

# The FAST Galactic Plane Pulsar Snapshot Survey. VII. Six Millisecond Pulsars in Compact Orbits with Massive White Dwarf Companions

## Postprint

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### Abstract

Binary millisecond pulsars with a massive white dwarf (WD) companion are intermediate-mass binary pulsars (IMBPs). They are formed via the Case BB Roche-lobe overflow evolution channel if they are in compact orbits with an orbital period of less than 1 day. They are fairly rare in the known pulsar population; only five such IMBPs have been discovered before, and one of them is in a globular cluster. Here we report six IMBPs in compact orbits: PSRs J0416+5201, J0520+3722, J1919+1341, J1943+2210, J1947+2304 and J2023+2853, discovered during the Galactic Plane Pulsar Snapshot survey by using the Five-hundred-meter Aperture Spherical radio Telescope, doubling the number of such IMBPs due to the high survey sensitivity in the short survey time of 5 minutes. Follow-up timing observations show that they all have either a CO WD or an ONeMg WD companion with a mass greater than about 0.8  $M_{\odot}$  in a very circular orbit with an eccentricity in the order of  $10^{-5}$ . PSR J0416+5201 should be an ONeMg WD companion with a remarkable minimum mass of 1.28  $M_{\odot}$ . These massive WD companions lead to a detectable Shapiro delay for PSRs J0416+5201, J0520+3722, J1943+2210, and J2023+2853, indicating that their orbits are highly inclined. From the measurement of the Shapiro delay, the pulsar mass of J1943+2210 was constrained to be 1.84

### Full Text

### Preamble

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## New Orbital Parameters of 850 Wide Visual Binary Stars and Their Statistical Properties

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### Abstract

Based on positional observations and measurements of radial velocities, the orbits of 850 wide visual binary stars have been determined. The parameters of the log-normal distributions for the histograms of orbital periods, stellar masses, and semimajor axes in astronomical units have been obtained. The eccentricity histogram for binary stars with orbital periods less than 400 yr follows a normal distribution centered at  $e = 0.545 \pm 0.029$ . For stars with longer periods, this distribution obeys the law  $f = 2e$ , within the accuracy of errors. The mass-to-luminosity relation for stars with well-determined masses is given by:  $\log(L_{\odot}) = 4.33 \log(M_{\odot})$ , where  $L_{\odot}$  and  $M_{\odot}$  are the luminosity and mass of the star in units of the solar luminosity and mass, respectively.

**Key words:** (stars:) binaries: visual -stars: statistics -stars: fundamental parameters

### 1. Introduction

The United States Naval Observatory maintains a comprehensive and continuously updated catalog of visual double stars (Mason et al. 2001), which currently contains over 154,000 records. However, as outlined in the Sixth Catalog of Orbits, approximately 3500 binary star orbits have been determined (Hartkopf et al. 2001). Additionally, these orbits have been determined by various authors using various methods, which complicates the application of data for different investigations such as orbital parameter distribution analysis for certain samples of the stellar population.

The catalog categorizes all orbits into five grades, from 1 (definitive) to 5 (indefinite). Initially, the grade assignment involved a significant subjective element (Worley & Heintz 1983). However, a well-formalized technique is now used, based on the convergence between positional observation data and ephemerides calculated using orbital elements. Additionally, the coverage of a visible ellipse by observational data should be taken into account. When the duration of

the observational series is short compared to the orbital period, this results in considerable uncertainties in estimating the orbital elements, even with highly precise data.

Formal application of the orbital element standard error estimates in statistical treatments may distort conclusions due to peculiarities of the computational approaches. For example, the same observational data set may yield quite different standard error values for two different techniques. This inhomogeneity in the approaches used for orbital element determination guarantees incorrectness in conclusions since the statistical weights are based on the error estimates.

The use of radial velocity differences between components derived from spectroscopic observations significantly decreases orbital parameter uncertainty for long-period binaries. It is reasonable to apply the method of apparent motion parameters (AMP; Kiselev & Kiyeva 1980) for this case. Recent improvements in radial velocity determination techniques in the context of exoplanet research allow astronomers to decrease radial velocity standard errors and detect the relative radial accelerations of binary components. Moreover, Gaia catalog astrometric data (positions and proper motions) are available for both components of most visual double stars (Gaia Collaboration et al. 2023). Gaia data have proven valuable for orbital solutions thanks to their high accuracy compared to ground-based measurements, despite providing just one measurement on the visible ellipse and in velocity space.

