

Relay Observation Scheduling of Globally Distributed Telescope Array Based on Integer Programming (Postprint)

Authors: Junhan Ju, Ce Yu, Yi Hu, Yajie Zhang, Chao Sun and Jizeng Wei

Date: 2025-02-25T00:00:00+00:00

Abstract

Certain transients require regular observations over several days at intervals of hours or shorter, which cannot be accomplished by telescopes at a single site. The deployment of globally distributed telescopes at geographic locations of different longitudes enables the periodic monitoring of transients through relay observation. However, the simultaneous relay observation of numerous targets requires a telescope array of multiple telescopes that can be efficiently coordinated, and an automated scheduler for the array. This paper proposes IPROS, an integer programming model relay observation scheduler for a telescope array, that accounts for the entire process of relay observation and is consistent with the practical scenarios. We introduce the integer programming mathematical model for the relay observation scheduling problem with the telescope array, upon which the scheduler is based. Additionally, we propose an algorithm to provide a comprehensive formulation of the optimization objective of minimizing cadence deviation in the model. Experimental results demonstrate that the relay observation scheduler based on the integer programming model can effectively address the telescope array relay observation problem. It shows superiority over a scheduler with non-specific consideration of relay observation in the modeling and a scheduler based on greedy thought.

Full Text

Preamble

Research in Astronomy and Astrophysics, 25:015004 (13pp), 2025 January

© 2025 National Astronomical Observatories, CAS and IOP Publishing Ltd. All rights, including for text and data mining, AI training, and similar technologies, are reserved. Printed in China.

<https://doi.org/10.1088/1674-4527/ad981d>
CSTR: 32081.14.RAA.ad981d

Analysis of Nuclear Structure Properties and Stellar β -decay Rates of Even-Even 106-120Zr Isotopes

Abdul Kabir¹, Jameel-Un Nabi², Syeda Anmol Rida², Izzah Anwaar¹, Noor-Ul Ain Raza¹, and Hamad Almujiabah³

¹ Department of Space Science, Institute of Space Technology, Islamabad 44000, Pakistan; kabirkhanak1@gmail.com

² University of Wah, Quaid Avenue, Wah Cantt 47040, Pakistan

³ Department of Civil Engineering, College of Engineering, Taif University, P.O. Box 11099, Taif 21974, Saudi Arabia

Received 2024 October 14; revised 2024 November 24; published 2025 January 2

Abstract

The nuclear ground state properties of even-even 106-120Zr nuclei have been investigated within the framework of the relativistic mean field (RMF) approach. The RMF model with density-dependent DDME2 and DDPC1 interactions is utilized for the calculation of potential energy curves, the nuclear ground-state deformation parameters (β_2), neutron separation energies (S_n and S_{2n}) and neutron skin thickness (r_{np}) of selected Zr isotopes.

Subsequently, the β -decay properties of Zr isotopes were studied using the proton-neutron quasi-particle random phase approximation (pn-QRPA) model. These include Gamow-Teller strength distributions, β -decay half-lives and stellar electron emission/positron capture rates. The β_2 values computed from the RMF model were employed in the pn-QRPA model as an input parameter for the calculations of β -decay properties for even-even 106-120Zr nuclei. The stellar rates were computed using the pn-QRPA framework with three different types of deformation parameters. Only at high temperature ($T_9 \geq 2$) and low density ($Y_e \leq 10^7 \text{ g cm}^{-3}$) values does the sum of electron emission and positron capture rates have a sizeable contribution (with positive exponents) to the stellar rates.

Key words: astroparticle physics -elementary particles -nuclear reactions - nucleosynthesis -abundances -(stars:) supernovae: individual (r-process)

1. Introduction

The rapid neutron capture process (r-process) is the primary nucleosynthesis mechanism responsible for the production of nuclei more massive than iron (Burbidge et al. 1957). Though the astrophysical regions of this process are contested, it occurs in environments with exceptionally high neutron density. The nucleosynthesis route involves neutron-rich isotopes that are far away from

the β -stability valley. Important nuclear characteristics (Cowan et al. 1991) for describing the r-process are the nuclear masses and beta-decay properties, specifically the β -decay half-lives ($T_{1/2}$) and the β -delayed neutron-emission probabilities (P_n).

