The FAST Galactic Plane Pulsar Snapshot Survey. VI. The Discovery of 473 New Pulsars postprint

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Abstract

The Five-hundred-meter Aperture Spherical radio Telescope (FAST) is the most sensitive telescope at the L-band (1.0–1.5 GHz) and has been used to carry out the FAST Galactic Plane Pulsar Snapshot (GPPS) survey in the last 5 yr. Up to now, the survey has covered one-fourth of the planned areas within $±$10 from the Galactic plane visible by FAST, and discovered 751 pulsars. After the first publication of the discovery of 201 pulsars and one rotating radio transient (RRAT) in 2021 and 76 RRATs in 2023, here we report the discovery of 473 new pulsars from the FAST GPPS survey, including 137 new millisecond pulsars and 30 new RRATs. We find 34 millisecond pulsars discovered by the GPPS survey which can be timed with a precision better than 3 s by using FAST 15 minute observations and can be used for pulsar timing arrays. The GPPS survey has discovered eight pulsars with periods greater than 10 s including one with 29.77 s. The integrated profiles of pulsars and individual pulses of RRATs are presented. During the FAST GPPS survey, we also detected previously known pulsars and updated parameters for 52 pulsars. In addition, we discovered two fast radio bursts plus one probable case with high dispersion measures indicating their extragalactic origin.

Full Text

Preamble

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Modeling a Relativistic Star in Multi-layered Settings

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Abstract

This paper presents a new exact solution for dense stellar objects by employing the Einstein–Maxwell system of differential equations. The established model comprises three interior layers with distinguishable equations of state (EoSs): the polytropic EoS at the core layer, the quadratic EoS at the intermediate layer, and the modified Van der Waals EoS at the envelope layer. The physical features indicate that the matter variables, metric functions, and other physical conditions are viable for dense astrophysical objects. Significantly, this model extends the two-layered model generated by Sunzu and Lighuda. The layers are matched smoothly across the junctions with the Reissner–Nordström exterior spacetime. Utilizing our model, we generate star masses and radii compatible with observations and satisfactorily known objects. The findings from this paper may be useful for describing purported strange stars such as SAX J1808.4-3658 and binary stars such as Vela X-1.

Key words: stars: interiors – stars: general – stars: massive – stars: neutron

1. Introduction

The establishment of multi-layered stellar models has attracted considerable attention among researchers in astrophysics and related fields. A description of massive stellar bodies with several layers emerges from the general theory of relativity, developed by Albert Einstein between 1907 and 1916. This theory provides a satisfactory description of interior structure, focusing on modeling relativistic stellar bodies governed by sets of differential equations. From the general theory of relativity, Einstein field equations are constructed, relating metric functions, energy density, and the momentum tensor. Furthermore, the majority of researchers in astronomy and astrophysics have been utilizing Einstein equations to model relativistic stars through static or non-static spherically symmetric spacetime approaches (Fulara & Sah 2018). Some studies affirm that massive stellar bodies may automatically undergo disintegration after attaining an excited state that triggers transformation (Abreu et al. 2007; Sunzu

& Lighuda 2023). Other authors have reported that the collapse of interstellar bodies may consolidate to yield a white dwarf state, a neutron star state, or further collapse into black holes (Misner et al. 1973). The studies by Bonnor (1960) and Oppenheimer & Volkoff (1939) show that a dense object can change when a particular thermonuclear energy point is overly generated at the interior of the stellar sphere. The study by Ipser (1969), who constructed the theory for massive star clusters, indicates that clusters may lead to collapse and produce a black hole. The work of Gedela et al. (2018, 2019, 2021) affirmed that multi-layered stars may comprise complicated interior configurations that require more gravitational explanations. The arrangement of physical quantities inside dense objects may result in various physical features such as energy density, radial and transverse pressures, metric functions, balancing forces, and other physical acceptabilities in astrophysical studies (Mafa Takisa et al. 2019; Lighuda et al. 2021a, 2021b, 2023; Sunzu & Lighuda 2023).

Many studies have been performed with several layers to explain the interior structure of dense stellar objects. Some research affirms that a composite stellar object may contain sub-layers referred to as core and intermediate layers (Pant et al. 2019, 2020, 2021; Bisht et al. 2021; Lighuda et al. 2021a). The inner layer is strongly held by baryonic materials, while intermediate and envelope layers are composed of neutron fluids and Coulomb liquids respectively (Gedela et al. 2019; Mafa Takisa et al. 2019; Pant et al. 2020). In the study of astrophysical objects, it is essential to examine the dynamic effect of multi-layered objects for a thorough understanding of the physical features inside stellar spheres. Modeling anisotropic stellar objects requires equations of state that should be specified to explain the particular type of material carried in each layer. An EoS is a crucial aspect utilized to explain the physical characteristics of the interior of dense stars. Several multi-layered models have been constructed with distinguishable EoS in particular layers, relying on the density profile they control (Mardan et al. 2021; Sunzu & Lighuda 2023; Mathias et al. 2024a, 2024b). Many studies in general relativity utilize linear EoS, quadratic EoS, polytropic EoS, Chaplygin EoS, and the Van der Waals EoS. Some models employ these EoS, including the work of Sunzu & Mashiku (2018), Ngubelanga & Maharaj (2015), Pant et al. (2019, 2020), Lighuda et al. (2021a, 2021b), Bisht et al. (2021), Rahaman et al. (2010), Mafa Takisa & Maharaj (2016), Mafa Takisa et al. (2019), Maharaj & Mafa Takisa (2013), Gedela et al. (2019), Ngubelanga et al. (2015), Komathiraj & Maharaj (2007), and Kumar et al. (2019).

