Simulation-Based Image Reconstruction Methods for Lunar Hydrogen Distribution: A Postprint

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Abstract

Obtaining accurate lunar water-hydrogen distribution maps holds significant scientific value for water ice detection research and subsequent deep space exploration. Based on the detection principle of neutron detectors and the satellite image degradation process, a kappa function-based point spread function with shift-variant characteristics was constructed, and simulated detection images were obtained by blurring and adding noise to the images. The Maximum Entropy algorithm and Richardson-Lucy algorithm were primarily employed to reconstruct the simulated detection images, with visual effect, chi-square test, and authenticity verification serving as evaluation criteria for comparative study. Experimental results demonstrate that direct reconstruction cannot achieve satisfactory results under either low-noise or high-noise conditions. After denoising preprocessing, under the premise of chi-square test safety, the reconstruction results are overall superior to those before preprocessing. The number of points with large deviations in the authenticity verification of the reconstructed images is significantly reduced, indicating that this method can obtain more accurate and reliable reconstruction results.

Full Text

Research on Image Reconstruction Method for Lunar Hydrogen Distribution Based on Simulation

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Abstract

Obtaining accurate maps of lunar water/hydrogen distribution holds significant scientific value for both water ice detection research and subsequent deep space exploration. Based on the detection principle of neutron detectors and the degradation process of satellite imagery, we construct a shift-variant point spread function (PSF) founded on the kappa function, which is used to generate simulated detection images through blurring and noise addition. This study focuses on reconstructing these simulated detection images using the maximum entropy algorithm and the Richardson-Lucy (RL) algorithm, with evaluation based on visual assessment, chi-square testing, and authenticity verification. Experimental results demonstrate that direct reconstruction fails to achieve satisfactory results under both low- and high-noise conditions. However, after denoising preprocessing and ensuring the safety of chi-square tests, the overall reconstruction quality surpasses that of the unpreprocessed case. Notably, the number of points with large deviations in the authenticity test decreases significantly, indicating that the proposed method yields more accurate and reliable reconstruction results.

Keywords: space vehicles: instruments, planets and satellites: detection, techniques: image processing

1. Introduction

Since the 1960s, scientists have predicted that substantial water ice may exist on the Moon. Due to the Moon' s unique geographical environment, certain crater regions remain permanently shadowed from solar illumination, maintaining average temperatures below -180°C and forming cold traps where unvolatilized water ice may persist. Over millions of years of accumulation, these regions could contain enormous quantities of water ice, estimated by scientists to reach tens of billions of tons [1–3]. This water ice not only holds scientific value for detection research but also presents tremendous potential for establishing human bases for deep space exploration. Recent observational data have further confirmed the existence of water ice at the lunar poles [4–5], and samples returned by the Chang' e-5 mission have verified the presence of water or hydroxyl groups [6], refocusing planetary exploration efforts on the Moon. China, Japan, Russia, the United States, and India have all proposed subsequent lunar exploration programs, making lunar water ice detection a key scientific objective.

Without atmospheric or magnetic field protection, the lunar surface is continuously bombarded by solar wind and cosmic rays, generating abundant secondary neutrons. The epithermal neutron flux is highly sensitive to surface hydrogen content, enabling neutron spectrometers to detect epithermal neutron flux and identify hydrogen-rich regions, thereby indirectly mapping lunar water ice distribution. Although neutron spectrometers cannot distinguish between different hydrogen compounds, they can identify areas enriched in hydrogen. Neutron flux spectroscopy is a common technique for detecting surface composition of celestial bodies [7]. When high-energy cosmic rays interact with lunar regolith, they produce neutrons and other particles. During this process, epithermal neutrons (with energies between 1 eV and 10 eV) lose most of their energy upon collision with hydrogen atoms of similar mass, becoming thermal neutrons (with energy approximately 0.025 eV) [8–10]. This results in lower epithermal neutron flux and higher thermal neutron flux in hydrogen-rich regions. Conversely, lunar mid- and low-latitude regions, receiving direct sunlight with high surface temperatures, make water ice existence nearly impossible, thus exhibiting higher epithermal neutron flux and lower thermal neutron flux. By comparing neutron flux relationships between high-latitude and mid/low-latitude regions, we can indirectly obtain lunar surface water/hydrogen distribution.