The β -decay half-life is an observable physical quantity for rare-isotope beam facilities that determines the timescale of r-process nucleosynthesis. Short half-lives observed at the $A = 110$ mass region enhance the flow of r-matter. Neutron-rich nuclei in the mass range of $A = 110$ – 120 have attracted substantial attention from both theoretical and experimental perspectives due to their fascinating properties. For example, they exhibit rapid structural variations in their ground state and low-lying excited states (Wood et al. 1992). At $Z = 40$, the shell structures show a wide range of forms: spherical, prolate, oblate, and other rare tetrahedral shapes (Skalski et al. 1997). Therefore, variations in shape and maximal deformation at deformed subshell closures are expected at $Z = 40$. Both relativistic (Xiang et al. 2012) and nonrelativistic (Bender et al. 2008) analyses of structural evolution for $Z = 40$ and $A = 110$ – 120 indicate rapid variation of nuclear shapes with changing nucleon numbers. Shape coexistence is observed, with nuclei exhibiting competing spherical, axially symmetric prolate and oblate, as well as triaxial morphologies at energies close to each other.

Investigating the nuclear masses and charge-changing transition rates of neutron-rich species at radioactive ion-beam facilities has been a major focus in recent decades (Sumikama et al. 2011). However, many exotic nuclear species cannot be studied in terrestrial laboratories, and addressing this issue is best accomplished through theoretical approaches. In neutron-rich environments, electron neutrino captures significantly amplify the effects of β -decays, and the subsequent neutrino-induced neutron spallation profoundly alters the r-abundance distribution pattern (McLaughlin & Fuller 1997). In neutron-rich Zr and Mo isotopes, deformation is a common characteristic that significantly influences β -decay rates and double β -decay matrix elements in neutron-rich nuclei (Sarriguren et al. 2003). Therefore, it is crucial for theoretical calculations to incorporate these nuclear deformations (Ni & Ren 2014).

Sumikama et al. (2011) investigated the low-lying states in ^{106}Zr and ^{108}Zr , including isomers with $T_{1/2} = 620 \pm 150$ ms in ^{108}Zr , and discussed different shaped isomers for ^{108}Zr . Kumar et al. (2014) studied the ground and excited state properties for $Z = 40$ isotopes employing relativistic and non-relativistic mean field approaches, predicting spherical shapes for most Zr isotopes in their ground states and large deformations in low-lying excited states. Nomura et al. (2016) demonstrated rapid structural changes between neutron numbers $N = 58$ and $N = 60$ in isotopes ranging from ^{94}Zr to ^{110}Zr using the SCMF-to-IBM mapping procedure with the Gogny-D1M effective density functional. Furthermore, β -decay properties aid in understanding isotopic abundances and potential r-process pathways.

Pereira et al. (2009) measured the β -decay properties of certain neutron-rich isotopes ($A \leq 110$) including Zr. Sarriguren & Pereira (2010) analyzed the Gamow-

Teller (GT) strength distributions and β -decay properties of neutron-rich nuclei including Zr within the framework of deformed quasi-particle random-phase approximation (QRPA). Yoshida et al. (2023) investigated the β -decay half-lives in Zr isotopes with shape changes, evaluating GT strength distributions in the proton-neutron QRPA and quasiparticle-vibration coupling models to investigate the effect of shape transition on half-lives.