The inclusion of charge in the construction of anisotropic stellar models assists in producing stimulating physical features (Bhatia et al. 1969). Gravitating objects may abruptly gain or lose charge in their position in the Universe. The presence of charge inside the multi-layered sphere assists in balancing the gravitational field (Malaver 2017a, 2017b). Some researchers have reported that the existence of charge enhances the balance of stellar bodies against gravitational collapse, increases the mass and central curvature, and affects the redshift, compaction, and luminosity of stellar bodies (Varela et al. 2010; Mardan et al. 2021). The effect of electric fields in the study of multi-layered stars has also been investigated in the works of Maharaj et al. (2014), Sunzu et al. (2014a, 2014b, 2019), Sharma & Maharaj (2007), Lighuda et al. (2021a, 2021b), Lighuda et al. (2023), Sunzu & Lighuda (2023), and Mafa Takisa & Maharaj (2016).

The existence of pressure anisotropy in the interior of a stellar body has been demonstrated and discussed in many relativistic models (Murad 2016; Maurya et al. 2022). It has been reported that the presence of pressure in the interior of a stellar sphere relies on the flowing matter configuration (Maurya & Ortiz 2019). Some authors have asserted that it is not necessary for pressure to comply with the isotropic limit ($p_r = p_t$); instead, anisotropic criteria ($p_r \neq p_t$) may affect how the compact structures of dense stellar spheres like neutron stars, white dwarfs, and black holes appear (Komathiraj & Maharaj 2007; Thirukkanesh & Maharaj 2009; Bijalwan 2011). The work of Makalo et al. (2022) addressed the fact that pressure anisotropy could provide a variety of physical features due to a tough core, the existence of superfluid, or distinct transitions. The studies by Maurya & Ortiz (2019) and Maurya et al. (2022) affirmed that when the pressure anisotropy is positive (i.e., $\Delta = p_t - p_r > 0$), the stellar body experiences repulsion, and if it is negative (i.e., $\Delta = p_t - p_r < 0$), this results in attraction. Additionally, a positive measure of anisotropy helps to maintain stability and equalize the matter configuration inside anisotropic models. Models that describe the feasibility of pressure in relativity include the work of Bijalwan (2011), Komathiraj & Maharaj (2007), Bhar et al. (2017), Maharaj & Mafa Takisa (2013), Mathias et al. (2021), and Maurya et al. (2022).

Models that consist of multi-layers have been established regarding the inner and outer layers by employing the Einstein–Maxwell system of equations including EoSs. Recently, two-layered models have been formulated in the works of Mathias et al. (2024a, 2024b), Sunzu & Lighuda (2023), and Mardan et al. (2021), who inserted EoSs in the respective layers. The studies by Mafa Takisa & Maharaj (2016) and Mafa Takisa et al. (2019) developed models by substituting a linear EoS at the core and quadratic EoS at the envelope; results show that the envelope layer has the least concentration compared to the core layer. Thomas et al. (2005) constructed a stellar model that comprises an isotropic fluid state in the core layer and an anisotropic fluid state in the envelope layer. Tikekar & Jotania (2009) developed a relativistic model with anisotropic matter distributed in the core and isotropic matter in the envelope layer. The model formulated by Metcalfe et al. (2003) provides deep descriptions of the physical features of white dwarf objects. The works of Pant et al. (2019) and Gedela et al. (2018, 2019, 2021) elaborate on the feasibility of geometrical features of dense objects by applying a linear EoS in the core and quadratic EoS in the envelope layer. Similar features for two-layer models are also demonstrated in the works of Sharma & Mukherjee (2002), Hansraj et al. (2016), Montgomery et al. (2003), Tikekar & Jotania (2009), Ramesh & Thomas (2005), and Sunzu et al. (2019).

In studies of relativistic objects, fewer three-layered models have been constructed regarding density visibility from each layer. Some three-layered stellar models are found with the absence of an electric field describing the layers. The three-layered models developed by Bisht et al. (2021), Gedela et al. (2021), and Pant et al. (2020) have demonstrated and discussed features of the physical characteristics and geometrical properties of uncharged neutron stars. On the other hand, the charged core-intermediate-envelope models formulated by Lighuda et al. (2021a, 2021b) and Lighuda et al. (2023) have remarked on the physical properties of massive objects and highlighted the importance of electric charge in the construction of three-layered models.

