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Abstract

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Full Text

Preamble

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Constrain the Jerk Parameters with DESI 2024 Data

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Abstract

The deceleration coefficient q and the jerk coefficient j, obtained through Taylor expansion of the scale factor a(t), play an important role in cosmological studies. The current values of these coefficients for a cosmological model reflect the transition time between dark energy-dominated and matter-dominated phases and can be used to determine if and how much the universe is decelerating. Thus, these coefficient values offer a way to constrain particular cosmological models. However, some approaches in this method should be tested prudently because certain physical conditions may not be guaranteed.

In this study, we used the MAPAge model to reconstruct the jerk parameters (q_0 and j_0) with DESI 2024 data. Using the MAPAge model ensures that particular physical circumstances are satisfied in the approach for determining the jerk parameters. Compared to previous methods that used the Taylor expansion series q_0, j_0, and s_0 as model-independent parameters, we obtained more physical and slightly different results for the jerk parameters. Our results suggest that the DESI 2024 BAO data set favors different jerk parameters compared to those in the standard ΛCDM model.

Key words: (cosmology:) dark energy – (cosmology:) large-scale structure of universe – (cosmology:) cosmological parameters

1. Introduction

A cosmological model with zero spatial curvature, cold dark matter (CDM), and time-independent dark energy is called the standard cosmological model or ΛCDM model. The ΛCDM model has been remarkably successful over the past decades (Baumann 2009; Eisenstein & Hu 1998), but problems including the Hubble tension and the σ_8 tension remain unsolved within this framework (Perivolaropoulos & Skara 2022; Riess et al. 2022; Hu & Wang 2023). Consequently, numerous alternative schemes have been proposed to address these issues (Bamba et al. 2012; Perivolaropoulos & Skara 2022; Di Valentino et al. 2021). Recently, with the release of high-precision DESI 2024 data, a series of model-checking studies have been conducted to test different cosmological models (Adame et al. 2024; Calderon et al. 2024; Carloni et al. 2024; Colgáin et al. 2024; Marina et al. 2024; Yang et al. 2024).

One method to verify the feasibility of each model is through the deceleration and jerk hierarchical terms in the Taylor expansion of the scale factor a(t) (Visser 2004; Luongo & Muccino 2024). However, some approaches in this method should be tested prudently because certain physical conditions may not be guaranteed (Huang 2020). In this paper, we used an alternative model to verify this fitting approach and ensure that specific physical conditions are fulfilled. To assess the fitting results for the deceleration coefficient q_0 and the jerk coefficient j_0 (where the subscript 0 indicates current values), we employed the "More Accurate Parameterization based on cosmic Age (MAPAge)" model (Huang et al. 2021b). In the MAPAge model, a new degree of freedom $_2$ is included to improve the fitting accuracy of the "Parameterization based on cosmic Age (PAge)" model suggested by Huang (2020), which approximates a broad class of beyond-ΛCDM models with typical accuracy of 1% in angular diameter distances at redshift z 10. The study shows that as long as the model's parameters are constrained, a series of cosmological conditions can be guaranteed, which will be exhibited in detail later in our article.

In this research, we processed the r_d parameter as a constant spanning the range [144, 152] Mpc to ensure that both the Planck satellite and DESI-BAO expectations fall within this interval, with a fixed step δr_d = 2 Mpc, to enable effective comparison with the earlier research by Orlando Luongo and Marco Muccino (Luongo & Muccino 2024; Brieden et al. 2023).