From the existing literature, we find that no previous work has addressed stellar weak interaction rates for $Z = 40$ within $A = 110\text{--}112$. For our analysis of stellar rates we utilize both the RMF (Walecka 1974) and pn-QRPA models to provide a reliable description of the nuclear structure and β -decay properties for these isotopes. The pn-QRPA theory is capable of performing microscopic calculations of weak rates for any heavy nuclides. Klapdor et al. (1984) were the first to conduct calculations of β -decay rates for many nuclei from the line of stability using microscopic nuclear theory. This model was later utilized to calculate the β -decay half-lives of 6000 neutron-rich nuclei, covering a broad range from the neutron drip line to the line of stability (Staudt et al. 1999). The same pn-QRPA model, with a separable and schematic interaction, was used to perform microscopic calculations of β^+ electron capture rates of neutron-deficient nuclei with atomic numbers ranging from Zr nuclei of $A = 106\text{--}120$, extending up to the proton drip line for over 2000 nuclei (Hirsch et al. 1993). The weak interaction rates have contributed significantly to our understanding of the r-process, including instability effects studied in dense electron-positron plasmas (Usman & Mushtaq 2023).

Fuller et al. (1982) conducted a thorough effort to compile these rates at stellar temperatures and densities, with specific emphasis on decays from excited states of parent nuclei. The pn-QRPA model is extensively employed for its robust and precise computation of β -decay properties in unstable nuclei under both terrestrial and stellar conditions. Spanning a remarkable range of roughly 6000 nuclei between the neutron drip line and the stability line, the β -decay characteristics have been meticulously investigated using the pn-QRPA framework. This model, employing a schematic and separable interaction, has been applied under both terrestrial (Hirsch et al. 1993) and stellar conditions (Nabi & Klapdor-Kleingrothaus 1999). The pn-QRPA technique was harnessed to unravel the β -decay properties of waiting point nuclei, focusing on their relevance for astrophysical applications (Nabi et al. 2019). Recent studies have explored the dependence of GT strength on nuclear deformation in neutron-deficient isotopes of $178\text{--}192\text{Hg}$, $185\text{--}194\text{Pb}$, $67\text{--}80\text{As}$ (Nabi et al. 2024) and $196\text{--}206\text{Po}$ (Nabi et al. 2023).

In this study, we concentrate on neutron-rich nuclei with masses around $A = 106\text{--}120$, which are highly significant for the astrophysical r-process. Additionally, neutron-rich isotopes within $A = 106\text{--}120$ are recognized as intriguing examples where the equilibrium shape of the nucleus undergoes rapid changes, and shape coexistence occurs with competing prolate, oblate, and spherical shapes at closely spaced energies.

This work is organized as follows: in Section 2, we provide a brief explanation of our models including the RMF and pn-QRPA framework. Section 3 presents the findings and discussion. Section 4 contains a summary and conclusions.

2.1. The RMF Model

The RMF model depicts nuclei such that nucleons interact with each other by exchanging various mesons and photons (Walecka 1974). The initial RMF model had several issues in expressing nuclear surface features and incompressibility, for which purpose a nonlinear variant was developed by Ring (1996). Further versions of the model, such as density-dependent (DD) meson exchange (ME) and point coupling (PC), were added, and the concept was referred to as covariant density functional theory (Meng et al. 2006). In this study, we utilized both the DDME and DDPC versions of the model. The DDPC version uses the isoscalar scalar (σ) meson, the isoscalar vector (ω) meson, and the isovector vector (ρ) meson fields to describe nuclear matter and single-particle nuclear characteristics. The DDPC variant begins with a Lagrangian density that incorporates the coupling of protons to the electromagnetic field, PC interaction terms, and the free-nucleon Lagrangian. Varying the Lagrangian with respect to the nucleon fields produces the single-nucleon Dirac equation with nucleon self-energies. For further discussion one can refer to Nikšić et al. (2008).