This study aims to develop a charged three-layered model as an extension of the two-layered model constructed by Sunzu & Lighuda (2023). Our model comprises distinct layers: the core layer is fitted with a polytropic EoS, an intermediate layer is described by a quadratic EoS, and the envelope obeys a modified Van der Waals EoS. The work by Olengeile et al. (2023) generated a three-layered model where the Van der Waals EoS is inserted at the intermediate layer. The work by Lighuda et al. (2021a) developed three-layered models by utilizing a linear EoS at the intermediate layer and Chaplygin EoS at the envelope region. The model by Lighuda et al. (2023) utilized a polytropic EoS at the core, quark matter at the intermediate layer, and Chaplygin EoS at the envelope layer. A remarkable aspect of our paper is the third layer fitted with the Van der Waals EoS, a significant feature that distinguishes this work from Lighuda et al. (2021a, 2023), Olengeile et al. (2023), and Sunzu & Lighuda (2023). The geometrical features of the three-layered model are subsequently observed.

To develop a relativistic stellar model, we consider a static spherically symmetric spacetime described using Schwarzschild coordinates ($x^\iota = t, r, \theta, \phi$) as

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

The basic line element describing the exterior spacetime for a charged object is expressed in terms of Reissner–Nordström geometry:

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

where $\nu(r)$ and $\lambda(r)$ define the metric functions, Q stands for total charge, and M expresses the total mass of the object. The energy-momentum tensor for the charged body is regarded as

$$T_{ij} = \text{diag}\left(-\rho - \frac{E^2}{2}, p_r - \frac{E^2}{2}, p_t + \frac{E^2}{2}, p_t + \frac{E^2}{2}\right),$$

where ρ expresses energy density, E is electric field, and p_r and p_t denote radial and transverse pressures respectively. If $G = c = 1$ are set due to geometrical reasons, the Einstein–Maxwell field equations may be expressed in the form

$$\frac{1}{r^2}\left(1-e^{-\lambda}\right)+\frac{e^{-\lambda}\lambda'}{r}=\rho+\frac{E^2}{2},$$

$$\frac{1}{r^2}\left(e^{-\lambda}-1\right)+\frac{e^{-\lambda}\nu'}{r}=p_r-\frac{E^2}{2},$$

$$e^{-\lambda}\left(\frac{\nu''}{2}+\frac{\nu'^2}{4}-\frac{\nu'\lambda'}{4}+\frac{\nu'-\lambda'}{2r}\right)=p_t+\frac{E^2}{2},$$

$$\sigma=\frac{1}{r^2}e^{-\lambda/2}(r^2E)',$$

where primes ($'$ and $''$) signify the first and second derivatives respectively.

In our study, we transform and simplify the system of field equations by utilizing new variables obtainable from Durgapal & Bannerji (1982, 1983) written in the form

$$x=Cr^2,\quad Z(x)=e^{-\lambda(r)},\quad y^2(x)=e^{\nu(r)},\quad E^2(x)=\frac{Q^2(x)}{r^4}.$$

Putting Equation (5) into the system of differential equations (4), the transformed system of field equations is expressed as

$$\frac{1-Z}{x}-2\dot{Z}=\frac{\rho}{C}+\frac{E^2}{2C},$$

$$\frac{Z-1}{x}+\frac{2Z\dot{y}}{y}=\frac{p_r}{C}-\frac{E^2}{2C},$$

$$4xZ\frac{\ddot{y}}{y}+(4Z+2x\dot{Z})\frac{\dot{y}}{y}+\dot{Z}=\frac{p_t}{C}+\frac{E^2}{2C},$$

$$\frac{4C}{x}\frac{d}{dx}\left(\frac{x^2E^2}{C}\right)=\frac{2\sigma^2}{C}.$$

The basic line element (1) is now expressed in three layers as

$$ds^2=-y_i^2dt^2+\frac{dx^2}{4CxZ_i}+\frac{x}{C}(d\theta^2+\sin^2\theta d\phi^2),$$

where $i = a, b, c$ denotes the core, intermediate, and envelope layers respectively.

2.1. Region a (Core Layer)

The fundamental hypothesis of polytropes affirms that the pressure inside the sphere is normally countered by the force of gravity which depends on the density profile. The core layer is compact, consisting of solid matter configuration. The polytropic EoS is applied in the core layer since it is convenient for explaining solid materials (Mardan et al. 2021; Lighuda et al. 2023; Sunzu & Lighuda 2023). Notable here are the geometrical features that may result from polytropes.

This EoS is given as

$$p_r = \upsilon \rho^{1+\frac{1}{n}},$$

where υ and n correspond to arbitrary real constants, meeting the condition $n > 0$. Placing (6a) in (10) results in

$$\rho = \left(\frac{p_r}{\upsilon}\right)^{\frac{n}{n+1}}.$$

For ease, we have stated

$$p_r = Ax^n,$$

where A is an arbitrary constant. Equations (6b) and (11) are equated to give

$$\rho = \left(\frac{A}{\upsilon}\right)^{\frac{n}{n+1}} x^{\frac{n^2}{n+1}}.$$

Assembling (7) and (8) into Equation (12) results in the differential equation

$$4xZ\frac{\ddot{y}}{y} + (4Z + 2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} = \frac{A}{C}x^n + \frac{E^2}{2C}.$$

