Magnetic field of atomic nuclei

Authors: Y.M. Zhao, H.Y.Wang, X.Yin, Y.M. Zhao

Date: 2025-01-04T00:00:00+00:00

Abstract

In this paper we evaluate typical values of magnetic field energy and electromagnetic angular momentum of atomic nuclei with very simple scenarios, for the first time. We point out possible situations in which magnetic energy and angular momentum might play essential roles in nuclear physics.

Full Text

Preamble

Magnetic field of atomic nuclei

Y. M. Zhao[1,2,†], H. Y. Wang[1], and X. Yin[1]

[1]Shanghai Key Laboratory of Particle Physics and Cosmology, School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China
[2]Collaborative Innovation Center of IFSA (CICIFSA), Shanghai Jiao Tong University, Shanghai 200240, China

(Dated: January 4, 2025)

We evaluate, for the first time, typical values of magnetic field energy and electromagnetic angular momentum of atomic nuclei using very simple scenarios. We point out possible situations in which magnetic energy and angular momentum might play essential roles in nuclear physics.

PACS numbers:
Keywords:

Introduction

Electric field is very important in atomic nuclei. It is the Coulomb repulsive force that leads to the fact that the number of elements is limited. The energy of electric field is one essential component of nuclear binding energy (in particular for heavy mass nuclei) [?]. On the other hand, the role of magnetic field has been by far less investigated in nuclear physics hitherto.

It is therefore one of the purposes of this paper to evaluate both the magnetic energy and electromagnetic angular momentum in atomic nuclei. The results for angular momentum are easily generalized to the case of one charged particle such as a proton or an electron in a magnetic field, in which case the angular momentum can be much larger than $\hbar$ if the space is large or the magnetic field is strong enough.

This paper is organized as follows. In Sec. II we investigate the magnetic energy of nuclei. In Section III we study the electromagnetic angular momentum. We summarize this paper in Sec. IV.

II. Magnetic Energy

We first exemplify magnetic energy by the deuteron, which is the weakly bound state of a proton and a neutron. One quickly obtains a crude panoramic view with this simple case. The deuteron is reasonably approximated as a proton and a neutron in an s-wave orbit with spins aligned. Under this assumption, the magnetic interaction energy between the proton and the neutron is given by

$$W_{m_{1m}2} = \frac{\mu_0}{4\pi r^3}\left[\vec{m}_1 \cdot \vec{m}_2 - 3(\vec{m}_1 \cdot \vec{e}_r)(\vec{m}_2 \cdot \vec{e}_r)\right]$$

where μ_0 is the permeability of vacuum, $\vec{m}_1$ and $\vec{m}_2$ are the magnetic moments of a free proton and free neutron respectively, and r is the distance between the proton and neutron. We assume that both spins are parallel to $\vec{e}_r$, and we obtain $W_{m_{1m}2} = -2|\mu_p\mu_n|/r^3 \simeq 0.0027$ MeV. One sees that the magnetic interaction energy between a neutron and a proton in the deuteron is on the order of keV. We note that this energy does not include the "self" energy of individual nucleons, for which we shall use a simple model to evaluate later in this paper.

For nuclei with many protons and neutrons, we simplify the entire nucleus as a uniformly magnetized ball with total magnetic moment $\vec{m}$ and radius R_0. In this case the distribution of magnetic induction $\vec{B}(\vec{r})$ is as follows [?]. Inside the nucleus,

$$\vec{B}(\vec{r}) = \frac{2}{3}\mu_0\vec{M}_0 = \frac{2}{3}\mu_0 M_0\vec{e}_z,$$

where $\vec{M}_0 = M_0\vec{e}_z$ is the magnetization strength (constant). Correspondingly, the magnetic energy inside (denoted by W_i) is given by

$$W_i = \frac{1}{2\mu_0}\int_{\text{in}} d\tau B^2 = \frac{2\mu_0}{9}M_0^2 V = \frac{\mu_0\vec{m}^2}{8\pi R_0^3}.$$