We employed the DIRHB code (Nikšić et al. 2014) to construct the potential energy curves (PECs), $^{106-120}\text{Zr}$ as a function of Z employing both DDME (Lalazisis et al. 2005) and DDPC interactions. To achieve this, RMF calculations with quadrupole moment constraints were carried out. For open-shell nuclei, pairing correlations are crucial, and the BCS approximation was employed to address these correlations. Moreover, all ground state computations were performed utilizing the constant G approximation (Karatzikos et al. 2010).

2.2. The pn-QRPA Model

This section discusses the formalism used to calculate beta decay properties including stellar weak rates at all temperatures and densities relevant to stellar environments using pn-QRPA theory. The pn-QRPA framework was used to compute the GT strength and terrestrial β -decay half-lives. The Hamiltonian in the pn-QRPA model is structured with contributions from the single-particle Hamiltonian, nucleon-nucleon pairing interaction, particle-particle (pp) and particle-hole (ph) GT forces.

The Nilsson model was utilized to assess single-particle wave functions and energies (Nilsson 1955). The oscillator constant is analyzed employing standard procedures. The Nilsson-potential variables were selected based on Ragnarsson & Sheline (1984). Mass excess values were taken from AME2020 (Wang et al. 2021) for Q-value analysis. We used $^{106-120}\text{Zr}$ values adopted from Möller et al. (2016) and RMF-based analysis.

The occupancy amplitudes were determined using the BCS equations, with the time-reversed state of k expressed by \bar{k} . Ground-state correlations are introduced by adding a residual interaction to the Hamiltonian, which is then treated within the RPA framework.

The creation operators of QRPA phonons are defined as follows, where p (proton) and n (neutron) indices represent quasiparticle states. The summation was over p - n pairs satisfying the conditions: $k_p - k_n = (-1, 0, 1)$ and $\pi_p \cdot \pi_n = 1$. The amplitudes of the pn -QRPA phonon are $(X\omega, Y\omega)$ and the RPA equation's eigenvalue is expressed by the excitation energy ω . The pp (\cdot) and ph (\cdot) are GT forces known as residual interactions in the pn -QRPA model. The pp interaction was incorporated in our Hamiltonian using standard formulations, and similarly for the ph interaction.

The pairing force was calculated using the BCS approximation, performed separately for both protons and neutrons. We employed a constant pairing force of strength G (G_p and G_n for protons and neutrons, respectively). The spherical nucleon basis was transformed into a deformed basis using the Bogoliubov transformation. The inclusion of the GT residual interaction significantly alters the calculated GT strength distribution and is crucial for accurately reproducing observed GT strength functions.

The pp and ph GT residual strengths were calculated using the ratios $0.58/A^0 \cdot 7$ and $5.2/A^0 \cdot 7$, respectively (Homma et al. 1996). Our findings supported the model-independent Ikeda sum rule (Ikeda et al. 1963). The GT transition probabilities were determined utilizing QRPA.

The partial half-lives $t_{1/2}$ were determined employing the equation involving phase space integrals and transition probabilities, where $K = 6143$ s (Hardy & Towner 2009) and the ratio gA/gV was taken as -1.2694 (Nakamura & Particle Data Group 2010). The phase space integrals $fV(A, Z, E)$ (vector transition) and $fA(A, Z, E)$ (axial vector transition) are used with BGT (GT transition probabilities) and BF (Fermi transition probabilities). Analysis of BF is straightforward and may be observed from Hirsch et al. (1991). The terrestrial half-lives of β -decay were computed accordingly, and for further analysis of the equations one can refer to Staudt et al. (1999).

The stellar β -decay (subsequently referred to as electron emission, EE) and positron capture (PC) rates between parent level n and daughter state m have been determined using the pn -QRPA framework. The construction of low-lying excited levels and computation of nuclear matrix elements in our analysis may be found in Muto et al. (1992). The phase space factor f_{mn} depends on core temperature (T), core density (ρ) and Fermi energy (E_f). It was calculated using standard equations for EE decay rates, while f_{nm} for PC were computed using the appropriate relation where E_k is the kinetic energy of the electron and E_l is the total capture threshold energy. The Fermi functions $F(\pm Z, E_k)$ were calculated using the method described in Gove & Martin (1971).

