Reconstruction of High-energy X-ray Source Using L-shaped Imaging Device

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Abstract

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Full Text

Preamble

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Abstract

L-shaped imaging devices are used to reconstruct the intensity distribution of high-energy X-ray sources. A physical model accounting for the penetration effect of X-rays through the imaging device is established, the transmission imaging matrix is constructed, and an algebraic solution method for source intensity reconstruction is presented. X-ray sources with Gaussian distribution are reconstructed. The reconstruction results demonstrate that artifacts and discontinuities in the center of images reconstructed using the L-Edge device

can be improved by the L-Rolled Edge device, while the L-Cylinder device can further enhance reconstruction quality.

Keywords: High-energy X-ray source, Source intensity reconstruction, L-shaped imaging device, Penetration, Transmission imaging matrix

1. Introduction

High-energy X-ray flash radiography can penetrate the structure, state, and evolution process of high-speed moving objects, and is widely used to study the transient evolution of internal structures under impact loading. It serves as an indispensable diagnostic tool for fast transient processes such as hydrodynamic experiments [?, ?, ?, ?, ?]. However, high-energy X-ray flash imaging systems are complex in composition, involve numerous physical processes, and imaging quality is constrained by many factors. Geometrical blur caused by the focal spot size of high-energy X-ray sources is one of the primary contributors to radiographic image degradation [?].

Consequently, the intensity distribution of high-energy X-ray sources and focal spot size are critical parameters in image analysis. The accuracy of focal spot parameters directly determines whether high-precision reconstructed images can be obtained theoretically. Focal spot measurement constitutes an important aspect of high-energy X-ray flash radiography research [?]. Major laboratories engaged in this research have developed various spot size measurement techniques, including the pinhole method, slit method, edge method, roll-bar method, and others [?, ?].

The Atomic Weapons Establishment (AWE) employs the cylindrical edge method to measure focal spots on its SuperSwarf flash imaging device [?]. Sandia National Laboratory (SNL) and Lawrence Livermore National Laboratory (LLNL) in the United States use conical pinhole imaging devices to measure focal spots on the ETA-II accelerator [?], while LLNL has also conducted measurements using the cylindrical edge method on ETA-II [?, ?]. Los Alamos National Laboratory (LANL) uses the pinhole method on the DARHT-I device [?], and SNL has performed focal spot measurements on the RITS-3 accelerator using a Rolled Edge imaging device [?].

In 2016, Fowler et al. [?] designed the L-Rolled Edge imaging device based on the "opaque" physical model proposed by Barnea [?]. By utilizing only one corner of a square-hole imaging device, they obtained two-dimensional light and shadow information and derived the two-dimensional intensity distribution of the focal spot through image reconstruction. Building upon this "L" configuration device, this paper proposes an imaging physical model that considers transmission effects. Using the L-Edge imaging device, we first obtain the two-dimensional distribution of the source penetrating the imaging device and then derive the source intensity distribution through image reconstruction. Through analysis of the reconstructed images, we subsequently propose two improved imaging device designs—the L-Rolled Edge and L-Cylinder devices—to perform source intensity reconstruction.

The remainder of this paper is organized as follows. Section 2 presents the mathematical and physical modeling and solution methodology. Source intensity reconstruction using L-shaped imaging devices is described in Section 3. Finally, conclusions are summarized in Section 4.

2.1. Mathematical Model of Source Intensity Reconstruction

In the energy range (10 keV–100 MeV) of high-energy X-ray flash radiography, X-rays exhibit strong penetrability. Therefore, the penetrability of high-energy X-rays must be considered when measuring source intensity using imaging devices. The attenuation of X-rays penetrating materials follows the Lambert-Beer law. Assuming the X-ray source is ideal—an isotropic monochromatic point source—the intensity of X-rays emitted from the source, passing through an object, and reaching each point on the detector plane can be expressed as:

$$I(x,y) = I_0 e^{-\int_{d(x,y)} \mu(l)\rho(l)dl}$$

where I and I_0 are the X-ray intensities received by the detector with and without objects, $d(x,y)$ is the distance from point (x,y) on the imaging plane to the source, and $\rho(l)$ and $\mu(l)$ are the material density and mass attenuation coefficient of X-rays in the material at distance l from the source, respectively.