In the core layer, the total mass is expressed in the form

$$M_a(x) = 4\pi \int_0^x \frac{\rho(x)}{2\sqrt{Cx}}\frac{x}{C}dx.$$

Applying Equations (7), (8), and (13) and the system of differential equations (6), the metric functions and matter variables of the core layer are obtainable in the form

$$y_a(x) = \exp\left[\int \frac{1}{2Z_a}\left(\frac{Z_a - 1}{x} + \frac{p_{ra}}{C} - \frac{E_a^2}{2C}\right)dx\right],$$

$$\rho_a(x) = \left(\frac{A}{\upsilon}\right)^{\frac{n}{n+1}} x^{\frac{n^2}{n+1}},$$

$$p_{ra}(x) = Ax^n,$$

$$p_{ta}(x) = C\left[4xZ_a\frac{\ddot{y}_a}{y_a} + (4Z_a + 2x\dot{Z}_a)\frac{\dot{y}_a}{y_a} + \dot{Z}_a\right] - \frac{E_a^2}{2}.$$

2.2. Region b (Intermediate Layer)

Here, a quadratic EoS is inserted to describe the physical trends of the matter arrangement in the intermediate layer that is characterized by less fused substances, namely neutron fluids and Coulomb liquids. The quadratic EoS is assigned to emphasize that the radially directed force in the intermediate layer is less than in the core layer (Lighuda et al. 2021b; Sunzu & Lighuda 2023). This is written as

$$p_r = L\rho^2 + N\rho + \Upsilon,$$

note that L, N, and Υ are arbitrary real constants.

Regarding Equations (6a) and (16), we have acquired

$$\rho_b(x) = \frac{-N \pm \sqrt{N^2 - 4L(\Upsilon - p_{rb})}}{2L}.$$

Combining Equations (6b) and (17), and making use of Equations (7) and (8), we produce a differential equation in the form

$$4xZ_b\frac{\ddot{y}_b}{y_b} + (4Z_b + 2x\dot{Z}_b)\frac{\dot{y}_b}{y_b} + \dot{Z}_b = \frac{p_{rb}}{C} + \frac{E_b^2}{2C}.$$

Plugging (7), (8), and (18) into the field equation (6), matter variables and metric functions are obtainable as

$$y_b(x) = \exp\left[\int \frac{1}{2Z_b}\left(\frac{Z_b - 1}{x} + \frac{p_{rb}}{C} - \frac{E_b^2}{2C}\right)dx\right],$$

$$\rho_b(x) = \frac{-N + \sqrt{N^2 - 4L(\Upsilon - p_{rb})}}{2L},$$

$$p_{rb}(x) = \frac{1}{2}\left[\frac{Q_b^2(x)}{x^2} - \frac{E_b^2}{C}\right],$$

$$p_{tb}(x) = C\left[4xZ_b\frac{\ddot{y}_b}{y_b} + (4Z_b + 2x\dot{Z}_b)\frac{\dot{y}_b}{y_b} + \dot{Z}_b\right] - \frac{E_b^2}{2}.$$

The total mass of a stellar object in the intermediate layer is written in the form

$$M_b(x) = 4\pi\int_{x_a}^{x}\frac{\rho_b(x)}{2\sqrt{Cx}}\frac{x}{C}dx + M_a(x_a),$$

where c_1 denotes an integration constant.

2.3. Region c (Envelope Layer)

The envelope layer may be considered as gaseous matter that has the lowest density of all layers. We insert the modified Van der Waals EoS that is adequately suitable to describe the outermost layer (Malaver 2017a, 2017b). This is written as

$$p_r = \frac{\rho}{1-\beta\rho} - \alpha\rho^2 - \gamma,$$

where α, β, and γ are arbitrary real constants. The energy density and radial pressure are subsequently respectively given by

$$\rho_c(x) = \frac{1 + \sqrt{1 + 4\beta(\alpha + \frac{p_{rc}+\gamma}{\rho_c^2})}}{2\beta(\alpha + \frac{p_{rc}+\gamma}{\rho_c^2})},$$

$$p_{rc}(x) = \frac{\rho_c}{1-\beta\rho_c} - \alpha\rho_c^2 - \gamma.$$

Plugging Equation (22) into (21) yields

$$p_{rc} = \frac{\rho_c}{1-\beta\rho_c} - \alpha\rho_c^2 - \gamma.$$

Regarding Equations (23) and (24), the results are

$$\rho_c(x) = \frac{1}{2\beta}\left[1 \pm \sqrt{1 - 4\beta(\alpha+\gamma)}\right].$$

.

Employing the chosen metric function Z and the electric field E^2, we have arrived at a differential equation in the form

$$4xZ_c\frac{\ddot{y}_c}{y_c}+(4Z_c+2x\dot{Z}_c)\frac{\dot{y}_c}{y_c}+\dot{Z}_c=\frac{p_{rc}}{C}+\frac{E_c^2}{2C}.$$

Solving Equation (26), the matter variables and metric functions of the envelope layer are obtainable as

$$y_c(x)=\exp\left[\int\frac{1}{2Z_c}\left(\frac{Z_c-1}{x}+\frac{p_{rc}}{C}-\frac{E_c^2}{2C}\right)dx\right],$$

$$\rho_c(x)=\frac{1}{2\beta}\left[1-\sqrt{1-4\beta(\alpha+\gamma)}\right],$$

$$p_{rc}(x)=\frac{\rho_c}{1-\beta\rho_c}-\alpha\rho_c^2-\gamma,$$

$$p_{tc}(x)=C\left[4xZ_c\frac{\ddot{y}_c}{y_c}+(4Z_c+2x\dot{Z}_c)\frac{\dot{y}_c}{y_c}+\dot{Z}_c\right]-\frac{E_c^2}{2}.$$

The envelope layer composed of a gaseous fluid has total mass written in the form

$$M_c(x)=4\pi\int_{x_b}^{x}\frac{\rho_c(x)}{2\sqrt{Cx}}\frac{x}{C}dx+M_b(x_b),$$

where c_2 denotes an integration constant.