The total β -decay energy was determined using mass differences and excita-

tion energies. The distribution functions were determined with the Boltzmann constant $k\beta$. The electron number density, related to nuclei and protons, is determined using the relationship involving the Avogadro number N_A , the ratio of electrons to baryon number Y_e , and the momentum of positron/electron. The total stellar weak rates are computed by summing over initial and final states until the rate computation reaches the necessary degree of convergence, where P_m is the occupancy probability of parent excited states calculated using the Boltzmann distribution.

3. Result and Discussions

We have provided detailed calculations of binding energy, one- and two-neutron separation energies, neutron skin thickness (r_{np}), and nuclear deformation (β_2) for 106-120Zr isotopes using the RMF DDME2 and DDPC1 functionals. We investigated β_2 in depth by performing PEC calculations within the RMF framework with constraints on the quadrupole moment to compute binding energy. The PECs were obtained by calculating the difference between the binding energy for specific β_2 values and the lowest binding energy for the nuclei under investigation. Nuclear shapes were determined by the PEC minima: prolate nuclei resulted from minima located on the positive side of β_2 , whereas oblate nuclei were found on the negative side.

Figure 1 [Figure 1: see original paper] shows the potential energy curves for even-even 106-120Zr isotopes plotted as a function of β_2 . The PECs obtained from RMF calculations demonstrate that both DDME2 and DDPC1 exhibit well-developed oblate and spherical shapes at the energy minima. Transitions occur from oblate to spherical shapes for 106-108Zr. For example, the ground states of 106-116Zr are located in the oblate sector with $\beta_2 = -0.36$ for DDPC1 interactions. Similarly, DDME2 gives an oblate morphology for 106-108Zr and a prolate minimum for 110Zr. In the isotopic region 112-120Zr via DDME2 and 118-120Zr via DDPC1, spherical shapes are exhibited. The general trend observed is the gradual disappearance of both prolate and oblate minima, collapsing into spherical shapes in the heavier isotopes at the magic number $N = 82$.

The present computed β_2 values, along with those from Möller et al. (2016), are displayed in Table 1. Neutron separation energies for one-neutron (S_n) and two-neutron (S_{2n}) are defined as:

$$\begin{aligned} S_n(Z, N) &= E_b(Z, N) - E_b(Z, N - 1) \\ S_{2n}(Z, N) &= E_b(Z, N) - E_b(Z, N - 2) \end{aligned}$$

where $E_b(Z, N)$, $E_b(Z, N - 1)$, and $E_b(Z, N - 2)$ are the binding energies for the Zr nuclei obtained by employing the RMF model with DDME2 and DDPC1 interactions. S_n indicates the stability of a nucleus against emission of a single neutron, with higher values meaning greater stability. S_{2n} provides information on stability against emission of two neutrons, which is particularly important for

defining the neutron drip lines that mark the limits where neutron-rich isotopes can exist.

Generally, S_{2n} decreases with increasing neutron number, indicating reduced stability as more neutrons are added. However, a sharp change in the S_{2n} slope is observed at magic numbers due to neutron shell closures, which confer extra stability. This is illustrated in Figures 2 and 3 [FIGURE:2, FIGURE:3]. The S_n values are close to FRDM-based results and experimental data, while the S_n computed via RMF DDPC1 is on average 0.45 MeV lower than measured data. However, the computed S_{2n} values are in good agreement with both FRDM calculations and experimental data (Wang et al. 2021). At magic numbers, S_n shows discontinuities that reduce neutron capture rates, causing the r-process matter flow to move closer to stability where nuclei have substantially larger β -decay half-lives.