If the source intensity I_0 follows a certain distribution $I_0(x',y')$, the intensity of X-rays at any point (x,y) on the imaging plane after penetrating an object can be expressed as:

$$I(x,y) = \iint I_0(x',y') e^{-\int_{(x,y)}^{(x',y')} \mu(l)\rho(l)dl} dx'dy'$$

Note that the integration area covers the entire X-ray source region, and the integration path l is the straight-line path from the source plane point (x',y') to the imaging plane point (x,y).

If the X-ray source region is discretized into pixels, the integral form of Eq. (2) can be rewritten in summation form:

$$I(x,y) = \sum_{x'} \sum_{y'} I_0(x',y') e^{-\int_{(x,y)}^{(x',y')} \mu(l)\rho(l)dl}$$

Furthermore, if both the source plane and the imaging plane are discretized into pixels and regarded as one-dimensional vectors, Eq. (3) can be written in matrix-vector form:

$$Ax = b$$

where the element a_{ij} of matrix A reflects the overall attenuation of the j-th source intensity reaching the i-th pixel on the imaging plane after passing through the imaging device, hence it is called the transmission imaging matrix. x represents the source intensity to be reconstructed, and b is the X-ray intensity measured on the imaging plane.

The above equation represents an ideal case without imaging system noise. If the coefficient matrix A is full rank, the unknown source intensity vector can be obtained by solving the linear equations. In practice, noise is generally present, and the equation becomes:

$$Ax = b + n$$

where n represents the imaging system noise.

2.2. Construction of Transmission Imaging Matrix

In measuring high-energy X-ray sources, imaging layouts with geometric magnification ratio $M > 1$ are generally adopted. In this paper, the X-ray source is positioned 15 cm from the center of the L-shaped imaging device, and the detector is 120 cm from the center of the L-shaped imaging device, yielding a geometric magnification ratio $M = 8$.

Given the current beam control level of accelerators, the radial size of the X-ray source can be less than 0.5 cm, so the source region is set to 1 cm × 1 cm. To address practical source drift issues, the imaging region is expanded from 8 cm × 8 cm to 15 cm × 15 cm. Considering computational capacity, both the source region and imaging region are discretized to a size of 80 × 80, resulting in a transmission imaging matrix A of size $(80 \times 80) \times (80 \times 80)$, i.e., 6400 × 6400.

Assuming the L-shaped imaging device is a homogeneous single medium, the element a_{ij} of the imaging matrix based on the transmission model can be written as:

$$a_{ij} = e^{-\mu\rho L((x',y')\to(x,y))}$$

where $L((x', y') \to (x, y))$ is the geometric length of the X-ray path through the L-shaped imaging device from point (x', y') in the source plane to point (x, y) in the imaging plane. This can be derived from the geometric relative position relationships.

[Figure 1: see original paper] shows a schematic diagram of X-ray source measurement using an L-Edge imaging device.

Taking the L-Edge imaging device as an example (see Figure 1), the calculation of $L((x', y') \to (x, y))$ proceeds as follows. First, the geometric description is defined. Using the central ridge line of the L-Edge (the intersection line of two vertical planes) as the Z-axis, the plane where X-rays are incident on the L-Edge (the side near the source) is defined as the X_1Y_1 plane, which is divided into zones I, II, III, and IV according to the two vertical edges of the L-Edge. Similarly, the plane where X-rays exit the L-Edge (the side near the imaging plane) is defined as the X_2Y_2 plane, also divided into zones 1, 2, 3, and 4, as shown in Figure 2 [Figure 2: see original paper].

[Figure 2: see original paper] Geometric region partition of L-Edge imaging device.

[Table 1] Various cases of calculating geometric length when X-ray passes through L-Edge.

