An interesting equation yields the MOND but does not rule out the dark matter

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Abstract

Inspired by the idea in Ref. [16], which introduced a viscosity coefficient into the expansion equation describing the universe, we also attempt to introduce such a positive viscosity coefficient into the rotational motion equation describing the galaxies, and then studies what will happen. Surprisingly, we obtained all the formulas assumed in MOND, including a concrete interpolation function between the centripetal acceleration and the Newtonian acceleration. But at the same time, something different from MOND was also obtained, that is, the critical acceleration, a0 in MOND, does not need to be a constant, but increases with the mass of the galaxy increases, and under the action of viscosity coefficient, the rotational galaxies will gradually expand over time at the radial direction, just like the expansion of the universe. However, unlike MOND, the model in this paper cannot rule out the existence of dark matter. Instead, the mass of dark matter can be used to help to adjust the value of A0 (here it just to distinguish from a0 in MOND, and A0 and a0 have the same meaning in the equation), thereby helping to better fit the radial acceleration relation (RAR) curve of galaxies. However, unlike ΛCDM, even if dark matter exists, it does not need to be carefully adjusted to meet the asymptotically flat rotational velocity curve of galaxies. The rotational curve of galaxies with this characteristic can be also achieved under the viscous dynamics of the galaxy itself.

Full Text

Preamble

An Interesting Equation Yields MOND but Does Not Rule Out Dark Matter

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Abstract: Inspired by the approach in Ref. [16], which introduced a viscosity coefficient into the expansion equation describing the universe, we attempt to introduce a similar positive viscosity coefficient into the rotational motion equation describing galaxies and study the consequences. Surprisingly, we obtain all the formulas assumed in MOND, including a concrete interpolation function between centripetal acceleration and Newtonian acceleration. However, we also find differences from MOND: the critical acceleration a_0 in MOND does not need to be constant but instead increases with the galaxy' s mass. Under the action of the viscosity coefficient, rotating galaxies gradually expand radially over time, analogous to cosmic expansion. Unlike MOND, our model cannot rule out dark matter. Instead, dark matter mass can help adjust the value of A_0 (distinguished here from MOND' s a_0, though they share the same meaning in the equations), thereby improving the fit to galaxies' radial acceleration relation (RAR) curves. However, unlike ΛCDM, even if dark matter exists, it does not require careful tuning to produce the asymptotically flat rotation curves observed in galaxies. Such rotation curves can also be achieved through the viscous dynamics of the galaxy itself.

Keywords: rotational motion; dark matter; MOND; ΛCDM; Newtonian dynamics; viscosity coefficient

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1. Introduction

Dark matter and dark energy represent two major puzzles in modern cosmology [1]. Dark energy was proposed to explain the observed acceleration of the universe' s expansion, while dark matter addresses the anomalous rotation curves of disk galaxies. In the standard cosmological model, Λ Cold Dark Matter (ΛCDM), dark energy constitutes approximately 70% of the cosmic energy budget, dark matter about 25%, and ordinary visible matter merely about 5% [2]. Although the ΛCDM model is relatively successful and widely accepted among physicists, it still faces unresolved issues, such as the fine-tuning problem and the origin of dark energy [3]. For dark matter, its composition remains uncertain [4], as no particle described or predicted by the Standard Model exhibits the required properties. More importantly, dark matter has not yet been directly detected experimentally. In this complex context, various alternative models for dark energy and dark matter have been proposed. Among alternatives to dark matter, the most well-known is MOdified Newtonian Dynamics (MOND), first proposed by Milgrom in 1983 [5].

Milgrom postulated that Newtonian dynamics break down at small accelerations and that the law of gravity must be modified. To date, MOND has been extensively studied and has successfully explained numerous astrophysical observations, including galaxy kinematics [6,7], dynamics of wide binary stars [8,9], the radial acceleration relation of galaxies [10,11], orbital velocities of interacting galaxy pairs [12], early-universe galaxy and cluster formation

[13], and galaxy-scale gravitational lensing [14,15]. For dark energy alternatives, distinguishing between models remains difficult, and no single alternative has emerged as clearly superior. However, we note an interesting model proposed in Ref. [16], which argues that cosmic fluid is not perfect but viscous and dissipative, introducing a viscous pressure proportional to the Hubble constant into the standard cosmological model. Although the origin of this viscous pressure remains debated, the model fits the combined SNe Ia + CMB + BAO + H(z) data significantly better than the ΛCDM model.