Monte Carlo Markov Chain (MCMC) analyses were used to obtain the parameters in the MAPAge model to fit the optimal values of q_0 and j_0. We incorporated three combinations of data catalogs: the first includes BAO with observational Hubble data, the second includes BAO with type Ia supernovae, and the last includes all three data sets. For comparison, similar analyses were performed with the Chevallier–Polarski–Linder (CPL) model. The code we used can be found in Huang et al. (2024). We reconstructed the deceleration parameter and the jerk parameter with the MAPAge model in the MCMC analyses, obtaining results similar to the method that uses Taylor expansion of a(t) (Luongo & Muccino 2024). However, slight differences appear, which will be discussed. We also conclude that dynamical dark energy better adapts to this catalog of BAO data, as previous studies have shown (Adame et al. 2024). Our results show severe disfavor for the concordance model in both the deceleration parameter and the jerk parameter.

The remainder of this paper is organized as follows: In Section 2, we describe the MAPAge model and the parameters that need to be constrained. In Section 3, we report our data set and its corresponding processing details. In Section 4, we illustrate the results and discuss the physical consequences of the analyses, including comparisons between different models. Finally, we highlight our conclusions in Section 5.

2. Model

In the PAge model (Huang 2020), two conditions were assumed: (i) the universe is dominated by matter at high redshift z 1, and (ii) the product of cosmological time t and the Hubble expansion rate H can be approximated as a t. These assumptions lead to a quadratic parameterization of the Universe's expansion rate H as:

$$H(z) = H_0 \left[p_{\text{age}} + \eta \ln(1+z)\right]$$

where $p_{\text{age}} = H_0 t_0$ is the product of the Hubble constant H_0 and the current age of the universe t_0, and the phenomenological parameter can be regarded as a quadratic fitting parameter. If the value of is required to be less than 1, the physical conditions can be guaranteed. It has been shown that the fitting errors of distance moduli are typically 1% at $z \lesssim 10$ (Luo et al. 2020; Huang et al. 2021a), and such accuracy is empirically good enough. Indeed, the PAge model has been applied to many currently available data sets and yielded fruitful results (Huang 2020; Cai et al. 2022).

Building upon this foundation, Huang et al. (2021b) proposed a new model called MAPAge, which provides higher measurement accuracy than the PAge model by adding a new parameter η_2 $(-1 < \eta_2 < 1)$, which can be regarded as a cubic-order correction to the PAge model. In the MAPAge model, the expansion rate of the Universe is expressed as:

$$H(z) = H_0 \left[p_{\rm age} + \eta \ln(1+z) + \eta_2 \ln^2(1+z)\right]$$

and Equation (2) degrades to Equation (1) when $\eta_2 = 0$.

A fitting of the Hubble parameter $H(z)$ at redshifts $z < 10$ is applied to illustrate the advantages of the MAPAge model over the PAge model. Table 1 shows the fitting accuracy of $H(z)$ for three models, demonstrating that the precision of the MAPAge approximation is typically an order of magnitude better than PAge, which is consistent with Huang et al. (2021b). The relative fitting errors of D_A for PAge and MAPAge for a few models are shown in Figure 1. The result again confirms MAPAge's superiority in fitting accuracy. This motivates our choice of the MAPAge model to fit the current high-precision DESI data.

With the definition of the scale factor $a(t)$ in the MAPAge model as $a(t) = (1+z)^{-1}$, we can express the deceleration and jerk coefficients, denoted as $q(t)$ and $j(t)$ respectively, from the Taylor expansion of $a(t)$ at $t = t_0$:

$$a(t) = a(t_0)\left[1 + H_0(t-t_0) - \frac{q_0}{2}H_0^2(t-t_0)^2 + \frac{j_0}{6}H_0^3(t-t_0)^3 + \cdots\right]$$

where we truncated the series up to a given order and used the conventional definitions of the deceleration coefficient:

$$q(t) = -\frac{\ddot{a}a}{\dot{a}^2}$$

and the jerk coefficient:

$$j(t) = \frac{\dddot{a}a^2}{\dot{a}^3}$$

Therefore, q_0 and j_0 can be expressed as functions of MAPAge parameters η, $p_{\rm age}$, and η_2 using Equations (2), (3), (5), and (6). From the MCMC fitting of the MAPAge model, we can obtain the distribution of the parameters q_0 and

j_0, thereby acquiring the preference of the observed results for specific models by comparing the values of q_0 and j_0 between the MAPAge model and other different models.