Case Description	Calculation Method
$L = 0$	Through $Y_1O_1O_2Y_2$ plane
Through $X_1X_2O_2O_1$ plane	Through $Y_1O_1O_2Y_2$ plane or $X_1X_2O_2O_1$ plane
Through $Y_1O_1O_2Y_2$ plane	Calculate distance between P_1 and P_2 directly
Through $X_1X_2O_2O_1$ plane	Calculate distance between P_1 and P_2 directly
Through $Y_1O_1O_2Y_2$ plane and $X_1X_2O_2O_1$ plane	Calculate distance between P_1 and P_2 directly
Through $Y_1O_1O_2Y_2$ plane or $X_1X_2O_2O_1$ plane	Calculate distance between P_1 and P_2 directly
Through $X_1X_2O_2O_1$ plane	Calculate distance between P_1 and P_2 directly
Through $Y_1O_1O_2Y_2$ plane and $X_1X_2O_2O_1$ plane	Calculate distance between P_1 and P_2 directly

If an X-ray intersects the X_1Y_1 plane at point P_1 and the X_2Y_2 plane at point P_2, the geometric length of the X-ray in the L-Edge can be obtained by examining the zones where P_1 and P_2 are located in the X_1Y_1 and X_2Y_2 planes, respectively. As shown in Table 1, various cases of X-ray penetration (or non-penetration) through the L-Edge are enumerated. Based on treatment complexity, these can be divided into four categories:

1. The simplest case occurs when the X-ray does not intersect the L-Edge, or the distance within the L-Edge can be calculated directly.
2. The second case involves the X-ray passing through either the $Y_1O_1O_2Y_2$ plane or the $X_1X_2O_2O_1$ plane, requiring determination of the intersection point before calculation.

3. A more complex case involves the X-ray passing through either the $Y_1O_1O_2Y_2$ plane or the $X_1X_2O_2O_1$ plane, which requires careful judgment.
4. The most complex case occurs when the X-ray passes through both the $Y_1O_1O_2Y_2$ plane and the $X_1X_2O_2O_1$ plane simultaneously. In this scenario, the X-ray path in the L-Edge is divided into two segments, and the distance in air must be subtracted during calculation, as shown in Figure 3 [Figure 3: see original paper].

For the "opaque" model adopted by Barnea et al. [**?**], this is equivalent to μ being infinite, and the matrix element based on the "opaque" model can be written as:

$$a_{ij} = \begin{cases} 0, & L > 0 \\ 1, & L = 0 \end{cases}$$

Figure 4 [Figure 4: see original paper] shows the imaging matrices for these two models under identical imaging conditions. The transition between light and dark regions in the opaque model is abrupt, as seen in Figure 4(a), whereas the imaging matrix in Figure 4(b) exhibits gradual changes when transmission effects are considered, more accurately reflecting the gradual variation of X-ray intensity when penetrating the L-Edge imaging device.

2.3. Numerical Solution Method

The source intensity reconstruction problem represented by Eq. (5) is a typical inverse problem. Due to the ill-posed nature of inverse problems, it is impossible to directly solve the linear equations (5) or its corresponding least-squares formulation. Only by establishing appropriate constraint criteria can the solution converge to a reasonable result [**?**, **?**]. Regularization techniques are introduced to constrain the solution space of the ill-posed inverse problem. By constructing a stable functional, the solution of the ill-posed inverse problem is transformed into a functional extremum problem with small regularization parameters. Although the solution may not be unique after introducing regularization, a relatively well-posed problem is realized for solving the original ill-posed problem. Regularization not only alleviates the ill-posedness of the inverse problem but also reduces the influence of noise in the solution and filters noise, thereby improving reconstruction quality. After introducing regularization, solving the algebraic equations (5) is transformed into solving the following optimization problem:

$$\min_x \|Ax - b\|_2^2 + \lambda R(x)$$

where $\|Ax - b\|_2^2$ is called the fidelity term, describing the degree of approximation between the reconstructed source intensity and the original measurement data, and $\lambda R(x)$ is the regularization term representing artificially imposed constraints.

In this paper, by assuming the unknown represents sampled values of a slowly varying function and using the minimum modulus of its first derivative as the criterion, we obtain the classical Tikhonov regularization model [?]:

$$\min_x \|Ax - b\|_2^2 + \lambda \|\nabla x\|_2^2$$

This is a nonlinear constrained optimization problem, which we solve using the practical Constrained Conjugate Gradient (CCG) method [?, ?].

