two planets exclusively. Accounting for this, we derive theoretical ratios $\Delta n_c/\Delta n_b = -0.90$ and $\Delta n_g/\Delta n_f = -0.71$ from the mean semimajor axes in the simulation to compare with the simulated ratios. They are closer to 1 than $\Delta n_{\mathrm{Sat}}/\Delta n_{\mathrm{Jup}}$. We focus on those pairs of planets to check if the Great Inequality is noticeable.

The simulated mean motion deviations (mean motions calculated with simulated orbital periods and then subtracted by the averages) are illustrated in Figure 15. A negative correlation between planets b and c is visible over 0–1000 yr, and their deviations stay at nearly the same level. The average ratio of smoothed deviation $\Delta n_c/\Delta n_b$ over this time is -0.69, comparable to its theoretical estimate. We attribute this rough consistency to their leading terms in Equation (5) due to the shortest semimajor axes as well as their masses. As for the pair of planets f and g, the correlation is more complex. A slice of 2500–3500 yr from the whole simulation is taken as an example in Figure 15. In general, planets f and g exhibit a negative correlation, which is most obvious from about 2740 to 3040 yr. However, their deviations do not maintain a consistent proportion from decade to decade; e.g., the ratio $\Delta n_g/\Delta n_f$ around year 2600 is apparently lower than in the following years around 2660. Meanwhile, as the outermost planet whose mass of 0.326 $M_\oplus$ is one magnitude smaller than its neighbors, planet h occasionally shows a negative correlation with g before 2700 yr and after 3400 yr, when the correlation between planets f and g becomes ambiguous. The average ratio of smoothed deviation $\Delta n_g/\Delta n_f$ over this time is -1.41, nearly double its theoretical estimate. We also calculate the correlation coefficient between the smoothed mean motion deviations to verify the negative correlations. Over those selected times in Figure 15, the correlation coefficient between planets b and c is -0.74, whereas that between f and g is -0.59, though they both become milder over the whole 5 kyr, i.e., -0.69 and -0.46.

Though the clues to the Great Inequality are present in the TRAPPIST-1 system, it has to be made clear that this phenomenon is more elusive than in the solar system. Because TRAPPIST-1 is more compact and has multiple planets with comparable masses, contrary to the dominance of Jupiter and Saturn in the solar system, the accompanying deceleration and acceleration can transfer to different pairs of planets at different times. The potential civilization on TRAPPIST-1e may need to take more than two planets into account simultaneously to confirm energy conservation (Equation (5)), giving rise to a higher requirement for observations. Nevertheless, within \$ \$100 yr when the Great Inequality is stable between some pairs of planets—probably planets b and c—there is still a chance to spot this phenomenon and trigger exploration.

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5. Perihelion Precession due to General Relativity

In Section 4, we verified Kepler' s laws in the TRAPPIST-1 system based on Newtonian mechanics. At the beginning of the 20th century, Einstein developed the theories of special and general relativity [?, ?]. From astronomical observations in the following decades, Einstein' s theory was validated by the observed perihelion precession of Mercury [?], as well as gravitational lensing events [?] and gravitational waves [?, ?].

In this section, we focus on the perihelion precession of inner planets (i.e., planets b, c, and d) to test the effects due to general relativity (hereafter GR) and their observability in the TRAPPIST-1 system. Taking GR perturbations into consideration, the perihelion shift caused by relativity can be expressed as [?, ?]:

$$P_{\mathrm{GR}} = \frac{3n^2a^2}{c^2(1-e^2)}$$

where P_{GR} represents the perihelion precession rate $d\omega/dt$ caused by relativistic precession, n is the mean motion of the planet, and c is the speed of light. Thus, we can calculate the rate of perihelion precession of each planet via Equation (7).

Meanwhile, simulated data in Section 2 provide the perihelion precession due to the gravity of other planets P_g. Given the complex dynamics of the system, a slice of 1500–2000 yr from the whole simulation—a typical period when the precession rates of inner planets are relatively stable—is selected to derive P_g in Table 7.

Table 7 shows that the precession of inner planets is dominated by gravitational perturbations, while relativistic effects contribute \$ \$0.1% of the total precession rate. The gravitational precession P_g of planets b and c is quite close and both are fast, on the order of 100 day^{-1}, while planet d is slightly slower with a rate of 71 day^{-1}. The GR precession rate P_{GR} of the innermost planet b is the largest, 0.197 day^{-1}, while planet d has the smallest rate at 0.038 day^{-1}. Compared to Mercury, although the GR precession rates of planets b, c, and d are 1-2 orders of magnitude larger, the gravitational precession in the TRAPPIST-1 system is even 3-4 orders of magnitude larger than Mercury' s due to the compactness. Thus, the relative contribution of GR, P_{GR}/P_g, is no more than \$ \$0.1% for all three inner planets in TRAPPIST-1, dozens of times smaller than Mercury' s 8.1%. The pericenter precession rate caused by GR in TRAPPIST-1 is insignificant.