For subsequent purposes, it is convenient to compute the cosmographic series for given dark energy models. Specifically, we focus on the following models: (1) the ΛCDM model, (2) the wCDM model, and (3) the CPL model. The q_0 and j_0 expressions for these models are as follows (Luongo & Muccino 2024) (the symbols have their conventional meaning):

ΛCDM:

$$q_0 = \frac{1}{2}\Omega_m - \Omega_\Lambda$$
$$j_0 = 1$$

wCDM:

$$q_0 = \frac{1}{2}\Omega_m(1+3w) - \Omega_\Lambda$$
$$j_0 = 1 + \frac{3}{2}\Omega_m w(1+w)$$

CPL:

$$q_0 = \frac{1}{2}\Omega_m(1+3w_0) - \Omega_\Lambda$$
$$j_0 = 1 + \frac{3}{2}\Omega_m\left[w_0(1+w_0) + w_a\right]$$

where the definitions of L_B, L_S, and L_O are introduced later in this section.

The total BAO log-likelihood is given by:

$$\ln \mathcal{L}_{\mathrm{BAO}} = -\frac{1}{2}\sum_{i,j}(X_i - \hat{X}_i)C_{ij}^{-1}(X_j - \hat{X}_j)$$

where $X = d_M/r_d$, d_H/r_d, d_V/r_d, and r_d is the sound horizon at the drag epoch (Adame et al. 2024). For data about d_V, the log-likelihood is represented as:

$$\ln \mathcal{L}_{d_V} = -\frac{1}{2}\frac{(d_V/r_d - \hat{d}_V/r_d)^2}{\sigma_{d_V}^2}$$

For data related to d_M and d_H, the corresponding log-likelihood can be expressed as:

$$\ln \mathcal{L}_{d_M,d_H} = -\frac{1}{2}\sum_{i,j}(X_i - \hat{X}_i)C_{ij}^{-1}(X_j - \hat{X}_j)$$

These expressions do not consider energy components other than matter for simplicity (Luongo & Muccino 2024). In Table 2, we exhibit q_0 and j_0 from the Planck and DESI data sets calculated by Orlando Luongo and Marco Muccino as a contrast.

3. Data Analysis

The general best-fit parameters are determined directly by maximizing the total log-likelihood function:

$$\ln \mathcal{L}_{\text{total}} = \ln \mathcal{L}_{\text{BAO}} + \ln \mathcal{L}_{\text{OHD}} + \ln \mathcal{L}_{\text{SNe}}$$

The parameters x, y, and Δ are defined as follows: [Note: The original text seems to have omitted the actual definitions here, but this doesn't affect the overall understanding.]

The DESI-BAO data set we used is illustrated in Table 3. Notice that the BGS and QSO tracers only provided d_V/r_d due to the lower signal-to-noise achieved, while other tracers have d_M/r_d and d_H/r_d and provide the value of the correlation r between them but no d_V/r_d data.

To calibrate the measures, OHD systematics mostly depend on stellar population synthesis models and libraries. Even the initial mass functions, taken into account together with the stellar metallicity of the population, may contribute further errors of (20–30)% (Moresco et al. 2022; Montiel et al. 2021; Muccino et al. 2023). Hence, the measures were not particularly accurate, although their determination was fully model-independent. The best-fit parameters are found by maximizing the log-likelihood:

$$\ln \mathcal{L}_{\text{OHD}} = -\frac{1}{2}\sum_i \frac{(H(z_i) - \hat{H}_i)^2}{\sigma_{H,i}^2}$$

and the OHD data set we used is from Table III of Luongo & Muccino (2024).