3. Reconstruction of Source Intensity

In this section, we validate the proposed source reconstruction model accounting for attenuation by reconstructing a Gaussian-distributed X-ray source. In nuclear engineering, Monte Carlo (MC) simulation of particle transport is considered the method closest to real experimental conditions [?, ?]. Therefore, we reconstruct MC-simulated images to test our mathematical and physical modeling and reconstruction methodology. As shown in Figure 5 [Figure 5: see original paper], the full-width at half-maximum (FWHM) of the Gaussian-distributed X-ray source used in the Monte Carlo simulation is set to 0.2 cm.

[Figure 5: see original paper] Ground truth of Gaussian distributed X-ray source.

To evaluate reconstruction quality, we can form a visual impression and also calculate the relative error between the reconstructed image and ground truth:

$$\text{Error} = \frac{\|x - x_0\|_2}{\|x_0\|_2}$$

where x is the reconstructed image and x_0 is the ground truth. Additionally, quantitative comparison of the FWHM between the reconstructed image and ground truth helps evaluate reconstruction performance.

3.1. Source Reconstruction Using L-Edge

First, we use the L-Edge imaging device for MC-simulated radiography, where the L-Edge is made of tungsten with a thickness of 3 cm. Figure 6 [Figure 6: see original paper] shows the simulated image and its corresponding reconstructed intensity distribution, with cross-sectional profiles of the reconstructed image in different directions shown in Figure 7 [Figure 7: see original paper].

[Figure 6: see original paper] MC simulated image and the corresponding reconstructed Gaussian source.

From the central transverse profiles in Figure 7, the size of the reconstructed X-ray source can be determined, yielding an FWHM of 0.2 cm in both horizontal and vertical directions, consistent with the true value. The relative error between the reconstructed image and ground truth is 0.0841. However, obvious "discontinuities" appear in the center of the reconstructed source intensity.

[Figure 7: see original paper] Transverse lines of reconstructed Gaussian source in horizontal and vertical directions.

3.2. Source Reconstruction Using L-Rolled Edge

Due to the right-angle edge of the L-Edge imaging device, sudden intensity changes near the edge cause serious artifacts in the reconstructed image. The "discontinuity" characteristic of edge devices has been recognized by researchers in the high-energy X-ray source measurement field [**?**], who note that Roll-bar imaging devices can improve this discontinuity. Drawing from the concept of developing Edge devices into Roll-bar devices, we improved the L-Edge imaging device into the L-Rolled Edge imaging device, as shown in Figure 8 Figure 8: see original paper. Specifically, the side surface changes from a planar surface to a circular arc surface, avoiding step discontinuities in source intensity near the side surface.

[Figure 8: see original paper] (a) L-Rolled Edge imaging device; (b) The transmission imaging matrix of the L-Rolled Edge device.

For selecting the curvature of the L-Rolled Edge device, as shown in Figure 9 [Figure 9: see original paper], the height between the highest point of the arc segment and the edge segment is set as h, equivalent to the focal spot size of the X-ray source. Based on geometric relationships, it is easy to derive:

$$R^2 = L^2 + (R - h)^2$$

$$L^2 + h^2$$

In this paper, h is set to 0.2 cm, and the arc radius of the L-Rolled Edge is 5.725 cm.

[Figure 9: see original paper] Radian setting of L-Rolled Edge imaging device.

The transmission imaging matrix of the L-Rolled Edge imaging device is shown in Figure 8(b). Compared with Figure 4(b), it is evident that the transition regions of the L-Edge device are primarily distributed on both sides of the central area of the transmission imaging matrix, with obvious discontinuities in the central matrix region. In contrast, the L-Rolled Edge device improves upon the discontinuities of the L-Edge, with the entire transition region distributed in a strip shape that is more uniform. Moreover, the transition region of the L-Rolled Edge device's transmission imaging matrix is significantly wider and exhibits better continuity, providing more information for source intensity reconstruction and theoretically being more conducive to reconstructed image continuity.