The significance of binary star investigation lies in the fact that these objects hold valuable information about the genesis of the stellar population, which is partially preserved in their kinematics and statistics. During the fragmentation of a protostellar cloud, various configurations of closely spaced stars can emerge, as they tend to form within a common gravitational potential well. Binary systems, in particular, are known to be among the most stable configurations. Analyzing the statistical parameters of binary star samples can enhance our understanding of the fragmentation process in protostellar clouds and the subsequent evolution of stellar populations. Investigation of long-period binary stars is especially significant, as a relatively short orbital period means a relatively short distance between the components during the early stages of their evolution. Therefore, objects that form simultaneously would have less influence on the binary being considered.

In this paper, we aimed to construct a comprehensive set of orbits of visual binary stars using a general technique for all objects. We relied on additional data regarding radial velocities, radial accelerations, precise positions, and proper motions obtained from the Gaia catalog. Furthermore, we propose to use sets of possible orbits for each binary star instead of orbit grades or error estimates of the orbital elements.

## 2. Data Sample Formation

We compiled a list of 850 visual binary stars to perform a new determination of their orbital parameters. All of these binary stars meet several requirements. First, the separation between components must be greater than 0.33 arcseconds. Second, the positional observations should cover a significant arc of the current orbit. Third, there should be a considerable amount of positional data at the level of accuracy of several milliarcseconds (mas) that have not been used before for orbital parameter determinations of these stars. These data have been obtained through ground-based (speckle and CCD) observations, available from the Gaia or Hipparcos databases. A portion of them resulted from astrometric observations (Izmailov et al. 2020) conducted with the Pulkovo 26-inch refractor ( $D = 65$  cm,  $F = 10.5$  m) equipped with an FLI ProLine09000 CCD (field of view =  $12 \times 12$ , scale =  $238$  mas  $\text{pix}^{-1}$ ).

The orbital elements of 745 stars in our sample are listed in the Catalog of Orbits and Ephemerides of Visual Binary Stars (Hartkopf et al. 2001). Many of these stars have a rich observational history. As noted in our third criterion, new highly accurate data for these stars (Pulkovo data, space-based, and speckle-interferometric measurements) are available. Therefore, refining the orbital elements is a logical next step. A significant portion of the raw positional data was obtained from the Washington Double Star database (WDS), provided by Rachel Matson upon our request.

### 3.1. General Notes on the Computation of Orbital Elements

There are many possible ways to construct orbital parameter determination techniques. In most cases, the algorithms are based on the well-known equations describing a transformation from coordinates in the orbital plane coordinate system ( $X, Y$ ) to the  $x, y$ -coordinates in the sky plane. The Thiele-Innes elements  $A, B, F$ , and  $G$  can be expressed as functions of the semimajor axis and the Campbell elements ( $i, \omega, \Omega$ ) that fix the orbital plane orientation relative to the sky plane. The corresponding formulas appear in almost every paper regarding the determination of orbital elements, from van den Bos (2016) to Halbwachs et al. (2022).

These equations are linear with respect to  $A, B, F$ , and  $G$ . Accordingly, the standard least squares approach is applied to calculate Thiele-Innes elements using astrometric  $x, y$ -measurements. Thus, the differences between diverse techniques primarily come from the method of estimating eccentricity, orbital period, and periastron epoch ( $e, P, T$ ) required to calculate  $X$  and  $Y$ . This depends on the properties of the observational data (e.g., orbit coverage by observations) and computational resource limits. For example, the Gaia team used Markov Chain Monte Carlo (MCMC) and Genetic Algorithm methods (Holl et al. 2022). For wide stellar pairs with relatively large orbital periods, the Gaia and/or Hipparcos proper motion and radial velocity differences between the com-

ponents of the binary system are taken into account (Pearce et al. 2020). The Bayesian rejection-sampling algorithm has been applied to compute posterior distributions of orbital elements (Blunt et al. 2017), with the authors demonstrating advantages of their technique compared to MCMC-based approaches.

