Correlation between SD Pair Structure and Interaction Strength in the Nuclear Pairing Shell Model

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Abstract

Based on the nucleon-pair shell model [Nucleon-pair shellmodel(NPSM)], truncated to the SD pair subspace, we investigate the energy spectra, wave function structures, and electromagnetic properties of low-lying excited states in Ba isotopes using two distinct SD pair construction schemes: the SDI method and the TDA approximation method. The results demonstrate that each approach possesses its own merits and drawbacks, with no absolutely superior scheme: both construction schemes for SD pair structures can produce results in close agreement with experimental values, and the agreement between experimental and theoretical values improves with increasing number of neutron hole pairs; both reveal an intrinsic correlation between the strengths of electromagnetic transitions and wave function structures in the Ba isotope chain. In describing spectral properties, the SDI method yields theoretical values for the yrast and quasi-bands that are overall closer to experimental data; the TDA method ameliorates the SDI method's deficiencies in describing the Ba isotope chain's energy spectra, specifically the excessively low energy of the second 0+ state and the excessively high energy of the first 3+ state. In describing electromagnetic properties, the TDA method provides superior fits. Additionally, when constructing SD pair structures with the SDI scheme, selecting an appropriate interaction strength can enhance the precision of theoretical calculations.

Full Text

Preamble

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Correlation between SD Pair Structure and Interaction Strengths in the Nucleon-Pair Shell Model*

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Abstract

Based on the nucleon-pair shell model (NPSM) truncated to the SD collective pair subspace, we investigate the low-lying energy spectra, wave function structures, and electromagnetic properties of Ba isotopes using two distinct schemes for constructing SD pairs: the SDI method and the TDA approximation method. Our results demonstrate that each approach has its own advantages and disadvantages, with no absolutely superior scheme. Both methods can produce results in close agreement with experimental data, and the agreement improves as the number of neutron hole pairs increases. Both approaches reveal an intrinsic relationship between the electromagnetic transition strengths and wave function structures along the Ba isotopic chain. For spectral properties, the SDI method generally yields theoretical values for the yrast band and quasi-γ band that are closer to experimental values overall, while the TDA method ameliorates the deficiencies of the SDI method—specifically, the overly low energy of the second 0^+ state and the overly high energy of the first 3^+ state. For electromagnetic properties, the TDA method provides better fits. Furthermore, when constructing SD pair structures with the SDI scheme, selecting appropriate interaction strengths can enhance the accuracy of theoretical calculations.

Keywords: nucleon-pair shell model; energy spectrum; electromagnetic transition; collective pairs

1 Introduction

Nuclear collective motion patterns have been extensively revealed through decades of experimental investigations, and various theoretical models have been developed to describe these collective properties with considerable success. The nuclear shell model has received particular attention in this regard. Since the shell model encompasses all nucleonic degrees of freedom and reflects the fundamental properties of atomic nuclei, it is, in principle, capable of describing nuclear collective motion. Indeed, the shell model represents one of the most successful models in nuclear structure research. With the rapid advancement of computer technology, the configuration space in which the shell model Hamiltonian can be diagonalized has gradually expanded. However, as the number of valence nucleons outside closed shells and the number of single-j shells increase, the shell model configuration space becomes prohibitively large. Beyond a certain point, even with modern high-performance computers, direct diagonalization of the shell model Hamiltonian becomes intractable [**?**]. To address this challenge, researchers have implemented model space truncations. The nucleon-pair shell model (NPSM) proposed by Chen Jinquan [**?**] allows for various forms of model space truncation.

It is noteworthy that the interacting boson model (IBM) [?], which approximates collective S and D pairs as s and d bosons, has achieved remarkable success [?], thereby demonstrating the feasibility of truncating the model space to an SD pair subspace [?, ?]. Consequently, truncating the shell model space to collective pairs with angular momentum quantum numbers 0 and 2 within the NPSM framework yields the SD-pair shell model (SDPSM) [?, ?].