[Figure 10: see original paper] MC simulated image and the corresponding reconstructed Gaussian source.

The MC-simulated image using the L-Rolled Edge imaging device and its corresponding reconstructed source image are shown in Figure 10. Compared with the right panel in Figure 6, the "L"-shaped grid in the center of the image reconstructed using the L-Edge device is significantly improved after using the L-Rolled Edge device, with the latter being closer to the ground truth in the central region. Additionally, the relative error of the source reconstructed using the L-Rolled Edge device is 0.0765, smaller than the 0.0841 obtained with the L-Edge device.

Figure 11 [Figure 11: see original paper] shows the comparison between the Gaussian source reconstructed using the L-Rolled Edge device and the ground truth in horizontal and vertical directions, demonstrating good agreement. The FWHM of the reconstructed Gaussian source is calculated to be 0.2 cm, consistent with the true value. Moreover, the reconstructed source using the L-Rolled Edge device significantly improves the discontinuity on the Gaussian peak that was present in the source reconstructed using the L-Edge device.

[Figure 11: see original paper] Transverse lines of reconstructed Gaussian source in horizontal and vertical directions.

3.3. Source Reconstruction Using L-Cylinder

After improving the L-Edge imaging device to the L-Rolled Edge device, imaging quality is greatly enhanced, particularly eliminating the "discontinuity" in the central portion of the reconstructed Gaussian source. However, careful observation of the transverse profiles of the reconstructed image in horizontal and vertical directions (see Figure 11) reveals that a slight "discontinuity" still exists at positions deviating from the central peak. Preliminary analysis suggests this discontinuity may be related to the junction between the arc section and edge section of the L-Rolled Edge device. Therefore, we propose a further improved design: the L-Cylinder imaging device, formed by directly splicing two cylindrical sections into an "L" shape, as shown in Figure 12 Figure 12: see original paper.

[Figure 12: see original paper] (a) L-Cylinder imaging device; (b) The transmission imaging matrix of the L-Cylinder device.

The transmission imaging matrix of the constructed L-Cylinder device is shown in Figure 12(b). Compared with Figures 4(b) and 8(b), the transition region of the L-Cylinder device's transmission imaging matrix is wider and more uniform than those of both the L-Edge and L-Rolled Edge devices, which theoretically provides greater benefit for reconstructed image continuity.

[Figure 13: see original paper] MC radiographic image with the L-Cylinder device and its corresponding reconstructed intensity distribution, with cross-sectional profiles of the reconstructed image in different directions shown in Figure 14 [Figure 14: see original paper]. It is evident that the L-Cylinder imaging device can accurately reconstruct the intensity distribution of the Gaussian source and precisely measure the FWHM of the Gaussian source as 0.2 cm. The L-Cylinder imaging device improves the non-smoothness in the lower portion of the Gaussian peak that was present in the source reconstructed with the L-Rolled Edge device. Furthermore, the relative error of the reconstructed Gaussian source is 0.0685, the smallest among the three imaging devices discussed in this paper.

[Figure 14: see original paper] Transverse lines of reconstructed Gaussian source in horizontal and vertical directions.

4. Conclusion

This paper presents a focal spot measurement method for high-energy X-ray sources based on source intensity reconstruction using L-shaped imaging devices. A physical model considering the penetration effect of X-rays through the imaging device is established, and the transmission imaging matrix is constructed. By introducing regularization techniques, an algebraic solution method for source intensity reconstruction is developed. The X-ray source with Gaussian distribution is reconstructed using the simplest L-Edge imaging device, though serious artifacts appear in the reconstructed image. To improve reconstruction quality, L-Rolled Edge and L-Cylinder imaging devices are subsequently proposed. The reconstruction results demonstrate that artifacts and discontinuities in the center of the reconstructed image can be improved using the L-Rolled Edge imaging device, while discontinuities can be further reduced using the L-Cylinder imaging device. Therefore, the L-Cylinder imaging device is more suitable for reconstructing high-energy X-ray sources.

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Note: Figure translations are in progress. See original paper for figures.