In this paper, inspired by the idea of introducing a dissipation term into the universe' s expansion equation [16], we attempt to introduce a similar dissipation term into the rotational motion equation describing galaxies and examine its effects on galactic rotation. Surprisingly, we find a high degree of coincidence between the derived motion properties and MOND predictions. This is particularly intriguing given that MOND is known to be an empirical model rather than a fundamental theory [14], suggesting that our approach may offer insight into the nature of MOND.

2. Newtonian Dynamics with Dissipation Term

In Ref. [16], the author introduced a dissipation term into the expansion equation describing the universe, making the evolution equation become

$$\ddot{a} + \zeta \dot{a} = -\frac{4\pi G}{3}\rho_m a + \frac{\Lambda}{3}a \tag{1}$$

where a denotes the scale factor of the universe, ρ_m is the total matter density, G is the gravitational constant, Λ is the cosmological constant, and ζ is the introduced viscosity coefficient, which is positive. Ref. [16] demonstrates that Eq. (1) fits astrophysical observational data significantly better than the ΛCDM model. Inspired by this, we introduce a similar dissipation term into the rotational motion equation describing galaxies to study the consequences, even though the physical basis for this term remains unclear at present.

Analogous to Eq. (1), we assume a dissipation term in the galactic rotational motion equation proportional to velocity, with a corresponding positive viscosity coefficient. Without considering relativistic effects, we obtain the force equilibrium equations in the radial and circumferential directions:

$$\frac{GM}{r^2} - \lambda \dot{r} = \frac{v^2}{r} \tag{2}$$

$$\frac{d}{dt}(mr^2\dot{\theta}) = -\lambda m r^2 \dot{\theta} \tag{3}$$

where M denotes the central mass of the galaxy, (r, θ) are the motion coordinates (for simplicity we consider only two-dimensional planar motion), and λ

is the assumed viscosity coefficient. The second equation states that the time derivative of angular momentum equals the rotational torque.

Equation (2) implies an assumption that the dissipation force depends on the mass m of the rotating object, as the mass cancels on both sides of the first equation—consistent with the approach in Ref. [16]. In fact, Eq. (2) is equivalent to introducing a dissipation potential ψ into Newtonian dynamics:

$$\psi = \frac{1}{2}\lambda r^2\dot{\theta}^2 \tag{4}$$

We can now use Eq. (2) to study galactic rotation. From the second equation of Eq. (2), we obtain

$$\dot{\theta} = \dot{\theta}_0 \left(\frac{r_0}{r}\right)^2 e^{-\lambda(t-t_0)} \tag{5}$$

where (θ_0, r_0) represents the initial condition. Substituting Eq. (4) into the first equation of Eq. (2) yields

$$\frac{GM}{r^2} - \lambda\dot{r} = \frac{v_0^2 r_0^4}{r^3} e^{-2\lambda(t-t_0)} \tag{6}$$

where $R = r/r_0$ and

$$v_0 = r_0\dot{\theta}_0 \tag{7}$$

An analytical solution for Eq. (5) is difficult to obtain, but numerical solutions are accessible. Before proceeding with numerical analysis, we can make a qualitative assessment. From the success of Newtonian dynamics, we know λ must be very small. Therefore, in strong gravitational fields, Eq. (2) reduces to standard Newtonian dynamics. In very weak gravitational fields where $r \gg GM/r^2$, the equation can be simplified as θ, and the first formula of Eq. (2) becomes $r\lambda \gg r$. The solution of Eq. (6) is , which combined with Eq. (4) yields , indicating that in very weak gravitational fields, the circumferential velocity of an object orbiting the galactic center remains constant as its orbital radius increases—similar to MOND predictions.

Since no clear physical basis currently exists to determine λ, we examine the motion properties predicted by Eq. (2) by arbitrarily setting λ values and initial conditions. Given that Eq. (2) produces MOND-like results in extremely weak gravitational fields, we next discuss Eq. (2) using MOND' s formalism. MOND' s formula has the following form:

$$g_N\mu = a \tag{8}$$

where g_N denotes gravitational acceleration, a is centripetal acceleration, and μ is an empirical or undetermined interpolation function satisfying $\mu \to 1$ when $a/a_0 \gg 1$ and $\mu \to a/a_0$ when $a/a_0 \ll 1$, with $a_0 \approx 1.2 \times 10^{-10}$ m/s^2 being a constant.