### 3.2. Brief Analysis of Our Data Set

The basic component of our source data is the time series of angular separation and positional angle ( , ) from ground-based astrometric observations. The x, y-coordinates were calculated using well-known expressions. The accuracy of these positional data ranges from several mas (current CCD or speckle-interferometric observations) to about a hundred mas (old photographic observations). High-accuracy relative coordinates and proper motions from the Hipparcos and Gaia catalogs are now available for most of our stars. These data correspond to one or two x, y points on the trajectory but play a significant role because of their low standard errors (about 0.01-1 mas and 0.1-1 mas yr<sup>-1</sup>). Furthermore, we utilized available relative radial velocities from Gaia Data Release 3 (DR3) or other sources.

### 3.3. Distinctive Features of Our Technique for Computation of Orbital Elements Based on the Thiele-Innes Approach

Our pipeline for calculating orbital elements is based on the properties of the data. We used x, y-data as discussed above, which are primarily the results of ground-based astrometric observations covering a considerable part of the orbit. Therefore, our procedure includes the following stages:

1. We preferred not to apply Markov Chain-based minimization algorithms. Instead, it is easy to form a grid of e, P, T triplets that covers all considered space with a certain step. In our case, there were 6000 triplets. We then solve the transformation equations for all triplets, which is very fast due to their linearity. For every triplet, we calculated the Residual Sum of Squares (RSS), which characterized the quality of the approximation and was calculated according to  $RSS = r^T r$ , where r is a column vector formed by the concatenation of  $x - (A \cdot X + F \cdot Y)$  and  $y - (B \cdot X + G \cdot Y)$  vectors. Finally, we chose ten triplets with the smallest RSS values for further calculations.
2. Each of these ten triplets is used as initial values for further improvement of the corresponding e, P, and T. The residuals  $\Delta x = x - (A \cdot X(e, P, T) + F \cdot Y(e, P, T))$  and  $\Delta y = y - (B \cdot X(e, P, T) + G \cdot Y(e, P, T))$  were expressed using linear equations with respect to the corrections  $\Delta e$ ,  $\Delta P$ , and  $\Delta T$ . These equations were solved iteratively. The new e, P, and T values for the nth iteration were calculated using update formulas, where  $\lambda > 0$  is the regularization factor. The condition for choosing  $\lambda$  was decreasing the RSS value. The initial value of  $\lambda$  was consecutively decreased while RSS

$< \text{RSS}_1$  and increased otherwise. The threshold value for  $\lambda$  was set at  $10^{-10}$ . If  $\lambda$  reached this limit, an attempt was made to assign random values for  $\Delta e$ ,  $\Delta P$ , and  $\Delta T$ . The calculations were finished if this attempt was unsuccessful. These iterations were interrupted when  $\Delta e = \Delta P = \Delta T = 0$ .

3. The solution with the lowest RSS value among these ten solutions was selected as the final result. It is natural that every source's data set consists of measurement result subsets taken at different observatories with diverse accuracy. Space-based measurements are significantly more accurate than ground-based ones, being 10-100 times better. Therefore, it makes sense to assign weights to each observation subset. This issue has been examined in numerous papers (e.g., Mendez et al. 2017; Anguita-Aguero et al. 2022), with authors noting that weight assignment procedures are often somewhat subjective. The idea of calculating weights according to telescope parameters (focal lengths and apertures) has become invalid. For instance, it has been shown that there are no significant differences between astrometric accuracy estimates reached with the Pulkovo 26-inch refractor and a 4-meter telescope for the same binaries (Izmailov et al. 2020). Thus, we developed a procedure for weight calculation based on positional differences between observations conducted by a specific observatory and ephemerides provided by an "etalon set of orbital elements."
  1. For pairs with periods greater than 500 yr, we calculate "etalon orbits" using median coordinates derived from 19th-century observations, Hipparcos positions, Gaia positions, and proper motions.
  2. The x, y residuals relative to these "etalon orbits" were computed for all individual observatory data sets.
  3. Standard deviations were calculated for all observatories with more than ten residuals. Corresponding weights were assigned as inverse squares of these standard deviations.
  4. All observatories with fewer than ten residuals were collected into one group with weight calculated according to the corresponding residuals.

The homogeneous coordinate subsets taken by the same observatory were used to reduce calculation complexity. For each subset, we formed a normal place consisting of coordinates and relative velocity values at the mean epoch. These data were used for orbit calculation. The equations incorporating velocity values are straightforward: we replace the coordinates with their time derivatives in the transformation equations.