Because the inner planets in the TRAPPIST-1 system are much closer to their host star than Mercury (indicating a stronger tidal effect), the perihelion precession due to the host star' s tides needs to be considered. We use the formula below to calculate the precession speed due to tidal effects [?]:

$$P_t = \frac{15}{2}\frac{k}{Q}\frac{M_*}{M_p}\left(\frac{R_p}{a}\right)^5 n$$

Here k is the Love number, describing the elasticity of a planet (dimensionless) [?]. For terrestrial planets, k can be set as 0.3 [?]. We thus calculate the tidal effect on planets b, c, and d, as shown in Table 7.

We see that the effect of tides on the precession of TRAPPIST-1 planets, P_t, is at least 10^7 times stronger than on Mercury, but still, all tidal effects are weaker than relativistic effects, not to mention gravity. For the innermost planet b, GR is about 4 times the tidal effect. As the distance to the host star increases, the tidal effect decreases more rapidly than the GR effect due to its distance-sensitive property, leading to the ratio of precession rates due to GR and tides, P_{GR}/P_t, increasing rapidly. This ratio increases to 88 for planet d.

We further generalize this calculation of P_{GR}/P_t to planets of different masses in nearly circular orbits with varying distances from the host star. Figure 16 shows an apparent positive correlation between this ratio and the semimajor axis of a planet, regardless of the mass of the host star the planet is orbiting. The GR effect is greater than the tidal effect for a terrestrial planet of one Earth mass as long as a is greater than 0.0077 au. A larger planet mass strengthens tides, which are still exceeded by GR for a compact system like TRAPPIST-1. Therefore, tides can usually be neglected compared to the GR effect.

Considering that the gravitational precession rate is enhanced more significantly than the GR effect in the TRAPPIST-1 system, despite the relatively weaker and even negligible tides, the probability for intelligence on TRAPPIST-1e to discover GR is largely weakened by observing the pericenter precession of inner planets.

6. Conclusion

In this paper, we produce planet ephemeris in the typical compact multiple planetary system around an M dwarf, TRAPPIST-1, and test the precision of measuring gravity theory. In Section 2, we simulate the TRAPPIST-1 system over 5 kyr (Figure 1) and establish a planetary ephemeris for planet e. The evolution of the orbital elements of the seven planets is stable at the timescale of 1 Myr (Figure 2) despite the complex dynamics of such a compact system.

In Section 3, we first validate the present-day synchronous status of planet e via its short damping timescale of 1 kyr. Its spin evolution leads to insolation close to Earth, supporting habitability, and is then used to produce the retrograde motions of other planets in the horizontal coordinates of planet e. The retrograde motions seen from planet e show different characteristics from the solar system due to compactness, including smaller duration and interval and much higher frequency (Table 4). We also simulate the transit events of three inner planets occurring in TRAPPIST-1, which are needed to estimate the semimajor axes of these planets when testing Kepler' s Laws (Section 4).

In Section 4, we test Kepler's laws and the Great Inequality in the TRAPPIST-1 system. The triangulation method for constructing orbits results in fitting errors of eccentricities of several tens of percent, but those of semimajor axes are only 0.1%. The swept area of each planet in the simulation turns out to be quite stable on a short timescale of hundreds of days (Figure 12). With observation accuracy of $ $1% (Table 5), similar to Kepler' s calculation of the solar system, the constant of Kepler' s third law in TRAPPIST-1 varies within $ $0.01% on a timescale of 10^3 yr (Figure 14). Therefore, we conclude that although Kepler' s first law is hard to verify due to the low eccentricities in the TRAPPIST-1 system, Kepler' s second and third laws can still be developed in such a compact system.

In addition, correlations between the mean motions of planets indicate that the Great Inequality is elusive in TRAPPIST-1, though the accompanying deceleration and acceleration of planets b and c are relatively visible.

In Section 5, we test GR. The pericenter precession rates due to GR of the inner planets in the TRAPPIST-1 system are larger than Mercury' s, but the contributed fractions due to the GR effect are only dozens of times smaller than Mercury' s (Table 7). Thus, although the tidal effects acting on these planets are weak relative to GR, the probability for intelligence on TRAPPIST-1e to discover this phenomenon is largely weakened.

In summary, multi-planetary systems around M stars are usually compact and suffer from stronger gravitational perturbation, but in the TRAPPIST-1 system, this effect does not block potential civilizations from discovering basic gravitational laws, e.g., heliocentric theory and Kepler' s Laws. However, the resonant chain in such compact systems is complex, so we do not eliminate the possibility that Kepler' s first law and GR are hidden in secular chaotic dynamical evolution.

Acknowledgments

We thank the referee for constructive comments. This work is supported by the National Key R&D Program of China (2019YFA0706601), the National Natural Science Foundation of China (grant Nos. 11973028, 11933001, 11803012, 12150009, and 42005045), Science and Technology Foundation of Zhejiang Ocean University (No. 2021C21021), and Key Laboratory of Modern Astronomy and Astrophysics (Nanjing University), Ministry of Education. We also acknowledge the science research grants from the Civil Aerospace Technology Research Project (D010102), as well as the China Manned Space Project with NO. CMS-CSST-2021-B12 and CMS-CSST-2021-B09.

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Note: Figure translations are in progress. See original paper for figures.