Neutron skin thickness $r_{np} = r_n - r_p$ must be well understood for astrophysics and nuclear physics challenges, as it is connected to the slope parameter of the symmetry energy of nuclear matter. The r_{np} for the Zr nuclei under investigation is plotted in Figure 4 [Figure 4: see original paper] along with the DDMEX and DDPCX predictions of Liu et al. (2024). From basic nuclear physics, r_{np} increases linearly with the addition of neutrons in the isotopic channel.

After analyzing the nuclear ground state properties, we focus on the detailed analysis of nuclear structure and stellar beta decay properties, including GT strength, half-lives, and stellar rates for even-even Zr nuclei using the pn-QRPA model. For the analysis of beta decay properties, we tested deformation parameters by applying β_2 to the deformed Nilsson potential as input. The β_2 employed in pn-QRPA from FRDM is designated pn-QRPA-FRDM, while those from RMF-DDPC1 and RMF-DDME2 are designated pn-QRPA-DDPC1 and pn-QRPA-DDME2, respectively.

Figure 5 [Figure 5: see original paper] shows the computed GT strength for ^{106}Zr along with experimental data (Ha et al. 2020). One can observe that the GT strength transitions computed with FRDM and RMF-based β_2 are in good agreement with available measured data. Figure 6 [Figure 6: see original paper] shows the computed GT for ^{108}Zr along with experimental data (Ha et al. 2020). The pn-QRPA-FRDM and pn-QRPA-DDPC1 results are well matched at lower and higher excitation energies, while pn-QRPA-DDME2 intensities are higher than the other predictions. The same trend is found for ^{110}Zr , as depicted in Figure 7 [Figure 7: see original paper]. Overall, the predicted GT strength shows good agreement with measured data (Ha et al. 2020).

The pn-QRPA model was used to calculate the β -decay half-lives of even-even Zr isotopes in the mass region $106 \leq A \leq 120$. The analysis of terrestrial half-lives, computed via the pn-QRPA model along with measured data (Wang et al. 2021), is shown in Figure 8 [Figure 8: see original paper]. Remarkably, our calculations via pn-QRPA-FRDM and pn-QRPA-DDPC1 agree with measured half-lives within a factor of 2, while calculations via pn-QRPA-DDME2 align

within a factor of 1.2. This underscores the robustness of the pn-QRPA model, renowned for its predictive accuracy in neutron-rich nuclei. The predictions based on pn-QRPA-DDME2 are more closely fitted to measured data.

After analyzing terrestrial half-lives, we examined the stellar PC and EE rates for Zr nuclei. The residual interaction parameters were optimized based on terrestrial half-lives and the Ikeda sum rule (where experimental half-lives were unavailable). We analyzed stellar rates at densities $Y_e = 10^4, 10^7$ and 10^{10} g cm^{-3} at temperatures $T_9 = 2-10$ (where T_9 is in units of 10^9 K). No explicit quenching factor was included in pn-QRPA calculations. The stellar rates increase with rising core temperatures because the occupation probability of parent excited states rises with temperature, leading to finite contributions to total stellar rates. Increasing stellar core density substantially reduces available phase space, leading to smaller rates (see Equation (15) in Kabir et al. 2024).

To our knowledge, stellar rates for 106-120Zr have not been calculated previously. We performed these calculations using the pn-QRPA model with three different deformation parameter values. The EE and PC rates for 106Zr to 120Zr are listed in Table 2. The pn-QRPA-DDPC1 and pn-QRPA-DDME2 values are higher than pn-QRPA-FRDM. Qualitatively, pn-QRPA-DDME2 is higher than pn-QRPA-FRDM by a factor of 1.2 at $T_9 = 10$ and $Y_e = 10^4$ g cm^{-3} for 106Zr. Similarly, pn-QRPA-DDPC1 is higher than pn-QRPA-FRDM by a factor of 1.85. This trend is similar for EE and PC rates up to 118Zr. However, for 120Zr, pn-QRPA-DDME2 is higher than pn-QRPA-FRDM by a factor of 2.62 at $T_9 = 10$ and $Y_e = 10^4$ g cm^{-3} . At $Y_e = 10^7$ g cm^{-3} and 10^{10} g cm^{-3} , this factor reduces to 1.02. The same factors are observed for pn-QRPA-DDPC1. Generally, at low densities and high temperatures the stellar weak rates are meaningful, possessing magnitudes with positive exponents, as seen in Table 2.

