Research on IMU-Based Positioning System for FAST Inspection (Postprint)

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Abstract

Routine inspection is an important task in the operation and maintenance of the 500,m Aperture Spherical radio Telescope (FAST). FAST comprises numerous components, and the inspection process suffers from heavy workload and time-consuming, labor-intensive component location. Implementing inspection positioning can guide maintenance personnel to fault locations rapidly, which is of great significance for improving inspection efficiency and ensuring the normal operation of the telescope. Considering that the reflector surface obstructs satellite signals, an inertial navigation approach is adopted, and a pedestrian navigation algorithm is designed based on dual foot-mounted IMUs (Inertial Measurement Units). The algorithm follows the zero-velocity update framework, fuses dual-foot information, and incorporates magnetic field information to correct heading errors. The effectiveness of the algorithm was tested on regular trajectories, with horizontal positioning errors within 3% of the walking distance. On irregular trajectories at the inspection site, combined with waypoint markers along the route, the horizontal positioning error is less than 11;m, which can meet the requirements of daily inspection positioning and support the practical implementation of subsequent inspection positioning systems.

Full Text

2 Zero Velocity Update Algorithm Framework

The overall algorithm framework is illustrated in [Figure 1: see original paper]. The system first uses gyroscope and accelerometer data to perform attitude, velocity, and position updates, yielding pure inertial navigation solutions. After each iteration, a one-step predicted state vector and its covariance matrix are computed, referred to as the time update. Simultaneously, the IMU data are used for zero-velocity detection, enabling zero-velocity updates during the stance phase. To improve positioning accuracy, the measurement update also incorporates auxiliary information such as geomagnetic fields, marker points, and inter-foot distances. Since different measurement updates have distinct triggering conditions, a sequential filtering approach is employed to facilitate program debugging and reduce matrix operation dimensions [3]. The time update and measurement update together constitute the complete Kalman filter algorithm framework.

The real system is nonlinear, so the state vector does not directly select quantities like velocity and position. Instead, errors in these quantities are modeled. The small error quantities reduce the impact of higher-order terms, addressing the system' s nonlinearity. The measurement update estimates the error state, which is then compensated into the inertial navigation solution, after which the error state is reset to zero. The Kalman filter framework can fuse inertial navigation calculations with external observation information to obtain more accurate navigation results.

2.1 Inertial Navigation Recursion

The inertial devices used in pedestrian navigation must meet wearable requirements and thus cannot have very high performance. Their noise levels are much larger than Earth' s rotation. Therefore, Earth' s rotation is ignored to obtain simplified inertial navigation equations, including attitude, velocity, and position propagation equations.

The attitude at the current moment is calculated from angular velocity and the previous attitude:

$$C^n_{b;k} = C^n_{b;k-1} \frac{2I_{3\times3} + \Omega_k \Delta t}{2I_{3\times3} - \Omega_k \Delta t}$$

where $C^n_{b;k}$ is the coordinate transformation matrix from the body frame (b-frame) to the navigation frame (n-frame) at sampling time t_k, $I_{3\times3}$ is the three-dimensional identity matrix, Ω_k is the skew-symmetric matrix of gyroscope readings at time t_k, and Δt is the update time interval.

The velocity at the current moment is calculated from specific force and the previous velocity:

$$v^n_k = v^n_{k-1} + C^n_{b;k} f^b_k \Delta t + g^n \Delta t$$

where v^n_k is the velocity in the n-frame at time t_k, f^b_k is the specific force measured by the accelerometer at time t_k projected in the b-frame, and g^n is the gravity acceleration vector projected in the n-frame, taking the value $[0, 0, -g]^T$ in the East-North-Up coordinate system, where g is the local gravity acceleration magnitude.

The position at the current moment is calculated from velocity and the previous position:

$$p_k^n = p_{k-1}^n + \frac{v_{k-1}^n + v_k^n}{2}\Delta t$$

where p_k^n is the position in the n-frame at time t_k.

2.2 Dynamic System Model

For low-cost IMUs, pure inertial navigation solutions diverge rapidly within a short time and must be corrected using other auxiliary information. The fusion of inertial navigation with auxiliary information adopts the Kalman filter framework. The design of the Kalman filter algorithm involves constructing discrete system error state models and observation equations, after which the Kalman filter formulas can be applied for computation.