Studies have shown that the NPSM truncation can well reproduce the classical limit spectra of the IBM [?, ?], indicating its excellent truncation effect for describing collective properties of low-lying nuclear spectra [?, ?]. Progress in this area has led to the following conclusion: for the SDPSM, the collective properties of low-lying spectra are strongly influenced by the structural coefficients of SD pairs. In a two-particle system (two neutrons or two protons), diagonalizing the surface δ interaction and taking the first 0^+ state and the first 2^+ state as the S pair and D pair, respectively, provides an effective description of collective properties in low-lying nuclear states [?]. This approach is referred to as the SDI method. However, applications of this method have revealed certain issues, such as the second 0^+ state being too low in energy and the first 3^+ state being too high. We have refitted the relevant parameters using the SDI method, improved the fitting results, and further analyzed the influence of pair structures in the NPSM on nuclear collective properties through parameter fitting. Recent advances in the selection of collective pairs have been summarized in review literature [?], which outlines the latest developments in shell model pairing theory and collective pair selection methods for deformed nuclei. References [?] and [?] have proposed the Hartree-Fock method and the conjugate gradient method as new approaches for selecting collective pairs. The aforementioned problems with the SDI method arise partly because many-body effects are neglected when constructing SD pair structures. To address this, researchers have employed variational methods that consider many-body effects and the proton-neutron Tamm-Dancoff approximation (TDA) to improve the fitting results. The TDA method first determines the S-pair structure coefficients through the variational principle and then determines the D-pair structure coefficients via the broken-pair method. Reference [?] combined a similar approach with shell model effective interactions to determine collective pair structure coefficients. Compared with the methods in references [?, ?, ?, ?], the advantage of the TDA method used in this work lies in its variational treatment that simultaneously considers contributions from both valence protons and valence neutrons, whereas the variational approaches in [?, ?, ?, ?] consider only identical valence nucleons. In this paper, we truncate the NPSM to the SD pair subspace to obtain the SD-pair shell model (SDPSM) and employ both the SDI and TDA methods to construct SD pair structures. We compare and analyze the relationship between these two methods and the interaction strengths [?, ?] to further discuss the truncation effect of the NPSM.

2 Theoretical Model

To highlight the essential physics while avoiding unnecessary complexity, we adopt a simple Hamiltonian comprising three terms: single-particle energies, interactions between identical nucleons, and quadrupole-quadrupole interactions between protons and neutrons. Denoting the single-particle energy levels by ε_a and the particle number operator by $\hat{n}_\sigma$, the single-particle energy term H_0 takes the form

$$H_0 = \sum_{\sigma=\pi,\nu} \sum_a \varepsilon_a \hat{n}_{a\sigma}$$

The interaction V_σ between identical nucleons is taken as the surface δ interaction [?]

$$V_\sigma = -\frac{G_\sigma}{2} \sum_{L=0,2} \sum_M P^\dagger_{LM\sigma} P_{LM\sigma}$$

where $P^\dagger_{LM\sigma}$ creates a pair of nucleons with total angular momentum L and projection M, and G_σ represents the interaction strength. The surface δ interaction's ideal pairs are denoted as $A^\dagger_{LM\sigma}$ [?].

The quadrupole operator is defined as

$$Q_{2\mu} = \sum_{\sigma=\pi,\nu} e_\sigma \sum_{ab} \langle a \| r^2 Y_2 \| b \rangle c^\dagger_{a\sigma} c_{b\sigma}$$

In second-quantized form, the pair creation operator is

$$P^\dagger_{LM\sigma} = \sum_{ab} C_{LM}(ab) c^\dagger_{a\sigma} c^\dagger_{b\sigma}$$

If e_π and e_ν represent the effective charges for protons and neutrons, respectively, the E2 transition operator is

$$T(E2) = \sum_\mu (-1)^\mu Q_{2\mu} Q_{2-\mu}$$

The M1 transition operator is expressed as

$$T(M1) = \sum_{\sigma=\pi,\nu} g^{\text{eff}}_{l\sigma} L_\sigma + g^{\text{eff}}_{s\sigma} S_\sigma$$

where $g^{\text{eff}}_{l\sigma}$ and $g^{\text{eff}}_{s\sigma}$ denote the orbital and spin g-factors, respectively. Currently, insufficient experimental data exist to uniquely determine these factors; based on previous work [?], we adopt the values $g_{l\pi}^{\text{eff}} = 1$, $g_{l\nu}^{\text{eff}} = 0$, $g_{s\pi}^{\text{eff}} = 5.586$, and $g_{s\nu}^{\text{eff}} = -3.826$ (in units of μ_N^2). The orbital angular momentum and spin operators can be written in collective dipole operator form.