3. Matching Conditions

When trying to satisfy continuity, the matching of the radial pressure and metric functions should meet the requirements at the interface of the star boundary. The matching rules may be stipulated as:

The junction criterion at the core-intermediate interface: $e^{\nu_a}=e^{\nu_b}$, $e^{\lambda_a}=e^{\lambda_b}$, $p_{ra}=p_{rb}$ at $r=R_a$.

The junction criterion at the intermediate-envelope interface: $e^{\nu_b}=e^{\nu_c}$, $e^{\lambda_b}=e^{\lambda_c}$, $p_{rb}=p_{rc}$ at $r=R_b$.

The junction criterion at the envelope-surface needs the interior and exterior line elements (1) and (2) to match smoothly at the interface $r=R_c$. We get

$$y_c^2(R_c)=1-\frac{2M}{R_c}+\frac{Q^2}{R_c^2},$$

$$Z_c(R_c) = 1 - \frac{2M}{R_c} + \frac{Q^2}{R_c^2},$$

$$p_{rc}(R_c) = 0.$$

Substituting the required equations yields

$\zeta(10^{-3})$	$\vartheta(10^{-5})$	$\varepsilon(10^{-1})$	$M/M_\odot$	Model Applied
0.0200	0.0000685	0.675	1.43	Mathias et al. (2024a)
0.0198	0.0000680	0.670	1.42	Maurya et al. (2022)
0.0201	0.0000690	0.676	1.44	Sunzu & Lighuda (2023)
0.0199	0.0000683	0.672	1.43	Ngubelanga & Maharaj (2015)
0.0202	0.0000692	0.677	1.45	Mafa Takisa et al. (2019)

Stellar Masses and Radii Consistent with Other Findings

The occurrence of a reasonable number of unconstrained parameters $(A, M, Q, p_r, R_c, \alpha, \beta, \zeta, \omega, \varepsilon, \vartheta)$ in the system (32) affirms that the matching criterion has met the needs of our study.

4. Star Quantities Relevant with Astronomical Observations

Based on physical grounds, it is essential to compute the star mass, radius, and surface redshift for comparison with the geometrical features contained inside stellar bodies. The calculated values have been presented in Tables 1–3.

[Figure 1: see original paper] Energy density vs. radial distance.
[Figure 2: see original paper] Radial pressure vs. radial distance.
[Figure 3: see original paper] Transverse pressure vs. radial distance.

5. Physical Conditions

To avoid discontinuity inside the star, matter variables, metric functions, and other physical viability must meet the following requirements:

(i) The energy density (ρ), and radial and transverse pressures (p_r, p_t) must be greater than zero, continuous, regular, and defined throughout the interior of the star.

(ii) The metric functions $e^{-\lambda}$ and e^{ν} must be positive and unlimited throughout the interior.

(iii) The radial speed of sound must be smaller than the speed of light to coincide with the causality limit (Itoh 1970; Herrera 1992). In our study, we have computed in each layer as

$$\nu_r^2 = \frac{dp_r}{d\rho} < 1,$$

$$\nu_t^2 = \frac{dp_t}{d\rho} < 1.$$

(iv) The energy conditions should be positive and continuous to satisfy the strong energy condition (SEC), the weak energy condition (WEC), and the null energy condition (NEC), i.e.,

$$\text{SEC: } \rho - p_r - 2p_t \geq 0,$$

$$\text{WEC: } \rho - p_r \geq 0, \quad \rho - p_t \geq 0,$$

$$\text{NEC: } \rho + p_r \geq 0, \quad \rho + p_t \geq 0.$$

(v) The adiabatic index of imperfect fluid must agree with the critical stability criterion $\Gamma > \frac{4}{3}$ (Maurya & Ortiz 2019). In our formulation, the computation of this condition in each layer has been done as follows:

$$\Gamma = \frac{\rho + p_r}{p_r}\frac{dp_r}{d\rho}.$$

(vi) In general relativity, the surface redshift has been thoroughly studied by many researchers from an astrophysical perspective (Gedela et al. 2019, 2021; Pant et al. 2020, 2021; Lighuda et al. 2021a, 2021b, 2023). Some authors have suggested that surface redshift must be less than 5.211 for imperfect fluids (Baraco & Hamity 2002; Ivanov 2002, 2010). The criterion for surface redshift is represented by

$$z_s = \left(1 - \frac{2M}{R_c}\right)^{-1/2} - 1,$$

where $M(r)$ is the effective mass–radius ratio. In this study we have solved the redshift as follows:

$$z_s = y_c^{-1}(R_c) - 1.$$

(vii) The compactification factor of dense stellar objects has been examined thoroughly in the study of astrophysical objects. The models developed by Jasim et al. (2018, 2020), Mathias et al. (2021), Jape et al. (2021), and Sunzu & Lighuda (2023) exhausted the mass–radius ratio of astrophysical objects in the interval $10^{-5} \leq \frac{M}{R} \leq 0.5\ M_{\odot}$ km ¹ for the group: normal stars, white dwarfs, neutron stars, ultra-dense compact stars, and black holes. The compactification factor is expressed in the form

$$u(r) = \frac{M(r)}{r}.$$

Applying the criterion in this model produces

$$u(R_c) = \frac{M(R_c)}{R_c}.$$

In our treatment, we have calculated that

$$u(R_c) = 0.7347.$$

(viii) Determination of the equilibrium forces in the study of relativistic stars has been enhanced by the Tolman–Oppenheimer–Volkoff (TOV) equation. Many authors have been applying the TOV equation to assess the viability of the matter configuration inside the star (Jasim et al. 2018, 2020; Maurya & Ortiz 2019; Maurya et al. 2019a, 2019b, 2019c, 2022; Jape et al. 2021; Mathias et al. 2021; Sunzu & Lighuda 2023). Referring to a charged object, the four equilibrium forces in astrophysical study are gravitational force (F_g), anisotropic force (F_a), hydrostatic force (F_h), and electric force (F_e). The sum of equilibrium forces in general relativity must be zero:

$$F_g + F_a + F_h + F_e = 0.$$

The TOV equation is stated in the form

$$\frac{dp_r}{dr} = -\frac{(\rho + p_r)(M + 4\pi r^3 p_r - Q^2/r)}{r(r - 2M + Q^2/r)} + \frac{2}{r}(p_t - p_r) + \frac{\sigma Q}{r^2}.$$

[Figure 4: see original paper] Measure of anisotropy vs. radial distance.
[Figure 5: see original paper] Metric functions vs. radial distance.
[Figure 6: see original paper] Energy conditions vs. radial distance.
[Figure 7: see original paper] Adiabatic index vs. radial distance.
[Figure 8: see original paper] Radial speed vs. radial distance.
[Figure 9: see original paper] Charge density vs. radial distance.
[Figure 10: see original paper] Electric field vs. radial distance.
[Figure 11: see original paper] Mass vs. radial distance.

[Figure 12: see original paper] Equilibrium forces vs. radial distance.
[Figure 13: see original paper] Redshift vs. radial distance.
[Figure 14: see original paper] Mass–radius ratio vs. radial distance.

6. Physical Analysis

In this paper, we have investigated the physical features inside stars that comprise three layers with the inclusion of charge. The geometrical features are yielded in the ranges (0–2.5) km, (2.5–6.0) km, and (6.0–10) km for the core region, intermediate region, and envelope region respectively. The graphs for metric functions and matter variables are drawn versus radial distance at identified radius values as applied in various past studies (Gedela et al. 2019, 2021; Pant et al. 2020, 2021; Bisht et al. 2021; Mardan et al. 2021; Sunzu & Lighuda 2023; Mathias et al. 2024b). The graphs are obtainable by applying the specified values of the constants:

$$A = \pm 2.43, \quad n = 3, \quad L = \pm 0.47, \quad \gamma = \pm 0.325, \quad N = \pm 0.02,$$

$$c_0 = c_1 = c_2 = 0.005, \quad \alpha = \pm 0.000071, \quad \beta = \pm 0.252, \quad \Upsilon = 6.5,$$

$$\zeta = \pm 0.0200, \quad \upsilon = \pm 0.000148, \quad \vartheta = \pm 0.0000685, \quad \varepsilon = \pm 0.675.$$

Figures 1 and 2 affirm that the energy density and the radial pressure are maximum at the center, decreasing toward the envelope layer. The same profiles are also found in the treatments of Pant et al. (2019, 2020), Gedela et al. (2019), Maharaj & Mafa Takisa (2013), Maharaj et al. (2014), Lighuda et al. (2021a, 2021b, 2023), Mardan et al. (2021), and Sunzu & Lighuda (2023). Figure 3 indicates the transverse pressure which demonstrates a decreasing profile. This physical trend is also marked in the studies by Thirukkanesh & Maharaj (2008), Jape et al. (2021), Maurya et al. (2019a, 2019b, 2019c), Mardan et al. (2021), Ngubelanga & Maharaj (2015), and Sunzu & Lighuda (2023). The measure of anisotropy in Figure 4 shows an increasing behavior, reaching a maximum peak where it starts to cease with the variation of the radial coordinate. We see that the behavior of anisotropy may be accelerated with the existence of the electric field and charge density. This viable characteristic is also demonstrated in previous studies (Thirukkanesh & Ragel 2014; Murad 2016; Bhar et al. 2017; Pant et al. 2020, 2021; Bisht et al. 2021; Sunzu & Lighuda 2023). In Figure 5, the metric functions ($e^{-\lambda}$, e^{ν}) are observed to intersect at the boundary, which is reasonable in the study of astrophysical objects (Lighuda et al. 2021a; Sunzu & Lighuda 2023; Mathias et al. 2024a, 2024b).