### 3.4. Application of the Apparent Motion Parameters Technique for Improvement of Orbital Parameter Values

We have radial velocities and estimates of radial acceleration for a sample of stars in our data set. AMP is a technique that allows us to incorporate these data, developed at Pulkovo Observatory (Kiselev & Kiyaeva 1980). This method is primarily used for determining binary star orbits when astrometric observations cover only a short arc of the trajectory.

The natural units for values in the AMP equations are astronomical units (au) and  $\text{au yr}^{-1}$  for spatial coordinates, distances, and velocities; years for time intervals (e.g., orbital periods); and solar masses for the masses of the primary and secondary components ( $M_1, M_2$ ). Astrometric observation results can be represented as a set of angular separations between the components of a binary system, with positional angles denoted as  $\theta_i(t_i)$  and  $\phi_i(t_i)$ . Here,  $t_i$  (where  $i = 0, K, N$ ) represents the moments of time corresponding to each observation, and  $N$  is the total number of observations. For a short arc of observations, these series can be approximated using a linear model:

In this model,  $r_0$  and  $\alpha_0$  can be derived using the linear least squares method at the mean epoch of observations ( $t_0$ ). The relative proper motion is defined by two values: the absolute value of the proper motion and the corresponding positional angle  $\alpha_0$ . The value of the local curvature of the relative trajectory  $\rho$  is equal to the radius of the circle that optimally represents the observed relative motion.

The basic equation of the AMP technique can be expressed as follows:

Here  $r_0$  is the spatial distance between the components of the binary system at the mean epoch  $t_0$ . Equation (4) allows us to estimate the distance between the primary and secondary at epoch  $t_0$ , made possible by the availability of high-quality parallax and photometric measurements from Gaia. The a priori mass estimations rely on modern isochrones, metallicity, and interstellar extinction data (e.g., Kiyaeva et al. 2021). Hence, the  $z_0$  value can be computed from the available data. The complete set of kinematic data ( $r_0, \alpha_0, \mu, \rho, c, \dot{z}_0$ ) and the sum of the masses  $M = M_1 + M_2$  can easily be transformed into the seven orbital elements. Therefore, having the relative radial velocity  $\dot{z}_0$  obtained from observations provides the final element needed to complete the kinematic data set.

It is important to note that the technique described does not provide a system of orbital elements based on all available data when there is good orbital coverage. However, it allows us to obtain a good initial approximation, which can be further refined using the Thiele-Innes method. In this case, we can avoid the MCMC-based procedures discussed above.

Recent spectroscopic observations of bright double stars have provided measurements of their relative radial velocities over the past few decades. These measurements include radial velocities from several epochs for a sample of binary

stars, allowing us to calculate values of relative radial acceleration  $\ddot{z}_0 = d\dot{z}_0/dt$ . The expression for  $W$  can be written in terms of AMP according to Newton's law of gravitation:

The combination of Equations (4) and (5) gives us the  $z_0$  estimate that is independent of the sum of stellar masses:

Therefore, it is logical to use  $\dot{z}_0$  and  $W$  in the procedure of orbital element calculation for the corresponding binary stars.

1. A preliminary set of orbital parameters was calculated through the Thiele-Innes procedure using all positional data. The mean values of  $\alpha$ ,  $\delta$ , and  $\rho$  with corresponding standard errors were computed for separate series of observations obtained from single telescope observations. Single observations of  $\alpha$ ,  $\delta$  were used as independent points. The relative positions and proper motions from Gaia were taken into account, with weights assigned according to the standard errors.
2. The values  $\dot{z}_0$  and  $W$  calculated at the epoch of radial velocity determination using elements of the preliminary orbit were substituted with the observed ones. Then, orbital elements were recalculated using a new set of AMP parameters.
3. The improvement of five elements ( $a_0$ ,  $b_0$ ,  $\alpha$ ,  $\delta$ ,  $c$ ) was performed with a nonlinear least squares procedure using the complete set of positional data, taking weights into account. Partial derivatives were computed numerically. If  $W$  was not available, the  $k^2$  value was added and improved in a similar manner.