To study the truncation effect of the NPSM, we analyze the feasibility of truncating the model space to the SD pair subspace. The model space is then spanned by collective SD pairs, defined as

$$S_\sigma^\dagger = \sum_{ab} y_{ab}^{(0)} C_{ab}^{(0)} c_{a\sigma}^\dagger c_{b\sigma}^\dagger$$

$$D_{\mu\sigma}^\dagger = \sum_{ab} y_{ab}^{(2)} C_{ab}^{(2)} c_{a\sigma}^\dagger c_{b\sigma}^\dagger$$

If many-body effects are considered, the S-pair structure is constructed via the variational method

$$\delta\langle\Psi|H|\Psi\rangle = 0$$

Considering both many-body effects and the chosen form of the Hamiltonian, the D-pair structure can be constructed according to the TDA approximation

$$D_{\mu\sigma}^\dagger = \frac{1}{\sqrt{2}} \sum_k \psi_k^\dagger A_{k\sigma}^\dagger$$

where n_σ is the number of non-collective pairs, and $D_{k\sigma}^\dagger$ represents non-collective pairs with $k = 1, 2, \ldots, n_\sigma$, $\sigma = \pi, \nu$. The indices i, j denote single-particle orbits within a major shell. Diagonalizing the Hamiltonian in the space $\{S^\dagger, D^\dagger\}$ yields the eigen wavefunctions

$$|\Psi\rangle = \sum_k C_k^\nu |\psi_k^\nu\rangle + \sum_l C_l^\pi |\psi_l^\pi\rangle$$

where C_k^ν and C_l^π are expansion coefficients for the 12^+ state, from which the D-pair structure coefficients can be obtained. The TDA approximation thus constructs the D-pair structure as

$$y_{ij}^{(2)} = \sum_k C_k^\nu \langle\psi_k^\nu|D_{\mu\sigma}^\dagger|0\rangle$$

In this way, the TDA method builds both collective S-pair and D-pair structures.

All matrix elements of the NPSM model Hamiltonian can be constructed through overlap integrals of multi-pair bases. The recursion relation for the N-pair basis overlap integral is

$$\langle\psi_N|\psi'_N\rangle = \sum_{i=1}^{N}\sum_{j=1}^{N}(-1)^{i+j}\langle\psi_{N-1}^{(i)}|\psi_{N-1}^{(j)}\rangle\langle\psi_1^{(i)}|\psi_1^{(j)}\rangle$$

where $\hat{P}$ is the permutation operator, [...] denotes Racah coefficients, and ψ_k is determined by the structure coefficients of $A^\dagger$ and $S^\dagger$. The quantity ir' represents new collective pairs $A^\dagger$ whose structure coefficients are determined by those of $A^\dagger$, $A^\dagger$, and $S^\dagger$. The intermediate angular momentum quantum numbers in the formula correspond to the angular momenta of the first i' pairs in the parentheses on the right-hand side. Through this recursive relationship, all required multi-pair basis overlap integrals can be obtained [**?**, **?**, **?**].

3.1 Trends of Interaction Strengths with Neutron Number

The single-particle energy term in the model Hamiltonian is H_0, with proton and neutron single-particle energies ε_π and ε_ν given in .

Proton and neutron single-particle energies [22,23] (MeV)

We select Ba isotopes for our study, constructing S-pair and D-pair structures using the surface δ interaction. By fitting low-lying state energies, $B(E2)$ values, and $B(M1)$ values, we determine the specific values of three interaction strengths: G_π, G_ν, and κ. These are compared with the three interaction strength values from reference [**?**] and with our fitting results. The collective properties of these nuclei were also fitted using the TDA approximation method from reference [**?**], yielding another set of three strength parameters. All three parameter sets are presented together in [Figure 1: see original paper] for comparative analysis, investigating the influence of interaction strengths on fitting results when using the surface δ interaction to construct S-pair and D-pair structures, and comparing these with the fitting results from the TDA approximation method [**?**].

[Figure 1: see original paper] Interaction strength values for Ba isotopes obtained from different fitting methods: SDI-1 denotes adjusted interaction strengths fitted using the SDI method; SDI-2 denotes interaction strength values from reference [**?**] fitted using the SDI method; TDA denotes interaction strength values fitted using the TDA approximation method [**?**].

[Figure 1: see original paper] clearly shows that in all three fitting scenarios (SDI-1, SDI-2, and TDA), the fitted parameter values for Ba isotopes vary almost monotonically with increasing neutron number, exhibiting smooth variations. Moreover, the interaction strength G_π for each nucleus is consistently greater than G_ν. This arises because the proton single-particle level splitting is larger than that of neutron holes, and such splitting can disrupt nucleon collective motion, necessitating a stronger SDI force for protons. During parameter fitting with the SDI approximation, we find that when the three interaction strengths are increased by the same amount, the proton-proton interaction strength has a greater impact on changes in the theoretical predictions for nuclear spectra and electromagnetic transitions. [Figure 1: see original paper] also reveals that as nucleon number varies monotonically, the proton-proton interaction strength curve is the steepest among the three interaction strengths, indicating that the proton-proton interaction strength changes more significantly for a given change in nucleon number.