The continuous system state equation is generally obtained first and then discretized. The continuous system state equation takes the form:

$$\delta\dot{x}(t) = F(t)\delta x(t) + G(t)w(t)$$

where δx is the state vector, F is the state transition matrix, G is the noise distribution matrix, and w is the system noise. All variables are functions of time t.

The navigation parameters and sensor errors constitute the state vector:

$$\delta x = [\phi^T \quad \delta b_g^T \quad \delta p^T \quad \delta v^T \quad \delta b_a^T]^T$$

where the five state components are attitude error ϕ, gyroscope bias error δb_g, position error δp, velocity error δv, and accelerometer bias error δb_a.

Through perturbation analysis of the attitude, velocity, and position propagation equations, and modeling sensor errors as first-order Gauss-Markov processes, the state equation is obtained:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\delta b}_g \\ \dot{\delta p} \\ \dot{\delta v} \\ \dot{\delta b}_a \end{bmatrix} = \begin{bmatrix} 0_{3\times3} & -C_b^n & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & -\frac{1}{T_{gb}}I_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} & 0_{3\times3} \\ (C_b^n f^b)^\times & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & -C_b^n \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & -\frac{1}{T_{ab}}I_{3\times3} \end{bmatrix} \begin{bmatrix} \phi \\ \delta b_g \\ \delta p \\ \delta v \\ \delta b_a \end{bmatrix} + \begin{bmatrix} -C_b^n & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & -C_b^n & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix} \begin{bmatrix} w_g \\ w_{gb} \\ w_a \\ w_{ab} \end{bmatrix}$$

where T_{gb} and T_{ab} are the correlation times for gyroscope and accelerometer error modeling, respectively; w_g and w_a are the white noises in gyroscope and accelerometer readings; w_{gb} and w_{ab} are the driving white noises for gyroscope and accelerometer biases; and $(\cdot)^\times$ denotes the skew-symmetric matrix of a vector.

The observation equation generally uses discrete form directly:

$$Z_k = H_k X_k + V_k$$

where Z_k is the observation at time t_k, H_k is the observation matrix at time t_k, X_k is the discretized state vector, and V_k is the observation noise at time t_k. The observation matrix is set according to the selected observation quantities. The difference between the actual observation vector and the system-estimated observation vector is called the observation innovation, which will be used for system state correction. When introducing measurement updates using different auxiliary information below, the corresponding observation matrices and innovations will be provided.

2.3 Zero Velocity Update

Zero Velocity UPdaTe (ZUPT) utilizes the characteristic that the foot periodically contacts the ground during walking, using zero velocity as observation information to correct errors. The observation quantity for ZUPT is velocity, so its observation matrix H_k is:

$$H_k = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} & 0_{3\times3} \end{bmatrix}$$

The foot is not absolutely stationary during ground contact but has 微小运动, which can be regarded as observation noise. Research in [4] indicates this noise has a standard deviation of approximately 0.017 m/s. The observation innovation m_k for ZUPT is:

$$m_k = v_k - \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

where v_k is the ground velocity obtained from inertial navigation calculation.

ZUPT is performed when the foot contacts the ground, which is detected using gyroscope and accelerometer data. When the magnitude of angular velocity ω_k is below threshold λ and the magnitude of acceleration a_k is close to gravity acceleration, the foot is determined to be stationary. λ is set to 0.6 rad/s [5]. The zero-velocity detection value T_k can be expressed as:

$$T_k = \begin{cases} 1, & \text{if } |\omega_k| < \lambda \text{ and } 9 < |a_k| < 11 \\ 0, & \text{otherwise} \end{cases}$$

In practice, individual misclassifications due to data fluctuations can affect the effectiveness of ZUPT and subsequent stride cycle determination. Mean filtering is applied to the zero-velocity detection sequence by calculating the average of detection values around a given moment. If the average exceeds 0.5, the detection value at that moment is set to 1; otherwise, it is set to 0:

$$T_k = \begin{cases} 1, & \text{if } \frac{1}{2N+1}\sum_{n=k-N}^{k+N} T_n > 0.5 \\ 0, & \text{if } \frac{1}{2N+1}\sum_{n=k-N}^{k+N} T_n < 0.5 \end{cases}$$

where N is the filter window size, set to 5. The filtering effect is shown in [Figure 2: see original paper], with accuracy comparable to the widely used Generalized Likelihood Rate Test (GLRT) method [6].