While neutron detectors offer high sensitivity for hydrogen detection, their spatial resolution is negatively correlated with orbital altitude [11]. Reducing orbital height to improve spatial resolution would inevitably increase satellite platform design complexity. Furthermore, during lunar surface detection, neutron detectors cannot completely shield neutron signals from outside the detection area, affecting the acquired images [12] and introducing significant noise that reduces effective information and degrades image quality. Consequently, employing image reconstruction techniques to enhance spatial resolution and image quality is essential.

In contemporary image processing, image reconstruction technology has attracted considerable attention, primarily addressing image "degradation" that occurs during acquisition, processing, and transmission due to factors such as defocusing, relative motion between equipment and objects, and system noise [13–15]. Reconstruction can be viewed as an "equation-solving" process that selects appropriate algorithms based on imaging system characteristics to recover the original image. The ultimate goal is to maximally restore the image' s authenticity, transforming imperfect images into clearer, more detailed representations that provide more accurate information.

2. Image Reconstruction Principles

The degraded image recovery process is illustrated in [Figure 1: see original paper]. Due to the lack of effective image data, this study employs simulated lunar water/hydrogen distribution detection images, which not only solves the shortage of detection images but also enables better algorithm analysis by providing original images for comparison. We focus on processing simulated lunar neutron detection images using the maximum entropy algorithm and Richardson-Lucy (RL) algorithm, comparing results to identify superior reconstruction methods. Additionally, we introduce Non-negative Least Squares (NNLS) and conjugate gradient methods for comparative analysis, though their basic principles are not elaborated here.

2.1 Maximum Entropy Principle Huang et al. [16] introduced the maximum entropy principle based on Shannon entropy, elaborating its classical fundamentals, discussing general forms of maximum entropy optimization under different constraints, and proposing specific improvements for classical limitations. When applying the maximum entropy principle, the following criterion applies: under limited constraints and uncertainty, maintain maximum possibility—in other words, maximize information content (entropy) under known conditions to derive probability distributions of random variables. This method has demonstrated significant practical value and research importance in finance, image processing, meteorology, and other fields [17-19].

Assuming the original image is X, the degraded image is Y, the imaging system's point spread function (PSF) is H, and noise is n, the degraded image can be expressed as:

$$Y = X \otimes H + n$$

where $\otimes$ represents convolution. Note that noise is not considered during formula derivation to avoid complex operations. Let the degraded image Y have M pixels and the original image X have N pixels, with:

$$\sum_{i=1}^{N} X_i = 1, \quad \sum_{k=1}^{N} X_k H_{k;i} = Y_k$$

where $1 \leq k \leq M$, X_i represents the i-th element in X, $H_{k;i}$ represents the element at position (k, i) in H, and Y_k represents the k-th element in Y. The maximum entropy method requires maximizing image entropy under the constraints of equations (2) and (3).

Using a successive approximation method based on the iterative formula proposed by Bonavito et al. [20], we construct the following algorithm:

$$\delta\lambda_k = \ln Y_k - \ln(H \otimes X_k)$$
$$\lambda_{k+1} = \lambda_k + \delta\lambda_k$$
$$X_k = c \cdot \exp(\lambda_k \otimes H)$$

where $\delta\lambda_k$ is the change in λ_k, λ_k is the k-th parameter introduced using the Lagrange multiplier method, starting from an initial $\lambda_0 = 0$ (which yields maximum entropy but does not satisfy constraint equations), and c is a proportional constant.

2.2 RL Algorithm The RL algorithm, proposed by Richardson and Lucy in the 1970s, is an iterative algorithm based on Bayesian theory. Its central idea is that through continuous iteration, the output image becomes the maximum likelihood estimate of the ideal image [21]. The RL algorithm follows a Poisson distribution, with the iteration equation:

$$X^{(r+1)}(x,y) = X^{(r)}(x,y)\left[\frac{Y(x,y)}{H(x,y)\otimes X^{(r)}(x,y)}\right]\otimes H(-x,-y)$$

This equation solves for the maximum likelihood probability solution, offering superior reconstruction precision among conventional algorithms. Here, r represents the iteration number, (x, y) is the two-dimensional representation of the image, $X^{(r)}(x, y)$ is the image obtained from the r-th reconstruction, and $H(-x, -y)$ is the conjugate of the PSF $H(x, y)$.

3. Construction of Detection Images

We construct an original image as shown in [Figure 2: see original paper]. Notably, two pixel points are constructed near the figure "7" to test the reconstruction algorithm' s ability to recover small information content.