Overall, the interaction strength parameters obtained with the TDA approximation method are relatively lower than those from the two SDI methods. This is because the TDA approximation incorporates many-body effects, reasonably including multi-body correlations between nucleon pairs. In contrast, the SDI methods corresponding to SDI-1 and SDI-2 in [Figure 1: see original paper] neglect many-body effects, requiring stronger interactions between identical nucleons and between protons and neutrons to compensate.

When fitting the spectra and transition strengths of Ba isotopes under the three scenarios, the three interaction strengths between identical nucleons are closest for ^{130}Ba, which has the largest number of nucleon pairs. As the number of nucleons decreases, the gap between the interaction strengths fitted by SDI and TDA methods gradually widens, with the difference in proton-proton interaction strength being most pronounced. This can be understood as follows: the effectiveness of the SD-pair approximation truncation is jointly influenced by single-particle energies and residual interactions. If single-particle energies are much larger than residual interactions, nucleons move relatively independently. In this case, nucleon pairs of various angular momenta become approximately degenerate and equivalent, requiring consideration of all angular momentum pairs to describe nuclear properties via NPSM, rendering the SD-pair approximation less ideal. As the number of nucleon pairs increases, the effective interaction between identical nucleons strengthens, making the SD-pair truncation increasingly effective. Simultaneously, in the TDA approximation, multi-body correlations between nucleon pairs become less significant than in nuclei with fewer nucleons.

As mentioned above, for ^{130}Ba, the parameter values fitted under the three scenarios in [Figure 1: see original paper] are closer than those for other Ba isotopes. Additionally, the difference between G_π-SDI-1 (the pairing strength fitted with the SDI approximation) and G_π-TDA (the pairing strength fitted with the TDA approximation from reference [**?**]) is smaller. The ^{130}Ba spectrum fitted with G_π-SDI-1 is slightly better overall than that obtained with G_π-SDI-2 [**?**] (see [Figure 1: see original paper]). The reason is that when constructing SD-pair structures with the SDI method, many-body effects between nucleons are neglected, requiring stronger interactions between identical nucleons and between protons and neutrons to compensate. Overall, SDI-1 shows the largest increase in both identical-nucleon interaction strength and proton-neutron interaction strength, and this enhancement partially compensates for the lack of many-body correlations considered in the TDA approximation.

Thus, to achieve results comparable to or even better than those from the TDA approximation method when using the SDI method, one can adjust the pairing strength parameters accordingly.

3.2 Influence of Interaction Strengths on Nuclear Spectra

Using the interaction parameter values from [Figure 1: see original paper], we calculated the energy spectra of Ba isotopes, as shown in [Figure 2: see original paper]. The results demonstrate that SDPSM can obtain calculated results close to experimental values for Ba isotopes using either the SDI or TDA approximation method, particularly for the yrast bands of each nucleus. The theoretical values for quasi-γ bands also basically agree with experimental values. Overall, the agreement between theory and experiment improves as the number of neutron hole pairs increases. As shown in [Figure 2: see original paper], for ^{136}Ba with one hole pair, the theoretical values for some yrast band levels are relatively close to experimental values, while others deviate significantly, such as the second 4^+ state, first 3^+ state, and 1^+ state. For ^{134}Ba with two hole pairs, the theoretical and experimental values agree well for $J^\pi \leq 4^+$, but diverge for $J^\pi = 6^+$. For ^{132}Ba with three hole pairs, agreement is good for $J^\pi \leq 6^+$, but deviations appear for $J^\pi = 8^+$. For ^{130}Ba with four hole pairs [FIGURE:2(a)], the yrast band theoretical values are very close to experimental values, and the quasi-γ band theoretical values also show good agreement. Particularly, the ^{130}Ba spectrum from SDI-1 shows the closest agreement with experimental values among the three calculations. The ^{130}Ba spectrum from SDI-2 is systematically lower than the experimental spectrum, while the ^{130}Ba energy levels calculated with the adjusted pairing parameters of SDI-1 are overall elevated and closer to experimental values. The yrast band 8^+, 10^+, 12^+, and 14^+ states calculated with SDI-2 for ^{130}Ba are somewhat low compared to experimental values, whereas the SDI-1 results for the ^{130}Ba yrast band are in excellent agreement with experiment. The SDI-1 theoretical values for ^{130}Ba are superior to those obtained with the TDA approximation method from reference [**?**]. For example, the TDA method yields a high 14^+ yrast state and high 8^+, 10^+, and 12^+ quasi-γ band states, which are improved in SDI-1.