For the supernova data, we chose the Pantheon data set, which comprises thousands of measures associated with SNe (Scolnic et al. 2018). The corresponding log-likelihood function is given by:

$$\ln \mathcal{L}_{\text{SNe}} = -\frac{1}{2}\sum_{i,j}(m_i - \hat{m}_i)C_{S,ij}^{-1}(m_j - \hat{m}_j)$$

where E_i represents the normalized Hubble rates, C_S is the covariance matrix, and $|C_S|$ is its determinant (Riess et al. 2017).

4. Results and Discussion

To compare with previous studies by Luongo & Muccino (2024), we set the comoving sound horizon at the drag epoch r_d to values varying between 144 and 152 Mpc with a uniform step $\delta r_d = 2$ Mpc. Table 4 shows the fitting parameters of the MAPAge model and the corresponding q_0 and j_0 obtained from Equations (5) and (6).

The results obtained from the BAO+OHD data set are less precise and less accurate compared to those from other data sets. For results from the BAO+SNe and BAO+OHD+SNe data sets, the accuracy of q_0 from the MAPAge model is consistent with the q_0 values in Table IV of Luongo & Muccino (2024), whereas j_0 is consistent only when s_0 is under consideration. Also, we found that the Hubble parameter h tended to decline with r_d.

Due to the OHD data set placing weaker constraints on the parameters compared to other data sets, there is generally a larger σ when examining q_0 and j_0 in the BAO+OHD data set. Nevertheless, all three types of measurements show obvious inconsistency with the ΛCDM model, as exhibited in Figure 2. To demonstrate the discrepancy with the ΛCDM model more rigorously, we also performed the same MCMC process for different data sets with the ΛCDM model and expressed the results in terms of probability density functions (PDFs), as shown in Figure 3.

Since the j_0 parameter in the ΛCDM model equals a constant value of 1, it is represented by a vertical line in the figures. Again, these results show strong disagreement between the CPL model and the ΛCDM model, and high agreement between the MAPAge and CPL models, which once again points to the rejection of the ΛCDM model by DESI-BAO data.

For a more detailed comparison between CPL and MAPAge, we performed the same fitting procedure with the CPL model as was done with the MAPAge model, and the corresponding results are listed in Table 5.

In general, compared to $p_{\rm age}$ and η_2, the accuracy of η shows higher sensitivity to the choice of data set, and j_0 shows the same behavior when compared with q_0. The fitting values of q_0 and j_0 obtained by the two models agree with each other when including the error bars. These consistencies could indicate that the MAPAge model satisfies fundamental physical conditions just as the CPL model does.

However, results using the BAO+OHD data set show some inconsistency due to the poorer quality of this data set compared to others. Besides, we notice that the error bars on q_0 and j_0 from the MAPAge model for specific r_d values are smaller than those from the CPL model, and the reduced Hubble parameter h obtained by the two models at specific r_d values agree precisely with each other.

In addition, there are some small but noticeable differences in our results: q_0 from the MAPAge model is slightly smaller than q_0 in the CPL model, whereas

j_0 in the CPL model is slightly smaller than j_0 in the MAPAge model. These differences could be attributed to the simplified formulas used in our calculations of q_0 and j_0.

In comparison with the work by Orlando Luongo and Marco Muccino, our approach ensures that a set of physical conditions are fulfilled, and the parameters in the MAPAge model show higher accuracy than the CPL model. Our results are consistent with the CPL model but show deviation from the standard ΛCDM model, which once again proves the inconsistency of the new DESI data with the standard model.

5. Conclusion

In this work, we reconstructed the jerk parameters with DESI 2024 data using the MAPAge model and the CPL model to fit the data sets. The fitting results for both models are as follows: For the MAPAge model, the best-fitting results of (q_0, j_0) are approximately $(-0.31, -0.45)$. For the CPL model, the best-fitting results of (q_0, j_0) are approximately $(-0.30, -0.51)$, with corresponding (w_0, w_a) approximately $(-0.75, -1.00)$.

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Note: Figure translations are in progress. See original paper for figures.