From a brief examination of Eq. (5), we observe that regardless of the value of λ ($\lambda > 0$) and the initial conditions, the velocity $v(= r\dot{\theta})$ eventually becomes almost constant (denoted as v_{ultimate}) over time. We can therefore define a physical quantity A_0 analogous to MOND' s a_0:

$$v^4_{\text{ultimate}} = GMA_0 \tag{9}$$

Similar to MOND, the relationship between gravitational acceleration and centripetal acceleration corresponding to Eq. (5) can be written as

$$g_N \mu' = a \tag{10}$$

We now conduct numerical analysis of Eq. (5). Table 1 shows the values of A_0 and μ' for different λ and initial conditions. We omit units in Table 1 as they are unnecessary—the conclusions from Eq. (5) remain unaffected by the presence or absence of units in the numerical analysis.

Figure 1 shows the obtained μ' corresponding to the values in Table 1. In Table 1, the groups (1,6,7,8), (9,10,11,12), (13,14,15,16), and (19,20,21) correspond to the same mass M, that is . From these groups we see that A_0 depends only on M and λ, not on initial conditions (v_0, r_0), provided the initial centripetal acceleration is sufficiently strong. This is crucial for Eq. (2) to yield MOND-like behavior. One might ask why the initial centripetal acceleration must be strong enough: if the initial centripetal acceleration a'_0 is too small while λ is relatively large, v_{ultimate} becomes very close to v_0, making $A_0 \approx a'_0$ according to Eq. (8). In this case, A_0 depends on M or initial conditions (v_0, r_0) rather than λ.

In other words, if numerous objects orbiting the galactic center initially reside in the strong gravitational potential region, and we assume λ is constant for that galaxy, then regardless of their initial orbits, all objects eventually reach the same rotational circumferential velocity under the dissipation potential (i.e., Eq. (3)). The fourth power of this ultimate velocity (Eq. (8)) is proportional to the galaxy' s central mass—known as the famous Tully-Fisher relation [17-19].

From groups (1,2,3,4,5) in Table 1, we see that if λ is assumed constant independent of central mass, larger galaxies exhibit higher A_0 values, echoing conclusions in Ref. [20]. From groups (1,9,13) and (6,14,19), we observe that for the same galaxy, higher assumed λ values yield higher A_0.

Most importantly, Table 1 and Figure 1 show that as long as the initial centripetal acceleration is sufficiently strong, the interpolation function μ' is essentially identical regardless of λ, initial orbit, or central mass. Figure 1 reveals that $\mu' \to x$ when $x \ll 1$ and $\mu' \to 1$ when $x \gg 1$, exactly as required by MOND' s empirical formula. The curve in Figure 1 can be fitted by

$$\mu' = \frac{x}{\sqrt{1+x^2}} \tag{11}$$

Unfortunately, we cannot provide a clear mathematical definition of "sufficiently strong initial centripetal acceleration" because Eq. (5) lacks an analytical solution, yielding only numerical solutions.

By simply introducing a viscosity coefficient λ into Newtonian dynamics, we have derived essentially all formulas assumed in MOND, including a nearly complete interpolation function. However, we also obtain differences from MOND: A_0 need not be constant, and rotating galaxies gradually expand over time, analogous to cosmic expansion, predicting "dynamic" rather than "static" galaxies as in ΛCDM and MOND.

Although these conclusions are based on Table 1' s data, we have privately expanded the dataset and found the conclusions remain unchanged, so we omit these lengthy results. We reiterate that the absence of units in Table 1 does not affect the conclusions drawn from Eq. (5).

3. Models Comparison

To compare observations with ΛCDM, MOND, and Eq. (9), we must separate the Newtonian gravitational acceleration due to ordinary matter (excluding dark matter) from the total acceleration. We assume baryonic acceleration originates from the bulge and disk (with no dark halo contribution):

$$g_b = \frac{v_b^2}{r} + \frac{v_d^2}{r} \tag{12}$$

where r is the distance from the galactic center, and v_b and v_d are rotational velocities due to the bulge and disk, respectively.