3.1 Point Spread Function In practical applications, an imaging system whose PSF model depends on the spatial position of objects is called a spatially variant system. Conversely, if the PSF model is largely independent of spatial position, the system is called spatially invariant. Currently, research on image restoration methods for spatially invariant systems is relatively mature, including frequency domain methods, iterative methods, and recursive methods. However, for spatially variant systems, restoration is more challenging because each point on the object plane corresponds to a specific PSF form.

To address this issue, appropriate methods can approximate spatially variant systems as spatially invariant problems. In this research, since the Moon can be approximated as a sphere, a detector scanning the lunar surface at different latitudes covers different areas per unit time—the lower the latitude, the longer the distance scanned and the larger the corresponding area. Considering that spherical images undergo deformation when projected onto a plane, each point on the lunar surface corresponds to a PSF value related to its latitude and longitude after passing through the satellite detector imaging system, making it a spatially variant system. Therefore, an appropriate PSF model is needed to obtain analytical PSF values for different lunar points, approximating the spatially variant system as spatially invariant for further processing.

Previous studies found that using a Gaussian function to model the satellite imaging system' s PSF resulted in poor tail fitting, whereas the kappa function avoids this deficiency [22-23]. The specific formula is:

$$W(D,h) = \frac{\kappa(h)-1}{2\alpha(h)^2}\left[1+\frac{D^2}{2\alpha(h)^2}\right]^{-\kappa(h)}$$

$$\alpha(h) = 0.704h + 1.39$$

$$\kappa(h) = -4.87\times 10^{-4}h + 0.631$$

where α and κ are fitting parameters, h is the detector' s orbital altitude, and D is the distance between a point on the Moon and the nadir point. Note that when calculating using equation (8), if the kappa function value $W(D, h)$ for a certain region is too small, it can be approximated as zero to improve reconstruction algorithm efficiency.

[Figure 3: see original paper] shows a schematic diagram for calculating distance D from known longitude and latitude. For convenience, we approximate the Moon as a sphere and establish an o-xyz coordinate system. Let B be the intersection point between the line connecting the satellite and the lunar center and the surface (the detector' s nadir point), and A be any other point. The arc length AB represents D. Point A has coordinates (θ_A, ϕ_A) and point B has coordinates (θ_B, ϕ_B). From [Figure 3: see original paper], we derive:

$$\Delta z = R[\sin\phi_B - \sin\phi_A]$$
$$\Delta y = R[\cos\phi_B \sin\theta_B - \cos\phi_A \sin\theta_A]$$
$$\Delta x = R[\cos\phi_B \cos\theta_B - \cos\phi_A \cos\theta_A]$$

where Δx, Δy, and Δz represent the differences between points A and B on the x, y, and z axes, respectively. The chord length AB, denoted as Q, is:

$$Q = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

From planar geometry, the relationship between arc length and chord length is:

$$D = 2R \arcsin\left(\frac{Q}{2R}\right)$$

where R is the lunar radius. Since detectors cannot acquire full lunar surface images at once, we must address non-conservation between received photons and emitted photons from corresponding regions. To handle this, we apply a coefficient correction to H:

$$H = F \otimes m$$

where F is the matrix composed of kappa function values and m is the correction coefficient matrix. The image after PSF addition is shown in [Figure 4: see original paper].

3.2 Noise Construction Noise in space remote sensing imaging originates from onboard background noise and photon statistical fluctuations (following Poisson distribution) measured by detectors. Given that background noise primarily comes from neutron signal background counts when cosmic rays interact with the detector and surrounding materials, we use the average pixel value u of the entire image as a basic noise addition quantity. After adding background noise to a given image, statistical fluctuations are added to each pixel value, with the noise value being the square root of the pixel value.

The noise-added image is shown in [Figure 5: see original paper]. When the background noise value reaches $5u$, image quality degrades noticeably, becoming almost dominated by noise. The chi-square test [24–26] reflects the deviation between reconstructed image X and degraded image Y:

$$\chi^2 = \sum \frac{(Y - X \otimes H)^2}{\sigma^2}$$

where σ represents the standard deviation matrix with the same size as Y. Ideally, the recovered image X equals the original image, making $(Y - X \otimes H)$ equivalent to n. The chi-square test first requires the distribution to satisfy normality; under this premise, we compare chi-square values across restoration methods—smaller values indicate better restoration when distributions are similar.