As previously mentioned, the SDI-2 fitting process suffers from problems such as the second 0^+ state being too low and the first 3^+ state being too high. Adjusting the interaction strengths in SDI-1 provides only modest improvement. For ^{136}Ba, SDI-1 lowers the first 3^+ state energy compared to SDI-2, but it still deviates from experiment. For ^{134}Ba, SDI-1 raises the second 0^+ state energy relative to SDI-2, but it remains below the experimental value. For ^{136}Ba, SDI-1 brings the second 0^+ state energy closer to experiment than SDI-2, but the deviation remains substantial. Overall, however, the TDA method improves upon the SDI deficiencies regarding the second 0^+ and first 3^+ state energies. For example, for ^{134}Ba, TDA lowers the first 3^+ state energy compared to SDI-2, bringing it closer to experiment. For ^{136}Ba, TDA significantly lowers the first 3^+ state energy, achieving agreement with experiment to within one decimal place. For ^{130}Ba and ^{132}Ba, TDA raises the second 0^+ state energy, improving agreement with experiment. For ^{134}Ba, TDA substantially raises the second 0^+ state energy, nearly matching the experimental value. Thus, even though SDI-1 adjusts interaction strengths, its fitting of the first 3^+ and second 0^+ state energies remains inferior to the TDA method.

In summary, within SDPSM, whether using the SDI or TDA method, theoretical values converge toward experimental values as the number of nucleon pairs increases. This occurs because the quality of agreement in the SD approximation depends on the relative magnitude of single-particle energies and residual interactions. When the number of nucleon pairs is small and the mean-field energy dominates over residual interactions, nucleons move relatively independently, and nucleon pairs of all angular momenta become nearly equivalent, reducing the accuracy of the SD-pair truncation. As the number of nucleon pairs increases, residual interactions strengthen, the SD pairs become increasingly low in energy, and the SD-pair truncation yields better results. For nuclei with many nucleon pairs, the effective interaction between identical nucleons is enhanced, allowing the SDI method to compensate for neglecting many-body correlations in SD-pair structure selection through adjustments to identical-nucleon and proton-neutron interaction strengths, sometimes even surpassing TDA results. For nuclei with fewer nucleon pairs, residual interaction strength decreases with nucleon number, and even considering many-body correlations in SD-pair structure selection does not yield significantly better results.

It is worth noting that for fitting the second 0^+ and first 3^+ state energies, the TDA method provides superior theoretical values compared to SDI, and even adjusting interaction strengths in SDI-2 does not overall produce better values for these states. We speculate that this may be because the many-body correlations considered in the TDA approximation have a special connection to the second 0^+ and first 3^+ state energies.

Although the SDI method gives slightly better overall results than TDA for spectral description from yrast to quasi-γ bands, the TDA method provides superior results for electromagnetic properties, as discussed in Section 3.4. This indicates that considering many-body correlations between nucleons in SD-pair structure construction is necessary for describing nuclear electromagnetic properties.

3.3 Wave Function Structure Analysis

Analyzing wave functions composed of normalized but non-orthogonal multi-pair basis vectors helps us understand their structural properties. presents the main components of wave functions for typical states in Ba isotopes. Basis vectors composed of multiple nucleon pairs are denoted as

$$|\psi\rangle = \prod_{i=1}^{N} A^{\dagger}_{L_i \sigma_i} |0\rangle$$

Coefficients in parentheses indicate basis vector amplitudes that appear more than once. For example, the first 2^+ state of ^{130}Ba obtained with the SDI method has the main component $(0.2486)S^\dagger D_\nu^\dagger|0\rangle$.

Since no underlying group structure exists in the shell model Hamiltonian, symmetric and mixed-symmetric states are defined in the shell model as follows: if a state's main component is $S^{N-1}D^\dagger|0\rangle$ with coefficient $a \cong +1$, it is called a symmetric state with a single D pair; if $a \cong -1$, it is called a mixed-symmetric state with a single D pair. Multi-D-pair symmetric and mixed-symmetric states are defined analogously. According to IBM [?] and NPSM [?] studies, E2 transitions are strong between two symmetric states, while M1 transitions are strong between mixed-symmetric states.