Outside the nucleus [?],

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi r^3}\left[3(\vec{m}\cdot\vec{e}_r)\vec{e}_r - \vec{m}\right],$$

where $\vec{r}$ is the space coordinate with respect to the center of the nucleus. By using the identity $[3(\vec{m}\cdot\vec{e}_r)\vec{e}_r - \vec{m}]^2 = m^2[(2\cos\theta)^2 + \sin^2\theta] = m^2(3\cos^2\theta + 1)$, where θ is the angle between $\vec{e}_z$ and $\vec{e}_r$, and integrating over the full space, one obtains the magnetic energy outside the nucleus (denoted by W_e) as

$$W_e = \frac{1}{2\mu_0}\int_{\text{out}} d\tau B^2 = \frac{\mu_0 \vec{m}^2}{24\pi R_0^3}.$$

Summing W_i and W_e, we obtain the total magnetic energy W:

$$W = W_i + W_e = \frac{\mu_0 \vec{m}^2}{6\pi R_0^3}.$$

According to Eq. (5), the value of W is proportional to R_0^{-3} or approximately $1/A$. Since the magnetic moments do not increase with A (typically a few μ_N), the value of W is larger for smaller A. This energy is largest for the proton: if one takes $m = 2.793\mu_N$ and $R_0 = 0.84$ fm (charge radius), the value of W is 210 keV according to this oversimplified classical model.

We now present typical values of W for a few light nuclei using the NNDC database [?] for magnetic moments and Ref. [?] for charge radii in this classical model. The magnetic energy is 27 keV for ^{3}H ($m = 2.98\mu_N$ and $R = 1.76$ fm), 9.5 keV for ^{3}He ($m = 2.13\mu_N$ and $R = 1.97$ fm), 12 keV for ^{7}Li ($m = 3.26\mu_N$ and $R = 2.44$ fm), and 11 keV for ^{41}Sc ($m = 5.43\mu_N$ and $R = 3.50$ fm). In most cases, the values of W are only a few keV or even well below. Currently, the uncertainty of nuclear mass formulas is well above 100 keV, thus the contribution from magnetic energy is not essential at all unless nuclear mass models could achieve accuracy of a few keV in the future.

III. Electromagnetic Angular Momentum

Atomic nuclei have strong electric fields. With further consideration of the magnetic moment, the momentum density is given by $\epsilon_0\vec{E}\times\vec{B}$, which is easily seen to yield an angular momentum parallel to the magnetic moment. Suppose that a given nucleus is (approximately) spherical with electric charge uniformly distributed. We denote the proton number of this nucleus by Z, and have the electric field strength

$$\vec{E}(\vec{r}) = \frac{Ze}{4\pi\epsilon_0 R_0^3}\vec{r}$$

inside the nucleus, and

$$\vec{E}(\vec{r}) = \frac{Ze}{4\pi\epsilon_0 r^3}\vec{r}$$

outside the nucleus. The resultant angular momentum density is $\vec{l} = \vec{r} \times (\epsilon_0 \vec{E} \times \vec{B})$, where $\vec{B}$ is given in Eqs. (3) and (4). Integrating $\vec{l}$ over the full space, we obtain the total electromagnetic momentum $\vec{L}$ of the system as

$$\vec{L} = \int_{\text{in}} d\tau\, \vec{l} + \int_{\text{out}} d\tau\, \vec{l} = \int_0^{R_0} r^{2dr} \int d\Omega\, [\vec{r}\times(\epsilon_0\vec{E}\times\vec{B})] + \int_{R_0}^{\infty} r^{2dr} \int d\Omega\, [\vec{r}\times(\epsilon_0\vec{E}\times\vec{B})].$$

Substituting the expressions for $\vec{E}$ and $\vec{B}$ gives

$$\vec{L} = \int_0^{R_0} r^{2dr} \int d\Omega\, \vec{r}\times\left(\frac{Ze}{4\pi R_0^3}\vec{r} \times \frac{2}{3}\mu_0\vec{M}_0\right) + \int_{R_0}^{\infty} r^{2dr} \int d\Omega\, \vec{r}\times\left(\frac{Ze}{4\pi r^3}\vec{r} \times \frac{\mu_0}{4\pi r^3}[3(\vec{m}\cdot\vec{e}_r)\vec{e}_r - \vec{m}]\right).$$