4. Conclusion

The key findings and conclusions are outlined as follows. In the present work, we analyzed nuclear ground state and beta decay properties using the RMF and pn-QRPA models for even-mass Zr isotopes ($A = 106-120$). The nuclear ground state properties including PECs, β 2, Sn, S2n, and rnp were analyzed using RMF-based calculations with DDPC1 and DDME2 interactions. These predictions show oblate shapes for 106-116Zr and spherical shapes for 118-120Zr using DDPC1, while DDME2 predicts oblate shapes for 106-108Zr and spherical shapes for the remaining isotopes. Furthermore, we analyzed Sn, S2n, and rnp for the same Zr nuclei, finding decent agreement with measured data and previous predictions.

The β 2 values computed via the RMF model were later used as input parameters in the pn-QRPA model to perform self-consistent calculations of β -decay properties for Zr isotopes in the mass region $106 \leq A \leq 120$. The calculated GT distributions and half-lives were found to be in reasonable agreement with mea-

sured data. The stellar weak interaction rates for Zr isotopes were computed in a fully microscopic fashion without assuming the Brink-Axel hypothesis in the analysis of excited state GT distributions. The pn-QRPA-DDME2 rates are higher than those of pn-QRPA-DDPC1 and pn-QRPA-FRDM, with the reported weak rates computed using pn-QRPA-DDME2 larger than those computed with pn-QRPA-DDPC1 and pn-QRPA-FRDM by as much as a factor of 1.3. Only at high temperature ($T9 \geq 2$) and low density ($Y_e \leq 10^7 \text{ g cm}^{-3}$) does the sum of EE and PC rates have a sizeable contribution to the total stellar rates. The predicted stellar rates could prove useful for r-process nucleosynthesis calculations and future simulations of late-time stellar evolution.

Acknowledgments

Dr. A.K. would like to acknowledge useful discussions with Prof. Peter Ring. The authors extend their appreciation to Taif University, Saudi Arabia, for supporting this work through project number (TU-DSPP-2024-33). J.-U.N. would like to acknowledge the support of the Higher Education Commission Pakistan through project 20-15394/NRPU/R&D/HEC/2021.

Funding: This research was funded by Taif University, Saudi Arabia, Project No. (TU-DSPP-2024-33).

Declaration of Competing Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

- Bender, M., Bertsch, G. F., & Heenen, P.-H. 2008, *PhRvC*, 78, 054312
Burbidge, E. M., Burbidge, G. M., Fowler, W. A., & Hoyle, F. 1957, *RvMP*, 29, 547
Cowan, J. J., Thielemann, F.-K., & Truran, J. W. 1991, *PhR*, 208, 267
Fuller, G. M., Fowler, W. A., & Newman, M. J. 1982, *ApJ*, 252, 715
Gove, N. B., & Martin, M. J. 1971, *ADNDT*, 10, 205
Ha, J., Sumikama, T., Browne, F., et al. 2020, *PhRvC*, 101, 044311
Hardy, J. C., & Towner, I. C. 2009, *PhRvC*, 79, 055502
Hirsch, M., Staudt, A., Muto, K., & Klapdor-Kleingrothaus, H. V. 1991, *NuPhA*, 535, 62
Hirsch, M., Staudt, A., Muto, K., & Klapdor-Kleingrothaus, H. V. 1993, *ADNDT*, 53, 165
Homma, H., Bender, E., Hirsch, M., et al. 1996, *PhRvC*, 54, 2972
Ikeda, K., Fujii, S., & Fujita, J. I. 1963, *PhL*, 3, 271
Karatzikos, S., Afanasjev, A. V., Lalazissis, G. A., & Ring, P. 2010, *PhLB*, 689, 72
Klapdor, H. V., Metzinger, J., & Oda, T. 1984, *ADNDT*, 31, 81
Kumar, B., Singh, S. K., & Patra, S. K. 2014, *IJMPE*, 24, 1550017
Kabir, A., Nabi, J.-U., Tahir, M., Muneem, A., & Abideen, Z.U. 2024, *ChPhC*,