3.3 Implementation Based on Section 2, we simulate degraded water/hydrogen distribution images and reconstruct them using the two described methods for comparative analysis. This work employs mixed programming in MATLAB and C++ for image restoration. Since MATLAB variables are vectorized, offering high efficiency for overall calculations but becoming complex and inefficient for single element operations or vector loops, C++' s high loop efficiency compensates for MATLAB' s limitations, thus improving runtime performance through mixed programming.

3.4 Reconstruction Evaluation Methods Our comparative study primarily uses reconstruction results, chi-square tests, and authenticity tests as evaluation criteria. Visual assessment of reconstruction results is the first criterion, but deep space exploration images require additional quantitative measures. The chi-square test [24–26] statistically reflects deviation between reconstructed image X and degraded image Y using equation (14).

Authenticity testing evaluates deviation between reconstructed and original images when chi-square tests are reasonable. Since no real lunar water/hydrogen distribution maps exist, authenticity testing is only applied to simulated degraded image recovery. The authenticity test formula is:

$$S = \frac{X - X}{\sigma}$$

where S is the matrix of pixel authenticity test values. Equation (16) shows that smaller differences between X and X yield S values closer to zero. If S values cluster near zero, the restored image closely matches the original. We define points with values beyond ± 3 standard deviations as significantly deviated points. Note that authenticity testing is only valid when solutions are reasonable, meaning chi-square distributions satisfy normality criteria.

4.1 Unpreprocessed Reconstruction Results

To compare reconstruction effects under different noise levels, we construct degraded images with background noise values of u and $5u$, then reconstruct them using maximum entropy and RL algorithms, evaluating through visual assessment, chi-square values, chi-square tests, and authenticity tests.

[Figure 6: see original paper] shows reconstruction results for both algorithms with background noise u. The results are nearly identical between maximum entropy and RL algorithms; due to the pixel-block representation, visual distinction is difficult, and background noise is not completely removed.

[Figure 7: see original paper] presents results for background noise $5u$. Compared to [Figure 6: see original paper], information contours are blurrier, background noise is greater, and reconstructed pixel values significantly exceed original values. However, both algorithms demonstrate good recovery capability for small information content under different noise conditions, with no loss of small-information pixel blocks.

While visual comparison is informative, we now evaluate reconstruction quality through chi-square and authenticity tests. [Figure 8: see original paper] shows chi-square distributions for noise u, and [Figure 9: see original paper] for noise $5u$. With $5u$ noise, both algorithms show nearly identical distributions that essentially satisfy normality. With u noise, distributions are generally consistent but show clear local detail differences, with chi-square value ranges reaching approximately ± 15 standard deviations.

provides chi-square values for both algorithms under different noise conditions. Notably, chi-square values for $5u$ noise are smaller than for u noise, with both algorithms producing similar values. This indicates that reconstruction effectiveness is suppressed under high noise—noise is barely removed, resulting in strong correlation between reconstructed and degraded images.

[Figure 10: see original paper] and [Figure 11: see original paper] show authenticity test distributions for u and $5u$ noise, respectively. Comparison reveals that reconstructed pixel values exceed original values, with greater deviations at higher noise levels. For $5u$ noise, most pixel blocks are significantly larger than original values, consistent with visual comparisons. Few pixel blocks match original values, indicating that reconstruction deviation increases with background noise level.

4.2 Preprocessed Reconstruction Results

The above analysis shows that reconstruction capability decreases as background noise increases, necessitating denoising preprocessing before reconstruction. Conventional denoising methods like Gaussian, median, or bilateral filtering remove noise but also affect image information, potentially generating erroneous data—a serious issue for deep space exploration. Based on background noise generation principles, we can select appropriate deduction values from the degraded image' s frequency histogram or directly measure background noise for removal before reconstruction, thereby improving high-noise degraded image reconstruction quality.

[Figure 12: see original paper] shows the frequency histogram of a degraded image with $5u$ background noise, revealing two main components: noise primarily distributed between 800-1000, and useful information near 1600. Due to photon statistical fluctuations, background noise values are nearly independent per pixel, with highest frequency between 865-870. We select 870 as the deduction value, which does not affect key image information after removal.

The degraded image after background noise deduction is shown in [Figure 13: see original paper]. Compared to [Figure 5: see original paper], most noise is removed while the PSF is well preserved.