### 3.5. Types of Orbital Solutions

In previous sections, the technique for determining orbital elements was briefly described. The calculation procedure takes into account the types and quality of observational data for each binary star in our sample. As a result, the systems of orbital solutions for different binaries are statistically unequal, which should be considered when using the presented data. Consequently, the obtained orbital solutions are classified into the following types:

- **Solution 1:** Binary systems for which only ground-based astrometric measurements were available. For these stars, we applied our modification of the Thiele-Innes approach.
- **Solution 2:** Stellar pairs that have both Hipparcos and Gaia data (relative position and proper motion) in addition to ground-based astrometric measurements. A method similar to the previous solution was applied, with weights assigned to the data according to their standard errors including space-based data.
- **Solution 3:** Stars for which measurements of radial velocity are available. The solution was obtained using the AMP technique as described above.

- **Solution 4:** Stars for which radial velocities and radial accelerations are available. The AMP algorithm branch based on Equation (6) was used.

The quality of the solutions generally improved from Solution 1 to Solution 4 due to the incorporation of new high-accuracy data. However, there were instances where this pattern was not met. In such cases, we selected the solution with the minimum standard errors as the final result. For example, a formal advantage of Solution 1 over Solution 2 could arise from the presence of an unknown third companion in the binary system, which would have perturbed the relative motion determined by the orbital parameters calculated in Solution 1.

In Tal-Or et al. (2019), radial accelerations for seven pairs of stars are given. 61 Cyg is a binary system with a rich history, discussed in over 100 research papers. We conducted a separate study on this object (Izmailov & Apetyan 2024). Unfortunately, for WDS 00491+5749 AB, while radial acceleration has been measured, there is no reliable measurement of radial velocity. Therefore, we determined the orbit using the Thiele-Innes method. For the remaining five stars, orbit determination based on acceleration significantly improved the accuracy of the orbital elements and the total masses of the components.

For WDS 00057+4549, also known as ADS 48, the sum of the masses in solar masses is: 1)  $1.859 \pm 2.025$  by the Thiele-Innes method, 2)  $1.639 \pm 1.192$  using Gaia data, 3)  $1.294 \pm 1.245$  by the AMP method, and 4)  $1.304 \pm 0.061$  by the AMP method using radial acceleration.

The errors in the orbital elements were calculated similarly to the method described in Izmailov (2019). First, we calculated ephemeris positions from the obtained orbit for all observation moments. Then, we added model random noise with standard error derived from the main orbit to these ephemeris positions. Additionally, for orbits calculated using these parameters, model noise was incorporated into the radial velocity and radial acceleration. We then re-determined the orbit based on these model data and repeated this calculation 100 times.

As a result, for each pair of stars we obtained a set of 100 orbits that described an area of possible orbital solutions. We sorted the arrays of 100 values for all elements, including mass, and determined the size of the interval that contains 68.27% of all estimates. Interpolation was applied to obtain the precise value of 68.27%, which corresponds to the probability of finding a random value within the  $\pm\sigma$  interval for normal random values.

We present the complete set of 100 possible orbits, containing one corresponding to the minimum rms value. It is important to note that the orbit with the minimum rms is not necessarily located at the center of the area of possible orbital solutions. From our perspective, this coincidence (the orbit with minimal rms aligning with the center of the region) occurs primarily for well-conditioned orbits, for example when the entire visible ellipse is well-covered by observations. Furthermore, in general cases this center may lie outside the area due to the non-convex cross-sectional shape. This statement has been validated with our

data: the orbit identified as the center of the region corresponds to the average median values derived from the 100 orbit solutions. Naturally, this central orbit is only one orbit from the region if its rms is less than the maximum rms of the original set of orbits. The center of possible orbital solutions lies outside this area for 540 cases out of 850 pairs.

The representation of an orbit using elements with standard errors generally assumes that the shape of the region of possible orbital solutions forms an ellipsoid in seven-dimensional space, with axes aligned with the coordinate axes. Alternatively, a representation can include paired covariations, similar to covariance matrix elements for five-parametric solutions in the Hipparcos and Gaia catalogs, suggesting an error ellipsoid with axes that can be oriented arbitrarily. Considered regions for binary orbital elements can have diverse shapes. While studying the properties of these forms would be theoretically interesting, a simpler approach is to utilize sets of orbits whenever uncertainty estimates for the elements are required.