3 Other Auxiliary Information

Heading is unobservable in ZUPT, causing gradual heading drift. The main road at the FAST site is spiral-shaped, and inspection paths are often irregular, making it impossible to use Heuristic Drift Elimination (HDE) [7] which assumes pedestrians walk along straight lines aligned with roads and buildings. In the pedestrian navigation domain, various researchers have proposed multiple correction methods. Through testing and comparison, the following methods suitable for FAST inspection scenarios were selected.

3.1 Geomagnetic Orientation

Most IMUs include magnetometers, and the geomagnetic field can provide effective heading information. However, the geomagnetic field is susceptible to interference from ferromagnetic materials, so outliers must be excluded before using magnetic field corrections:

$$40\,\mu\mathrm{T} < B_{m;k} < 60\,\mu\mathrm{T}$$

where $B_{m;k}$ is the magnetic field magnitude at the current moment. The geomagnetic field magnitude is approximately 50 µT, so measurements deviating too far from this value are excluded. To avoid vibration effects, geomagnetic orientation is also performed during the zero-velocity phase.

The magnetic heading angle ψ_{mb} is calculated from magnetometer readings:

$$\psi_{mb} = \psi_{nb} + \alpha_{nm} = \mathrm{atan2}\left(\frac{\cos\phi_{nb} B_x + \sin\phi_{nb} B_z}{\sin\phi_{nb}\sin\theta_{nb} B_x + \cos\theta_{nb} B_y - \cos\phi_{nb}\sin\theta_{nb} B_z}\right)$$

where B_x, B_y, B_z are the three-axis magnetometer readings; θ_{nb}, ϕ_{nb}, and ψ_{nb} are pitch, roll, and heading angles, respectively; α_{nm} is magnetic declination; and atan2() is the four-quadrant arctangent function. The magnetic declination is obtained from the International Geomagnetic Reference Field, and the true heading is obtained by subtracting the declination:

$$\psi_{nb} = \psi_{mb} - \alpha_{nm}$$

Since heading angle is used as the observation quantity, the observation matrix is:

$$H_k = [0_{1\times3} \quad 0_{1\times3} \quad 0_{1\times3} \quad 0_{1\times3} \quad 0_{1\times3}]$$

The observation innovation m_k is the difference between the heading angle ψ_k obtained from inertial navigation calculation and the heading angle ψ_{nb} obtained from geomagnetic orientation:

$$m_k = \psi_k - \psi_{nb}$$

3.2 Magnetic Angular Rate Update

Similar to ZUPT, Zero Angular Rate Update (ZARU) [8] can be performed during the zero-velocity phase, assuming zero angular velocity when the foot contacts the ground. However, the foot may have 微小转动 during contact, and the zero angular velocity assumption may introduce systematic errors. The geomagnetic field is approximately quasi-static locally, and changes in magnetometer readings are caused by magnetometer rotation. Therefore, angular velocity can be calculated from magnetometer reading changes to more accurately calibrate gyroscope bias, a process called Magnetic Angular Rate Update (MARU).

The quasi-static magnetic field condition is:

$$\Delta B_m = |B_{m;k} - B_{m;k-1}| < 0.15\,\mu\mathrm{T}$$

where ΔB_m is the change in magnetic field magnitude between adjacent moments. The observation innovation m_k and observation matrix H_k at time t_k are [9]:

$$m_k = B_k - B_{k-1} + [(\omega_k \Delta t)^\times] B_{k-1}$$

$$H_k = [0_{3\times3} \quad -(B_{k-1}\Delta t)^\times \quad 0_{3\times3} \quad 0_{3\times3} \quad 0_{3\times3}]$$

where B_k is the magnetometer reading at time t_k.

3.3 Dual-Foot Minimum Distance Correction

Theoretically, installing IMUs on both left and right feet can yield more accurate positions, but information from both feet must be fused. The state vectors $\delta x^{(1)}$ and $\delta x^{(2)}$ of the left and right foot inertial navigation systems are combined to form a new state vector δx:

$$\delta x = [\delta x^{(1)} \quad \delta x^{(2)}]^T$$

Using δx as the new system' s state vector, the establishment and discretization of state and observation equations are similar to the single inertial navigation system described above and are omitted here. The observation matrix and innovation remain key to algorithm design and are provided below.

The walking process can be simplified as one foot contacting the ground as a pivot while the other foot swings. The left and right feet alternately contact the ground, with their zero-velocity detection values shown in [Figure 3: see original paper]. During walking, the distance between the feet continuously changes. When the stance foot changes, the inter-foot distance reaches its maximum; around the middle of the swing phase, the distance reaches its minimum. Both maximum and minimum inter-foot distances can be used to correct navigation errors, with correction using the minimum value yielding better results [10].