Main components of partial wave functions for ^{134}Ba isotope

Studies show that in Ba isotopes, the first 0^+ state wave function is dominated by S-pair condensation, though with considerable mixing from two-D-pair states. The first 2^+, 4^+, and 6^+ states are essentially coherent states of one, two, and three D pairs, respectively, with all yrast states being symmetric. Consequently, strong E2 transitions occur between neighboring yrast states. As shown in , for ^{134}Ba, the first 4^+ state from SDI-1 with adjusted interaction strengths includes a coherent two-D-pair state with $2D_\nu$ expansion coefficient 0.3878 and $2D_\pi$ coefficient 0.3996. The first 4^+ state from SDI-2 also contains a coherent two-D-pair state, but the expansion coefficients are less balanced: $2D_\nu$ coefficient 0.5406 and $2D_\pi$ coefficient 0.3772. Overall, SDI-1 with adjusted interaction strengths yields more pronounced yrast band symmetric states, and correspondingly, the calculated $B(E2)$ values show some improvement. Combining results from [Figure 1: see original paper] and and comparing SDI-1 with SDI-2 reveals that for ^{134}Ba, the increase in interaction strength between identical nucleons is larger than that between protons and neutrons, making the yrast band symmetric states more pronounced.

3.4 Influence of Interaction Strengths on Electromagnetic Transitions

For $B(E2)$ calculations, according to reference [?], when nuclei are in the 50-82 shell, $e_\pi/e_\nu = 1.5$. Through fitting of Ba isotope experimental data, SDI-1 and SDI-2 select $e_\pi = 1.5e$, $e_\nu = 1.0e$, while TDA selects $e_\pi = 1.6e$, $e_\nu = 1.0e$. lists the theoretical $B(E2)$ predictions from SDI-1, SDI-2, and TDA. Comparison with experimental values shows that the theoretical $B(E2)$ predictions from all three cases are consistent with wave function structures: transitions between yrast states are strong, and the strength increases with neutron hole pair number, matching experimental trends. The study also shows that for certain nuclei, SDI-1 can produce the best results among the three. For example, the experimental $B(E2; 2_1^+ \to 0_1^+)$ for ^{134}Ba is $0.136(3)e^2b^2$, while the SDI-1 theoretical value is $0.13642e^2b^2$, extremely close to experiment (accurate to three decimal places). The experimental $B(E2; 6_1^+ \to 4_1^+)$ is $0.120e^2b^2$, with SDI-1 giving $0.12401e^2b^2$ (accurate to two decimal places), SDI-2 giving $0.1197e^2b^2$ (too low), and TDA giving $0.14939e^2b^2$ (too high). Referring to , the first 4^+ state from SDI-1 for ^{134}Ba has $2D_\nu$ coefficient 0.3878 and $2D_\pi$ coefficient 0.3996, showing significant coherence. This suggests that the better the symmetry of the yrast band wave function structure, the closer the corresponding $B(E2)$ theoretical value is to experiment. For ^{134}Ba, the experimental $B(E2; 2_2^+ \to 2_1^+)$ is $0.003e^2b^2$ [?], with SDI-1 giving $0.00264e^2b^2$ (consistent with experiment), while SDI-2 gives $0.001e^2b^2$ (too low). This occurs because for ^{134}Ba, the increase in interaction strength between identical nucleons is larger than that between protons and neutrons, making the yrast symmetric states more pronounced. From the perspective of interaction strengths, SDI-1 shows the largest overall increase in both identical-nucleon and proton-neutron interaction strengths, which partially compensates for the lack of many-body correlations in the TDA approximation. Consequently, even when TDA theoretical values are high, SDI-1 theoretical values can still agree well with experiment.

$B(E2)$ values (e^2b^2) and $B(M1)$ values (μ_N^2) for Ba isotopes. Experimental data taken from references [?, ?].

Analysis of $B(E2)$ relative values shows that theoretical ratios agree well with experimental values in all three cases. Among these, SDI-1 yields slightly better results for certain cases. For example, for ^{132}Ba, the ratio $B(E2; 3_1^+ \to 2_1^+)/B(E2; 2_1^+ \to 0_1^+)$ has an experimental value of 73, with SDI-1 giving 38, SDI-2 giving 36, and TDA giving 30.1. The ratio $B(E2; 4_1^+ \to 2_1^+)/B(E2; 2_1^+ \to 0_1^+)$ has an experimental value of 75, with SDI-1 giving 67, SDI-2 giving 66.5, and TDA giving 95.9. The ratio $B(E2; 4_2^+ \to 2_1^+)/B(E2; 4_1^+ \to 2_1^+)$ has an experimental value of 2.2, with SDI-1 giving 2.2 (identical to experiment), SDI-2 giving 2.1, and TDA giving 0.3.