For instance, instead of calculating the mass of a binary system based on a single central orbit and then deriving the error estimate using error propagation expressions, we can calculate the mass as the median mass from a set of 100 orbits, with the “standard error” estimated from the same set of calculated masses.

The star system designated as WDS 03520+0632 = 31 Tau illustrates the advantages of our approach. It is the most massive system with a reliably defined mass in our analysis. Observations of this binary star were conducted from 1937 to 2019. During this period, the positional angle changed from  $215.0^\circ$  to  $206.9^\circ$ , and the distance between the components varied from 0.3 to 0.8 , indicating that only a relatively small arc of the orbit was covered. The primary star is classified as B5V, and the lack of narrow absorption lines in its spectrum prevents us from obtaining accurate estimates of both radial velocities and accelerations. At first glance, it may appear that determining orbital elements and masses is impossible. Fortunately, there is a strong correlation between the orbital period and the semimajor axis in the potential orbits of this binary system (see Figure 1). Consequently, the mass dispersion derived from different possible orbits is less than the dispersion in semimajor axes and periods. Taking parallax error into account, the total mass of the components of this star system is estimated to be  $12.3 \pm 1.3$  solar masses.

Another potential application of our technique involves pairs of celestial bodies within a group of relatively close objects with similar masses. In this scenario, N-body simulations are essential to investigate the dynamical evolution of the system. This analysis could be particularly useful for studying the orbit evolution of exoplanets in binary systems or the dynamical evolution of stellar clusters. In such cases, separate numerical integrations for each orbit in the set are necessary.

## 4. Results

The primary outcome of this study is the orbital solutions for 850 visual binary stars. The associated data are presented in four tables, which can be accessed as ASCII files through the Pulkovo Observatory database (<http://izmccd.puldb.ru/vds.htm>) or the Strasbourg astronomical Data Center (CDS; <https://cds.u-strasbg.fr>).

1. **Table 1:** This table presents the sums of masses of the components for 184 stellar pairs, measured in solar masses.
2. **Table 2:** This table includes the sums of the component masses of 850 stars, each multiplied by the cube of their parallax.
3. **Table 3:** Here, the orbital elements for the solution with the minimum rms are provided for all binaries in our sample.
4. **Table 4:** Each entry in this table contains 100 orbits for each binary star, outlining the range of possible orbital solutions as described above.

Table 1 also provides the lower and upper bounds of the confidence interval for the sum of stellar masses calculated with a probability of 0.6827. Only pairs for which the sum of the mass estimate is three times greater than half of the confidence interval are included. Out of 850 pairs, 184 meet this condition. The corresponding parallax data were taken from the Gaia DR3 catalog. Additionally, since stellar magnitude and interstellar absorption are also determined by Gaia, the mass-luminosity relationship can be constructed for our sample, as shown in Figure 2.

The mass-luminosity relation is:  $\log(L_{\odot}) = 4.33 \log(M_{\odot})$ , where  $L_{\odot}$  is the luminosity in solar units in the G band, and  $M_{\odot}$  is the mass in solar masses. Each binary pair in this figure is represented by two dots, meaning one star corresponds to one data point. The individual masses of the components were determined through a simple iterative procedure based on the mass-luminosity law. The mass ratio for a pair of stars was derived from this law, and the individual masses were calculated using the known sum of masses.

In the iterative process, we initially set  $a = 4$  and  $b = 0$ . Using these values, we calculated the individual masses, then updated  $a$  and  $b$  based on the computed masses. This process continued until the values converged and stabilized. The results we obtained are as follows:  $a = 4.33 \pm 0.02$  and  $b = 0.01 \pm 0.01$ . In the paper by Eker et al. (2018), values of  $a = 4.329 \pm 0.087$  and  $b = 0.010 \pm 0.019$  for eclipsing variable stars of average mass range are presented, which agree with our values within standard errors. A theoretical estimate was produced by Malkov et al. (2022) for slightly more massive stars:  $\log(L_{\odot}) = 9.53 \log(M_{\odot}) - 4.33$ , which is also close to our results, although it is outside the  $1\text{-}\sigma$  error interval.