Using the minimum inter-foot distance for error correction: if both foot-mounted inertial navigation systems start 推算 from the same moment and both initial positions are set to zero, the minimum inter-foot distance can be assumed to be zero. After completing one swing cycle, the inter-foot distance is calculated using the system-estimated positions of both feet, and the minimum distance and corresponding moment are identified. Assuming the minimum distance occurs at time t_k, with the system-estimated left and right foot positions being (x_1, y_1, z_1) and (x_2, y_2, z_2), respectively, the minimum distance is:

$$d_{\min} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

The observation innovation m_k is the difference between the measured distance and the system-estimated distance:

$$m_k = d_{\min} - 0$$

Since the observation quantity is inter-foot distance, which has a nonlinear relationship with the system state, the observation matrix is also nonlinear and must be linearized. The linearized observation matrix is:

$$H_k = [0_{1\times 6} \quad H_1 \quad 0_{1\times 6} \quad 0_{1\times 6} \quad H_2 \quad 0_{1\times 6}]$$

where

$$H_1 = \frac{[x_1 - x_2 \quad y_1 - y_2 \quad z_1 - z_2]}{d_{\min}}$$

$$H_2 = \frac{[x_2 - x_1 \quad y_2 - y_1 \quad z_2 - z_1]}{d_{\min}}$$

The minimum inter-foot distance and its corresponding moment t_k can only be determined after completing one foot swing cycle. After obtaining t_k, the measurement update must be applied retroactively at that moment before 重新推算 subsequent moments.

3.4 Marker Point Position Correction

During inspection, personnel 停留 at actuators, equipment rooms, and other points to check and record equipment status or perform maintenance. Each inspection point has 事先测绘的精确位置坐标 that can serve as marker points to correct navigation position information. Using position as the observation quantity, the observation matrix is:

$$H_k = [0_{3\times3} \quad 0_{3\times3} \quad I_{3\times3} \quad 0_{3\times3} \quad 0_{3\times3}]$$

The observation innovation is the difference between the inertial navigation-estimated position p_k at that moment and the marker point position (x, y, z):

$$m_k = p_k - [x \quad y \quad z]^T$$

4 Experiments and Results

We verified the effectiveness of different error correction algorithms through experiments conducted in both regular rectangular trajectory scenarios and irregular trajectory scenarios at FAST inspection sites. Compared to irregular trajectories, the effects of different algorithms are more clearly contrasted in regular trajectories. The experimental results demonstrate the effects of geomagnetic orientation and magnetic angular rate update when used individually and in combination, followed by the effect of dual-foot trajectory fusion. Finally, tests were conducted at the FAST site, where the above three correction methods were combined with marker points for trajectory correction.

The sensor used for data collection was the IMU CMP10A produced by Yabo Intelligence, a 10-axis IMU including a 3-axis accelerometer, 3-axis gyroscope, 3-axis magnetometer, and barometer. The basic parameters of the gyroscope and accelerometer are listed in . In the table, g represents gravitational acceleration, and LSB represents Least Significant Bit. The bandwidth and output frequency were set to 256 Hz and 100 Hz, respectively. The IMU dimensions are 24 mm × 30 mm × 1.6 mm, weighing 6.4 g, and was mounted on the foot instep using shoelaces, with one unit on each foot.

Experiment 1 involved walking along a regular 200 m × 500 m rectangular trajectory at normal walking speed, covering a total distance of 1.4 km in approximately 20 minutes. The initial attitude was determined by the magnetometer, with initial velocity and position set to zero. A 10-second static period before walking allowed for bias calculation and subtraction. [Figure 4: see original paper] and [Figure 5: see original paper] show the trajectories of the left and right feet, respectively.

Compared to inertial navigation, satellite navigation positioning has smaller errors, so the satellite navigation trajectory serves as the reference. The red trajectory shows navigation results using only the ZUPT algorithm. Due to residual biases and measurement noise, heading gradually drifts. The trajectory obtained using geomagnetic orientation is relatively straight but still offset from the satellite navigation trajectory, indicating that geomagnetic orientation can effectively reduce heading drift, though local magnetic anomalies and magnetometer biases cause heading errors. Using magnetic angular rate update can also reduce heading drift but sometimes over-corrects, making it ineffective when used alone. The blue trajectory obtained by combining geomagnetic orientation with magnetic angular rate update is close to the yellow trajectory using only geomagnetic orientation, indicating that geomagnetic orientation plays a decisive role, while magnetic angular rate update can slightly reduce heading errors. The combination of both methods yields better results.