For $B(M1)$ calculations, to simplify, we adopt from reference [?] the values $g_{l\pi} = 1$, $g_{l\nu} = 0$, $g_{s\pi} = 5.586$, and $g_{s\nu} = -3.826$ (in units of μ_N^2). The calculated results from all three cases are consistent with wave function structures. For certain nuclei, SDI-1 provides values closer to experiment. For example, the experimental $B(M1; 0_2^+ \to 1_1^+)$ for ^{134}Ba is $0.022(6)\mu_N^2$, with SDI-1 giving $0.01809\mu_N^2$ and SDI-2 giving $0.00305\mu_N^2$. The experimental $B(M1; 1_1^+ \to 0_1^+)$ is $0.096(18)\mu_N^2$, with SDI-1 giving $0.08215\mu_N^2$ and SDI-2 giving $0.07537\mu_N^2$. Examining the interaction strength values in [Figure 1: see original paper] reveals that for ^{134}Ba, the increase in neutron-neutron interaction strength in SDI-1 is larger than the decrease in proton-proton interaction strength compared to SDI-2. The net effect is that in some cases, SDI-1 yields slightly larger $B(M1)$ values than SDI-2, in better agreement with experiment.

$B(E2)$ ratios for Ba isotopes. Experimental data taken from reference [?].

4 Conclusions

By truncating the NPSM to the SD pair subspace to obtain SDPSM, and focusing on constructing SD pair structures using both SDI and TDA methods, we conclude that these two approaches have complementary advantages and disadvantages, with no absolutely superior scheme for describing Ba isotopes. Both methods can produce results close to experimental values, with better agreement as the number of neutron hole pairs increases. Both can reveal the intrinsic relationship between electromagnetic transition strengths and wave function structures along the Ba isotopic chain. First, for describing nuclear spectral properties, the SDI method generally provides better fits for yrast and quasi-γ bands, while the TDA method yields superior results for the second 0^+ state energy and first 3^+ state energy. Second, for describing electromagnetic properties, the TDA method provides better fits due to its inclusion of many-body effects. Additionally, in the SDI method, adjustments to identical-nucleon and proton-neutron interaction strengths can compensate for the neglect of many-body correlations between nucleon pairs, sometimes even producing results superior to the TDA approximation method.

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References

[1] Caurier E, Martínez-Pinedo G, Nowacki F, et al. The shell model as a unified view of nuclear structure [J]. Reviews of Modern Physics, 2006, 77(2): 427. doi: 10.1103/RevModPhys.77.427

[2] Chen Jinquan. Nucleon-pair shell model: Formalism and special cases[J]. Nuclear Physics A, 1997, 626(3): 686. doi: 10.1016/S0375-9474(97)00502-2

[3] Otsuka T, Arima A, Iachello F. Nuclear shell model and interacting bosons[J]. Nuclear Physics A, 1978, 309(1-2): 1. doi: 10.1016/0375-9474(78)90532-8

[4] Iachello F, Arima A. The Interacting Boson Model[M]. New York: Cambridge University Press, 1987: 60-174.

[5] Otsuka T, Arima A, Iachello F. Shell model description of interacting bosons [J]. Physics Letters B, 1978, 76(2): 139. doi: 10.1016/0370-2693(78)90260-5

[6] Arima A, Iachello F. Interacting boson model of collective states. I. vibrational limit [J]. Annals of Physics, 1976, 99(2): 253. doi: 10.1016/0003-4916(76)90097-X

[7] Luo Yan-An, PAN FENG, DRAAYER JERRY P, NING PING-ZHI. SD-PAIR SHELL MODEL FOR EVEN-EVEN SYSTEMS. International Journal of Modern Physics E, 2008, 17(1): 245. doi: 10.1142/S0218301308011896

[8] Zhao Y.M., Yoshinaga N, Yamaji S. Relationship between the fermion dynamical symmetric model Hamiltonian and nuclear collective motion [J]. Physical Review C, 2000, 62(2): 024322-1. doi: 10.1103/PhysRevC.62.024322

[9] Luo Yan-An, Chen Jin-Quan, Draayer J.P. Nucleon-pair shell model calculations of the even–even Xe and Ba nuclei [J]. Nuclear Physics A, 2000, 669(1-2): 101. doi: 10.1016/S0375-9474(99)00818-0