Energy conditions in Figure 6 indicate reducing behavior toward the envelope layer, which complies with the suggested criteria. These physical trends are also seen in the models by Gedela et al. (2019, 2021), Mafa Takisa et al. (2019), Lighuda et al. (2021a), Mardan et al. (2021), and Mathias et al. (2021). Figure 7 represents the adiabatic index utilized to measure the stability condition of the stars ($\Gamma > 4/3$). In our model, we yield $\Gamma(x = 0) = 1.6736$, which affirms that the star is stable against gravitational collapse. A similar physical trend is also captured in Mathias et al. (2024a, 2024b), Sunzu & Lighuda (2023), Jasim et al. (2018, 2020), and Jape et al. (2021). Figure 8 affirms that the radial speed of sound has a lower range than the speed of light. In our study, we obtained values in the range $0.00798 < \nu_r < 0.225$, which is acceptable for the star. The same trend was also observed in various models constructed in the past (Gedela et al. 2019, 2021; Mafa Takisa et al. 2019; Pant et al. 2019, 2021; Mathias et al. 2021; Lighuda et al. 2023; Sunzu & Lighuda 2023). Figures 9 and 10 illustrate the charge density and electric field respectively. The two graphs of σ^2 and E^2 exhibit increasing behavior away from the physically acceptable center. Figure 11 indicates that the mass function increases from the origin toward the surface, which is consistent with the study of relativistic stars. In Figure 12, we observe that the hydrostatic force (F_h), the electric force (F_e), and the gravitational force (F_g) flow uniformly toward the surface where the anisotropic force (F_a) appears to increase. This can also be demonstrated in the work of Fulara & Sah (2018), Lighuda et al. (2021a), Das et al. (2016), Jape et al. (2021), Mathias et al. (2021), and Sunzu & Lighuda (2023).

Figure 13 depicts that the redshift increases monotonically versus radial coordinate (r), having the maximum value $z_s = 3.159$, which is valid in general relativity. We can also find this trend in the studies of Lighuda et al. (2021b), Maurya & Ortiz (2019), Maurya et al. (2019a, 2019b, 2019c, 2022), and Jasim et al. (2018, 2020). Figure 14 depicts the trend of the mass–radius ratio as an increasing function, reaching the peak value $\mu(x) = 0.7347$, which is consistent with the study of stars.

7. Conclusion

In our paper, we have yielded a new solution utilizing the Einstein–Maxwell system of differential equations. Our model comprises three interior layers, each satisfying a chosen EoS. Matching across the layers has been completed smoothly by utilizing exterior Reissner–Nordström spacetime. A thorough analysis of the physical features has been performed, affirming that all physical parameters are feasible and consistent with the study of massive stellar objects. Our solution extends the two-layered model developed by Sunzu & Lighuda (2023) into a three-layered model, yielding notable geometrical features in the third layer fitted with a modified Van der Waals EoS. These are physically demonstrated in various graphs.

In future work, we shall construct a charged model that will obey distinguishable multi-layered settings.

References

Abreu, H., Hernandez, H., & Nunez, L. A. 2007, CQGra, 24, 4631
Baraco, D. E., & Hamity, V. H. 2002, PhRvD, 65, 124028
Bhar, P., Singh, K. N., & Pant, N. 2017, InJPh, 91, 6
Bhatia, M. S., Bonazzola, S., & Szamosi, G. 1969, A&A, 3, 206
Bijalwan, N. 2011, Ap&SS, 336, 413
Bisht, R. K., Gedela, S., Pant, N., & Tewari, N. 2021, RAA, 21, 162
Bonnor, W. B. 1960, ZPhy, 160, 59
Das, A., Rahaman, F., Guha, B. K., & Ray, S. 2016, EPJC, 76, 654
Durgapal, M. C., & Bannerji, R. 1982, Ap&SS, 84, 409
Durgapal, M. C., & Bannerji, R. 1983, PhRvD, 27, 328
Fulara, P. C., & Sah, A. 2018, IJAA, 8, 46
Gedela, S., Bisht, R. K., & Pant, N. 2018, EPJA, 54, 207
Gedela, S., Pant, N., Upreti, J., & Pant, R. P. 2019, EPJC, 79, 566
Gedela, S., Pant, N., Upreti, J., & Pant, R. P. 2021, EPJP, 36, 215005
Hansraj, S., Maharaj, S. D., & Mlaba, S. 2016, EPJP, 131, 4
Herrera, L. 1992, PhLA, 165, 206
Ipser, J. R. 1969, ApJ, 158, 17
Itoh, N. 1970, PThPh, 1, 44
Ivanov, B. V. 2002, PhRvD, 65, 104001
Ivanov, B. V. 2010, IJTP, 49, 1236
Jape, J. W., Maharaj, S. D., Sunzu, J. M., & Mkenyeleye, J. M. 2021, EPJC, 81, 1057
Jasim, M. K., Deb, D., Ray, S., Gupta, Y. K., & Chowdhury, S. R. 2018, EPJC, 78, 603
Jasim, M. K., Maurya, S. K., & Al-Sawaii, A. S. M. 2020, Ap&SS, 365, 9
Komathiraj, K., & Maharaj, S. D. 2007, IJMPD, 16, 1803
Kumar, J., Maurya, S. K., Prasad, A. K., & Banerjee, A. 2019, JCAP, 11, 005
Lighuda, A. S., Maharaj, S. D., Sunzu, J. M., & Mureithi, E. W. 2021a, Ap&SS, 366, 76
Lighuda, A. S., Maharaj, S. D., Sunzu, J. M., & Mureithi, E. W. 2023, Prama, 97, 5
Lighuda, A. S., Sunzu, J. M., Maharaj, S. D., & Mureithi, E. W. 2021b, RAA, 21, 310
Mafa Takisa, P., & Maharaj, S. D. 2016, Ap&SS, 361, 262
Mafa Takisa, P., Maharaj, S. D., & Mulangu, C. 2019, Prama, 92, 40
Maharaj, S. D., & Mafa Takisa, P. 2013, GeReGr, 45, 1951
Maharaj, S. D., Sunzu, J. M., & Ray, S. 2014, EPJP, 129, 3
Malaver, M. 2017a, WSN, 86, 333
Malaver, M. 2017b, WSN, 86, 1
Makalo, P. D., Sunzu, J. M., & Mkenyeleye, J. M. 2022, NewA, 98, 10135
Mardan, S. A., Noureen, I., & Khalid, A. 2021, EPJC, 81, 912
Mathias, A. K., Maharaj, S. D., Sunzu, J. M., & Mkenyeleye, J. M. 2021, Prama, 95, 178
Mathias, A. V., Mkenyeleye, J. M., & Sunzu, J. M. 2024b, NewA, 110, 102216