After applying the above correction algorithms, the calculated positions of the left and right feet still show significant differences. The results after applying dual-foot minimum distance correction, shown in [Figure 6: see original paper], effectively fuse the data from both feet, making the fused trajectory closer to the satellite navigation results.

In closed-loop trajectory scenarios, error estimation is convenient: when the starting coordinates are zero, the ending coordinates represent the error. lists the endpoint coordinates using different correction methods. Using only ZUPT, position errors exceed 10% of walking distance, with right foot heading errors exceeding 100°. Using geomagnetic orientation reduces position errors to 4% and heading errors to around 10°. Using MARU alone still yields large position errors but significantly reduces heading errors. Combining both methods performs slightly better than using geomagnetic orientation alone. After data fusion, the position difference between the two feet is very small, with heading errors nearly zero.

Experiment 2 was conducted under the FAST reflector surface, walking along the FAST spiral inspection road to simulate the inspection process, 停留 at some actuators to record their numbers. Three datasets were collected, each lasting 40 minutes and covering approximately 3.3 km. Actuator numbers were used in post-processing, with the precisely surveyed coordinates of actuator anchor points serving as marker points to correct inertial navigation solutions.

In the figures, red dots represent all actuators recorded during multiple walks, solid dots represent actuators recorded and used for position correction during a particular walk, and three datasets contain 47 correction points total. The time interval between recording actuators was approximately 2 minutes, with spacing of about 150 m. The yellow dashed line shows the trajectory obtained using ZUPT combined with both magnetic corrections and dual-foot correction. The trajectory shape roughly matches the spiral road, but position deviations remain when compared with actuators along the route. The blue solid line shows the trajectory obtained by additionally incorporating marker point correction. After correction, the trajectory aligns well with actuator positions, with position jumps visible at some correction points. Comparing the blue and yellow trajectories demonstrates that marker point position correction can further reduce positioning errors.

Quantitative error analysis is performed by comparing calculated position coordinates with marker point coordinates. Marker point correction pulls the trajectory toward the marker points, so the position correction amount at correction points can be considered the maximum positioning error during the walk. The horizontal distance between the position before correction and the marker point is calculated for statistical analysis, yielding a frequency distribution histogram. The frequency distribution histograms for the three datasets are shown in the right column of [Figure 7: see original paper], with aggregated results in [Figure 8: see original paper]. The mean, RMS, maximum, and standard deviation of horizontal position errors for the three datasets are calculated and listed in . Considering that the IMU position does not exactly coincide with the actuator anchor position when recording numbers, parameters were adjusted to make the corrected IMU position close to the marker point position, with a standard deviation of approximately 2 m between them. Statistically, the mean horizontal positioning error is 5.4 m, with only one of 47 correction points having an error greater than 10 m (10.3 m). Most errors are within 10 m, meeting inspection requirements.

5 Discussion and Conclusion

In pedestrian navigation, the ZUPT algorithm enables low-cost MEMS IMUs to achieve usable navigation results, but position and heading errors grow with walking duration. Using multiple auxiliary information sources to correct errors, geomagnetic orientation plays a decisive role, MARU can reduce heading drift, and their combination yields better results. Dual-foot correction effectively fuses information from two foot-mounted inertial navigation systems. Using these methods in FAST inspection scenarios with marker point spacing of approximately 150 m enables relatively accurate positioning, with horizontal position errors of about 5.4 m and maximum errors less than 11 m. The positioning accuracy meets the requirements for positioning and trajectory recording in routine inspections.

This paper primarily verified the pedestrian navigation algorithm for inspection positioning. Future development of a practical inspection positioning system requires further research. In terms of hardware, electromagnetic compatibility must be considered for equipment use during telescope observations, and more wearable sensor devices must be designed to not interfere with inspectors' normal operations. In terms of software, real-time navigation processing, automatic marker point detection, and real-time navigation output must be implemented. Backend optimization can incorporate smoothing algorithms to eliminate position jumps during marker point correction and obtain higher-precision trajectories. In this study, closed-loop trajectories, satellite navigation, and actuator position comparison were used to verify algorithm accuracy, but these verification methods themselves have certain errors. Establishing more accurate reference values is needed to further improve positioning accuracy.

References

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Note: Figure translations are in progress. See original paper for figures.