The noticeable dispersion in the data points relative to the straight line is likely due to the different ages and metallicities of the stars. It would be essential to create a set of evolutionary tracks based on the metallicity and color of the

stars, allowing us to select the appropriate track for each star (see Bressan et al. 2012) to conduct a more detailed analysis. The authors intend to continue this research in the future.

A histogram displaying the distribution of the 850 studied binary stars by their eccentricities is shown in Figure 3. For this analysis, all 100 orbital solutions for each binary star were utilized. It is also evident from the figure that the histogram exhibits a slight upward deviation from the linear relationship represented by the equation  $f(e) = 2e$ .

Figure 4 illustrates the histogram for eccentricities, focusing only on orbits for which the eccentricity standard error is less than 0.1 and the period is under 400 yr. In contrast, Figure 5 presents orbits with periods exceeding 400 yr, applying the same criterion regarding eccentricity error. Unfortunately, only 85 orbits fall within this short-period set ( $P < 400$  yr). The peak of this histogram corresponds to an eccentricity of  $e = 0.545 \pm 0.029$ . This estimate, like all subsequent ones, is derived by approximating the data points using a normal distribution.

In the paper by Tokovinin & Kiyayeva (2016), similar calculations were based on a sample of 477 close binary stars without direct orbit determinations, yielding a histogram peak of  $0.59 \pm 0.02$ , which is within  $2\sigma$  agreement with our result. The paper also notes that this maximum tends to shift upward with an increase in orbital period. Furthermore, Marks & Kroupa (2011), which updated statistics obtained from radial velocities of 164 Sun-like stars in Duquennoy & Mayor (1991), showed that the distribution law  $f(e) = 2e$  accurately describes the analyzed data. Without limiting the eccentricity error, the histogram of eccentricity for stars with periods longer than 400 yr in our sample also confirms the  $f(e) = 2e$  law within standard error limits.

Figure 6 shows a histogram by period. It is unfortunately not feasible to impose a limitation on period errors, as was done for eccentricities, because such a restriction would reduce the sample of binary stars with long periods since the period value directly affects its relative error. This is explained by relatively poor orbit coverage by observations for binary stars with relatively large orbital periods. In comparison to the previous study by Izmailov (2019), a bimodal pattern in the distribution has been confirmed. However, the first maximum has shifted toward shorter periods from 200 to 87 yr. The exact position of this first maximum remains quite uncertain. The second maximum was retained at  $587 \pm 83$  yr. It is possible that the addition of a significant number of short-period binaries could shift the first maximum position even further, but the bimodal pattern of the distribution will likely persist.

Interestingly, in logarithmic scale (see Figure 7), the first maximum is still observed around the value  $P = 208 \pm 73$  yr. The two peaks in the histogram nearly merge, making it possible to interpret the distribution as single-modal, with a maximum at  $342 \pm 28$  yr. This finding is quite similar to the result reported in Raghavan et al. (2010), where a similar maximum is detected at  $P = 293$  yr.

The discussed bimodality in the distribution could result from both the intrinsic properties of the stellar population and a combination of selection effects, limitations from observation intervals, and limitations of methods used for orbit determinations. For instance, Offner et al. (2010) outlined two mechanisms for binary star formation.

Here, stars are selected for analysis only if the value of the semimajor axis is at least three times greater than its standard error. The distribution is single-modal, with a maximum at  $\log(a) = 1.81$ , which corresponds to 64.9 astronomical units (au). Since the distance to the binary (measured by parallax) was used to convert the size of the semimajor axis from arcseconds to astronomical units, it is possible to examine selection effects related to both distance and the apparent separation of components. The sample was divided into two equal groups based on distance: relatively close binaries to the Sun and those located relatively far from the Sun. A similar division was made for the values of the semimajor axes. All distributions corresponding to these four subsets show a similar shape to the original distribution. This consistency allows us to infer that the distribution we obtained reflects the stellar population's properties rather than selection effects.

A histogram of the semimajor axes in logarithmic scale is shown in Figure 8. The distribution of binary stars by mass reaches its maximum at  $\log(M) = 0.45$  for our sample, which corresponds to a mass ratio of  $M = 2.8$  solar masses (see Figure 9). The mass represented here is the sum of the masses of two stars, with one of the stars having a mass approximately equal to that of the Sun. Our sample of binary stars is likely lacking in low-mass stars, and the real peak of this distribution probably falls at lower mass values.