48, 094101

- Lalazissis, G. A., Nikšić, T., Vretenar, D., & Ring, P. 2005, PhRvC, 71, 024312
- Liu, Z. X., Lam, Y. H., Lu, N., & Ring, P. 2024, ADNDT, 156, 101635
- McLaughlin, G. C., & Fuller, G. 1997, ApJ, 489, 766
- Meng, J., Toki, H., Zhou, S., et al. 2006, PrPNP, 57, 470
- Möller, P., Sierk, A. J., Ichikawa, T., & Sagawa, H. 2016, ADNDT, 109, 204
- Muto, K., Bender, E., Oda, T., & Klapdor-Kleingrothaus, H. V. 1992, ZPhyA, 341, 407
- Nabi, J. U., Bayram, T., Riaz, M., et al. 2023, PhS, 98, 085314
- Nabi, J. U., Kabir, A., Khalid, W., Rida, S. A., & Anwaar, I. 2024, ChJPh, 92, 22
- Nabi, J. U., & Klapdor-Kleingrothaus, H. V. 1999, ADNDT, 71, 149
- Nabi, J. U., Ullah, A., Shah, S. A. A., Daraz, G., & Ahmad, M. 2019, AcPPB, 50, 1523
- Nakamura, K. & Particle Data Group 2010, JPhG, 37, 075021
- Ni, D., & Ren, Z. 2014, PhRvC, 89, 064320
- Nikšić, T., Paar, N., Vretenar, D., & Ring, P. 2008, PhRvC, 78, 034318
- Nikšić, T., Paar, N., Vretenar, D., & Ring, P. 2014, CoPhC, 185, 1808
- Nilsson, S. G. 1955, Dan. Mat. Fys. Medd., 29, 16
- Nomura, K., Rodriguez-Guzman, R., & Robledo, L. M. 2016, PhRvC, 94, 044314
- Pereira, J., Hennrich, S., Aprahamian, A., et al. 2009, PhRvC, 79, 035806
- Ring, P. 1996, PrPNP, 37, 193
- Ragnarsson, I., & Sheline, R. K. 1984, PhyS, 29, 385
- Sarriguren, P., Moya de Guerra, E., Pacearescu, L., & Faessler, A. 2003, PhRvC, 67, 044313
- Sarriguren, P., & Pereira, J. 2010, PhRvC, 81, 064314
- Skalski, J., Mizutori, S., & Nazarewicz, W. 1997, NuPhA, 617, 282
- Staudt, A., Bender, E., Muto, K., & Klapdor-Kleingrothaus, H. V. 1999, ADNDT, 44, 79
- Sumikama, T., Yoshinaga, K., Watanabe, H., et al. 2011, PhRvL, 106, 202501
- Usman, S., & Mushtaq, A. 2023, NatSR, 13, 15315
- Walecka, J. D. 1974, AnPhy, 83, 491
- Wang, M., Huang, W. J., Kondev, F. G., Audi, G., & Naimi, S. 2021, ChPhC, 45, 030003
- Wood, J. L., Heyde, K., Nazarewicz, W., Huyse, M., & Van Duppen, P. 1992, PhR, 215, 101
- Xiang, J., Li, Z. P., Li, Z. X., Yao, J. M., & Meng, J. 2012, NuPhA, 873, 1
- Yoshida, K., Niu, Y., & Minato, F. 2023, PhRvC, 108, 034305

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv – Machine translation. Verify with original.