[10] Chen Jin-Quan, Luo Yan-an. Nucleon-pair shell model: The effects of

the SD pair structure on collectivity of low-lying states[J]. Nuclear Physics A, 1998, 639(3-4): 615. doi: 10.1016/S0375-9474(98)00422-9
[11] Luo Yanan, Chen Jinquan. Shell model calculation in the S-D subspace [J]. Physical Review C, 1998, 58(1): 589. doi: 10.1103/PhysRevC.58.589
[12] Luo Yanan, Pan Feng, Bahri C, et al. SD-pair shell model and the interacting boson model[J]. Physical Review C, 2005, 71(4): 4304(1). doi: 10.1103/PhysRevC.71.044304
[13] Zhao YM, Yamaji S, Yoshinaga N, et al. Nucleon pair approximation of the nuclear collective motion[J]. Physical Review C, 2000, 62(1): 014315-1. doi: 10.1103/PhysRevC.62.014315
[14] Jia L Y, Zhao Y M. Systematic calculations of low-lying states of even-even nuclei within the nucleon pair approximation[J]. Physical Review C, 2007, 75(3): 034307-1. doi: 10.1103/PHYSREVC.75.034307
[15] Lei Y, Lu Y, Fu G J, et al. Advances in the study of nuclear collective rotation using pairing theory in the framework of the shell model [J]. Chin Sci Bull., 2024, 69(25): 3757.
[16] Fu G J, Johnson C W. From deformed Hartree-Fock to the nucleon-pair approximation[J]. Physics Letters B, 2020, 809(10): 135705-1. doi: 10.1016/j.physletb.2020.135705
[17] Kaneko K, Shimizu N, Mizusaki T, et al. Quasi-SU(3) coupling of (1h11/2, 2f7/2) across the N = 82 shell gap: Enhanced E2 collectivity and shape evolution in Nd isotopes[J]. Physical Review C, 2021, 103(2): L021301-1. doi: 10.1103/PhysRevC.103.L021301
[18] Xu Z Y, Lei Y, Zhao Y M, et al. Low-lying states of heavy nuclei within the nucleon pair approximation[J]. Physical Review C, 2009, 79(5): 054315-1. doi: 10.1103/PhysRevC.79.054315
[19] Arvieu R and Moszkowski S A. Generalized seniority and the surface delta interaction[J]. Physical Review C, 1966, 145(3): 830. doi: 10.1103/PhysRev.145.830
[20] Van Egmond A, Allaart Klaasa, Bonsignori G. Microscopic approach to magnetic excitations in IBM-2[J]. Nuclear Physics A, 1985, 436(3): 458. doi: 10.1016/0375-9474(85)90080-6
[21] Wu Cheng-Li, Feng Da Hsuan, Chen Xuan-Gen. Fermion dynamical symmetry model of nuclei: basis, Hamiltonian, and symmetries[J]. Physical Review C, 1987, 36(3): 1157. doi: 10.1103/PhysRevC.36.1157
[22] Fogelberg B, Blomqvist J. Single-hole and three-quasiparticle levels in 131Sn observed in the decay of 131g,m1,m2In[J]. Nuclear Physics A, 1984, 492(2): 205. doi: 10.1016/0375-9474(84)90205-7
[23] Baldrige W J. Shell-model studies for the 132Sn region I Few proton cases[J]. Physical Review C, 1978, 18(1): 530. doi: 10.1103/PhysRevC.18.530
[24] Bohle D, Richter A, Steffen W. New magnetic dipole excitation mode studied in the heavy deformed nucleus 156Gd by inelastic electron scattering [J]. Physics Letters B, 1984, 137(1-2): 27. doi: 10.1016/0370-2693(84)91099-2
[25] FAZEKAS B, BELGYA T, MOLNAR G. LEVEL SCHEME AND MIXED-SYMMETRY STATES OF BA-134 FROM IN-BEAM (N, N'GAMMA) MEASUREMENTS[J]. Nuclear Physics A, 1992, 548(2): 249. doi: 10.1016/0375-

9474(92)90011-8
[26] Iachello F. New Class of Low-Lying Collective Models in Nuclei[J]. Physical Review Letters, 1984, 53(15): 1427. doi: 10.1103/PhysRevLett.53.1427
[27] Casten R F. AN EXTENSIVE REGION OF O(6)-LIKE NUCLEI NEAR A=130 [J]. Physics Letters, 1985, 152(1,2): 22. doi: 10.1016/0370-2693(85)91131-1
[28] Zamick L. Collective magnetic dipole transitions in the rotational model: Orthogonalization with respect to the spurious states. [J]. Physics Letters B, 1986, 167(1): 1. doi: 10.1016/0370-2693(86)90533-2

Note: Figure translations are in progress. See original paper for figures.