The parameters of fitting the above distributions are summarized in Table 1.

**Table 1. Parameters of Distribution Fits for Orbital Elements**

Orbital Element	Solution Feature	Peak Position	Standard Deviation
Eccentricity	$P < 400$ yr	0.545	0.029
$\log(\text{Semimajor axis})$ [au]	All solutions	1.81	0.033
$\log(\text{Mass})$ [M]	All solutions	0.45	0.011
$\log(\text{Period})$ [yr]	1st maximum	1.94 (87 yr)	0.020
$\log(\text{Period})$ [yr]	2nd maximum	2.77 (587 yr)	0.009
Period [yr]	1st maximum	87	33
Period [yr]	2nd maximum	587	196

The distribution of the poles of our orbits over the celestial sphere was examined in this study. Such investigations have a long history, as referenced in Dommangeat (2014). The paper by Agati et al. (2015) highlights the concentration of poles near galactic coordinates  $l = 46^\circ$  and  $b = 37^\circ$ , based on 51 binary

systems in the solar vicinity. However, the authors caution that the reality of the deviation from isotropy cannot be concluded with certainty.

It is noted that there are two equally probable pole positions for all binary systems that lack spectral data. In a given tangential plane, two hypothetical binary systems differing only in the position of their ascending nodes by  $180^\circ$  will demonstrate identical motion. For real systems, however, one pole position is valid while the other is not. Furthermore, the distribution of incorrect pole positions will generally be random in a relatively large sample of binaries. These incorrect poles should appear as random noise, which complicates detection of any effect but does not make such detection impossible.

A straightforward calculation was conducted to analyze the distribution of orbital poles across the celestial sphere and assess any potential violation of isotropy. Only binaries with reliably determined pole positions were utilized (see Figure 10). The HEALPix equal-area pixelization method was applied (Hivon & Banday 2005). Pixel areas varied, covering values from  $1/2$ ,  $1/3$ ,  $1/4$ , ...,  $1/40$  of the total celestial sphere area. The pixel with the highest number of poles was identified for each specified pixel area value. Subsequently, the probability of obtaining the same or greater maximum for a particular pixel area was evaluated. A numerical simulation involving 1,000 random distributions was performed. In 538 out of these 1,000 cases, the maximum number of simulated poles exceeded the number of real poles.

The analysis of pole concentration close to a certain plane was performed in a similar manner, using spherical zones instead of HEALPix pixels. The real pole distribution maximum was exceeded in 766 cases out of 1,000. Therefore, neither the concentration of poles in a certain direction on a sphere nor in a plane can be confirmed.

## Conclusion

Coordinates, proper motions, and corresponding standard errors from Gaia data were used in the orbit determination of the selected stars. These data were introduced into orbital solutions with the highest weight values thanks to the high accuracy of Gaia astrometry compared to ground-based data. Therefore, the ephemeris for the relative position of the components is close to the Gaia results at the mean Gaia epoch, both in coordinates and in their standard errors. It is easy to calculate the accuracy of estimates of  $\alpha$ ,  $\delta$  at an arbitrary epoch using orbit sets constructed for each binary. The initial step is computing the ephemeris  $\alpha$ ,  $\delta$  for each binary orbit set. Next, the mean value over the set of  $\alpha$ ,  $\delta$  and their standard errors represent the ephemeris and the corresponding accuracy estimate of the relative position for the arbitrary epoch. The same procedure can be applied for radial velocities and accelerations. The considered technique improves the accuracy of the ephemeris several times compared to the standard method with error propagation.

Finally, we should note that our sample of binary stars has significant selection

biases from a statistical completeness point of view. There is a lack of binaries with low-mass components in the sample. The fraction of low-mass component binaries should be significant in contrast to the solar-mass component binary fraction for the complete sample of binaries. However, low-mass component binaries have no long observation history due to their low brightness. As a result, the astronomical community has focused heavily on these kinds of binary stars to collect sufficient astrometric data sets for accurate orbit determinations. Another limitation concerns long-period binary stars, where a relatively short observed arc leads to significant uncertainties in the orbital parameters. Hence, additional radial velocity and acceleration measurements will improve orbital parameter convergence. Simultaneous astrometric observations of well-known bright binaries are necessary to refine current orbital parameter sets and potentially discover hidden low-mass components.

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