Mathias, A. V., Sunzu, J. M., & Mkenyeleye, J. M. 2024a, NewA, 106, 101977
Maurya, S., Govender, M., Singh, K. N., & Nag, R. 2022, EPJC, 82, 49
Maurya, S. K., Banerjee, A., Jasim, M. K., et al. 2019a, PhRvD, 99, 044029
Maurya, S. K., Errehymy, A., Deb, D., Tell-Ortiz, F., & Daoud, M. 2019c, PhRvD, 100, 044014
Maurya, S. K., Maharaj, S. D., Kumar, J., & Prasad, A. K. 2019b, GeReGr, 51, 86
Maurya, S. K., & Ortiz, F. T. 2019, EPJC, 79, 85
Metcalfe, T. S., Montgomery, M. H., & Kawaler, S. D. 2003, MNRAS, 344, L88
Misner, W. C., Thorne, S. K., & Wheeler, A. J. 1973, Gravitation (San Francisco, CA: Freeman)
Montgomery, M. H., Metcalfe, T. S., & Winget, D. E. 2003, MNRAS, 344, 657
Murad, M. H. 2016, Ap&SS, 361, 20
Ngubelanga, S. A., & Maharaj, S. D. 2015, EPJP, 130, 211
Ngubelanga, S. A., Maharaj, S. D., & Ray, S. 2015, Ap&SS, 357, 74
Olengeile, L., Sunzu, M. J., & Mkenyeyele, M. J. 2023, NewA, 100, 102002
Oppenheimer, J. R., & Volkoff, G. M. 1939, PhRv, 55, 374
Pant, N., Gedela, S., Pant, R. P., Upreti, J., & Bishi, R. K. 2020, EPJP, 135, 180
Pant, N., Govender, M., & Gedela, S. 2021, RAA, 21, 109
Pant, R. P., Gedela, S., Bisht, R. K., & Pant, N. 2019, EPJC, 79, 602
Rahaman, F., Ray, S., Jafry, A. K., & Chakraborty, N. 2010, PhRvD, 82, 104055
Ramesh, T., & Thomas, V. O. 2005, Prama, 64, 5
Sharma, R., & Maharaj, S. D. 2007, MNRAS, 375, 1265
Sharma, R., & Mukherjee, S. 2002, MPLA, 17, 2535
Sunzu, J. M., & Lighuda, A. S. 2023, NewA, 100, 101977
Sunzu, J. M., Maharaj, S. D., & Ray, S. 2014a, Ap&SS, 1, 634
Sunzu, J. M., Maharaj, S. D., & Ray, S. 2014b, Ap&SS, 352, 719
Sunzu, J. M., & Mashiku, T. 2018, Prama, 91, 75
Sunzu, J. M., Mathias, A. K., & Maharaj, S. D. 2019, JApA, 40, 8
Thirukkanesh, S., & Maharaj, S. D. 2008, CQGra, 25, 235001
Thirukkanesh, S., & Maharaj, S. D. 2009, MMAS, 32, 684
Thirukkanesh, S., & Ragel, F. C. 2014, Ap&SS, 81, 1
Thomas, V. O., Ratanpal, B. S., & Vinodkumar, P. C. 2005, IJMPD, 14, 85
Tikekar, R., & Jotania, K. 2009, GrCo, 15, 129
Varela, V., Rahaman, F., Ray, S., Chakraborty, K., & Kalam, M. 2010, PhRvD, 82, 044052

Note: Figure translations are in progress. See original paper for figures.