In ΛCDM, dark matter is included alongside ordinary matter, with the dark halo function typically expressed as [21]:

$$\rho(r) = \frac{\rho_0}{\frac{r}{h}\left(1+\frac{r}{h}\right)^2} \tag{13}$$

where ρ_0 and h are the scale density and scale radius of the dark halo. The dark halo mass is then

$$M_h = 4\pi\rho_0 h^3 \left[\ln\left(1+\frac{r}{h}\right) - \frac{r/h}{1+r/h}\right] \tag{14}$$

yielding

$$g_h = \frac{GM_h}{r^2} \tag{15}$$

The total acceleration becomes

$$g_{\text{total}} = g_b + g_h \tag{16}$$

In MOND, no dark matter exists, and the relationship between gravitational and centripetal acceleration follows Eq. (7). Since interpolation functions are empirical, we adopt the "simple" function [22]:

$$\mu = \frac{a/a_0}{1 + a/a_0} \tag{17}$$

Figure 2 [Figure 2: see original paper] [23] plots centripetal acceleration versus baryonic gravitational acceleration for observational data, with fitting curves for various models.

Figure 2 shows centripetal acceleration versus baryonic gravitational acceleration for data and models. Black circles with error bars represent Milky Way data, purple squares represent M31. The short-dashed line is the ΛCDM fit for the Milky Way, the dash-dot line for M31. The dotted line is the reference line $a = a_B$. The dashed line shows MOND predictions with $a_0 = 1.2 \times 10^{-10}$ m/s^2, and the solid line shows predictions from Eq. (9) with $A_0 = 1.2 \times 10^{-11}$ m/s^2.

Note that predictions in Figure 2 assume isolated galaxies. For smaller radii (large accelerations), MOND and Eq. (9) do not significantly deviate from the expected ΛCDM predictions, but differences emerge at lower accelerations. The centripetal accelerations in M31 and the Milky Way are smaller than MOND predictions at low a_B but align with Eq. (9). This occurs because MOND' s radial acceleration relation (RAR) predictions have no free parameters, whereas A_0 in Eq. (9) is a parameter that can vary with galaxy mass, allowing adjustment to fit observational data.

One might ask whether dark matter was considered in Eq. (9) when fitting the Figure 2 data. Mathematically, it makes no difference to the fitting results. If dark matter exists near the galactic center (similar to MOND, our model does not require careful dark halo tuning to produce asymptotically flat rotation curves), the a_B coordinate in Figure 2 should be replaced by g_t. Since dark matter resides near the center and the maximum a_B values in Figure 2 correspond to radii far from the center, the g_t axis can be obtained by adding a constant value (determined by dark matter mass) to all a_B values—effectively shifting the a_B axis rightward. In Eq. (9), the left side relates to g_t and the right side to a. Since Table 1 shows μ' is independent of central mass (i.e., independent of a_B and g_t), dark matter' s presence near the galactic center does not mathematically affect Eq. (9)' s fitting results. Physically, however, dark matter may be needed as its mass can help adjust A_0.

4. Summary

Without clear physical basis, this paper follows Ref. [1] by introducing a viscosity coefficient into the universe' s expansion equation and applies a similar positive viscosity coefficient to galactic rotational motion. Surprisingly, this yields all formulas assumed in MOND, including a concrete interpolation function between centripetal and Newtonian acceleration—though MOND's function is empirical.

We also obtain differences from MOND. First, while a_0 is constant in MOND, our A_0 (distinguished from MOND' s a_0 though equivalent in the equations) is not constant but depends on galaxy mass: more massive galaxies have larger A_0 values for constant λ. This helps fit RAR data better than MOND. Second, the model predicts that rotating galaxies expand radially over time, analogous to cosmic expansion, predicting "dynamic" rather than "static" galaxies as in ΛCDM and MOND.

Many studies [24-27] find MOND fits galactic rotation data better than ΛCDM, attributing this primarily to MOND' s two extreme conditions (Newtonian and deep-MOND limits) rather than the specific interpolation function form. Our model can also fit these data well because these extreme conditions naturally emerge from our framework. In MOND, fitting observational data requires adjusting the galaxy' s mass-to-light ratio—the only free parameter [28]—thereby excluding dark matter, as its presence would affect this ratio. Our model cannot rule out dark matter: its mass can help adjust A_0, and our inspiration comes from Ref. [2], which supports dark matter' s existence. However, unlike ΛCDM, our model does not require careful dark matter tuning to produce asymptotically flat rotation curves; these can arise from the galaxy' s own viscous dynamics.

We have not discussed λ' s value because Eq. (9)' s extreme conditions fit many observational data well without specifying λ, and our model does not exclude dark matter, though dark matter' s role remains unclear. Additionally, limited detailed observational data prevents further analysis, but this represents a task for future research.

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Note: Figure translations are in progress. See original paper for figures.