Reconstruction results after preprocessing are shown in [Figure 14: see original paper] and [Figure 15: see original paper]. Both sets retain characteristics from [Figure 6: see original paper] and [Figure 7: see original paper]—reconstruction results within each set are consistent, and small-information pixel blocks remain intact. However, background region noise is greatly improved, with minimal noise values, and information-containing regions are well restored and reconstructed.

Beyond visual comparison, [Figure 16: see original paper] and [Figure 17: see original paper] show chi-square distributions for u and $5u$ noise after preprocessing, with comparing chi-square values. The distributions essentially satisfy normality, and in both low- and high-noise conditions, the maximum entropy algorithm performs slightly better than RL, particularly for u noise where RL shows more distribution in distal regions. Both algorithms perform better for $5u$ noise than u noise, with smaller chi-square values at higher noise levels.

Authenticity test distributions after preprocessing are shown in [Figure 18: see original paper] and [Figure 19: see original paper]. Comparison with unpreprocessed results shows substantial improvement, with distributions satisfying normality. The $5u$ authenticity test distribution is more concentrated than the u distribution, with more statistical values at the "0" point.

While preprocessing deducts background noise, it also removes some information content, causing preprocessed reconstructed pixel values to be slightly smaller than original values. In contrast, unpreprocessed reconstruction results show pixel blocks significantly larger than original values with substantial gaps. Both algorithms perform well in preserving small information content from the original image without information loss.

4.3 Discussion

We compared preprocessed and unpreprocessed reconstruction effects of both algorithms under different noise levels. Visual assessment of [Figure 6: see original paper], [Figure 7: see original paper], [Figure 14: see original paper], and [Figure 15: see original paper] shows clear improvement after preprocessing, particularly for high background noise reconstruction. Since preprocessing deducts background noise, pixel values become slightly smaller than original values, while unpreprocessed results are significantly larger.

To demonstrate preprocessing necessity, we derived two parameters from authenticity test distributions: peak value at zero and number of significantly deviated points. Additional tests with higher noise values of $10u$ and $15u$ were conducted, comparing preprocessed and unpreprocessed reconstruction, while also including NNLS and conjugate gradient algorithms for comparison.

Experimental results show that reconstruction quality degrades with increasing background noise, but denoising preprocessing substantially improves visual effects and authenticity test distributions while maintaining normal chi-square distributions. shows authenticity test statistics without preprocessing: as noise increases, distribution quality gradually deteriorates, with zero-value counts reaching zero for noise above $10u$ and predominantly large-deviation points appearing. Even at u and $5u$ noise levels, results are unsatisfactory with numerous large-deviation points.

In contrast, shows preprocessed reconstruction results are significantly superior, with increased zero-value peaks and decreased large-deviation points. Preprocessed degraded image reconstructions yield satisfactory authenticity test distributions that remain stable as noise increases, showing no substantial changes in zero-value peaks or large-deviation point counts across four algorithms. Comparative analysis reveals no fundamental difference between maximum entropy and RL algorithms, though maximum entropy shows slight advantages.

Conclusion

This study focuses on simulated planetary surface neutron detection distribution images, employing the kappa function model as PSF—used in multiple missions (Lunar Prospector, Mars Odyssey, etc.) for image reconstruction research. Based on this foundation, we reconstruct images under different noise conditions using maximum entropy and RL algorithms. Since space exploration image reconstruction differs from conventional image reconstruction, we employ three evaluation methods: visual assessment, chi-square testing, and authenticity testing. Key findings include:

1. Reconstruction quality significantly degrades with increasing background

noise. Chi-square values decrease under high background noise because reconstruction barely removes noise, creating strong correlation between reconstructed and degraded images.

2. Denoising preprocessing before reconstruction, while increasing chi-square values, maintains normal distribution characteristics. Authenticity testing shows substantial improvement, confirming preprocessing importance.
3. Multi-dimensional analysis reveals no fundamental difference between maximum entropy and RL algorithms, though the former shows slight advantages.

While this study cannot overcome all limitations of traditional algorithms, it provides a beneficial foundation for their improvement. Future work will further optimize both reconstruction algorithms, focusing on chi-square testing under different noise conditions to achieve more precise authenticity test distributions and consequently more accurate and clear lunar surface water/hydrogen distribution maps.

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Note: Figure translations are in progress. See original paper for figures.