The axial symmetry of the system yields $\vec{L} = L_z\vec{e}_z$, with

$$L_z = -\frac{\mu_0 ZeM_0}{6\pi R_0^3}\int_0^{R_0} r^{4dr} \int \sin^2\theta d\Omega + \frac{\mu_0 Ze}{16\pi^2}\int_{R_0}^{\infty} \frac{m\,dr}{r^2} \int \sin^2\theta d\Omega = \frac{\mu_0 Zem}{10\pi R_0}.$$

For typical nuclei, the value of L evaluated this way is found to be very small. For example, for the ^{41}Sc nucleus with $m = 5.43\mu_N$ and $R_0 = 3.50$ fm, this angular momentum is only $1.0 \times 10^{-2}\hbar$, and for the proton, $L \simeq 1.0 \times 10^{-3}\hbar$. Since the orbital angular momentum of any given system is always quantized in multiples of $\hbar$, we conclude that the electromagnetic angular momentum of atomic nuclei is effectively zero.

It is now interesting to consider another simple system: a point charged particle in a uniform magnetic field $\vec{B} = B_0\vec{e}_z$. In this case the electromagnetic angular momentum is simply given by

$$\vec{L} = \int d\tau\, \vec{r} \times \left(\epsilon_0 \frac{Ze}{4\pi\epsilon_0 r^3}\vec{r} \times B_0\vec{e}_z\right) = L_z\vec{e}_z,$$

with

$$L_z = -\frac{ZeB_0}{6\pi}R,$$

where Ze is the charge of the point particle and R is the radius of the space considered. If both R (i.e., the space to be considered) and B_0 are not very large, L is well below $1\hbar$. On the other hand, according to Eq. (6), the value of $\vec{L}$ increases with R and magnetic field strength B_0. In the case that B_0 is taken to be very large under an extreme condition, e.g., in a neutron star, the value of L in Eq. (6) could become much larger than $1\hbar$ and play an essential role in certain processes related to angular momentum of nucleons and electrons in neutron stars.

Another possible application of Eq. (6) is relativistic heavy ion collisions, in which the magnetic field is parallel to the orbital angular momentum. Because the electromagnetic angular momentum of the system $\vec{L}$ is antiparallel to the magnetic field $\vec{B}$, as shown in Eq. (6), the emergence of electromagnetic angular momentum $\vec{L}$ suddenly enhances the value of orbital angular momentum of the whole system during the collision if the colliding fire-ball is not electrically neutral. The value of $\vec{L}$ could be as large as $10^3\hbar$ ($\vec{L}$ is proportional to the strength of the magnetic field) or even larger, depending on the number of nucleons in the spectator part and the collision energy. Clearly, the larger the collision energy and the number of protons in the spectator of the collision, the larger the value of $\vec{L}$. This provides us with a possible mechanism and perspective on the fact that the fire-ball of relativistic heavy ion collisions becomes more and more electrically neutral with increasing collision energy.

IV. Summary

To summarize, we investigate, for the first time as far as we know, the magnetic energy and electromagnetic angular momentum of atomic nuclei with very simple assumptions about the electromagnetic field. All these quantities are obtained in compact formulas. We present a few typical values of magnetic energy and electromagnetic angular momentum for atomic nuclei, noting that the value of magnetic energy is negligibly small in current nuclear mass models and that the value of electromagnetic angular momentum is effectively zero for conventional atomic nuclei. On the other hand, it is also worth pointing out that the value of electromagnetic angular momentum for a nucleon might be larger than $1\hbar$ and might play an important role in physics under certain extreme conditions such as in neutron stars with very strong magnetic fields. This might be one of the mechanisms explaining the fact that the fire-ball of RHIC is electrically neutral.

Acknowledgments

We thank the National Natural Science Foundation of China (Grants No. 12375114 and No. 11961141003), and the MOE Key Lab for Particle Physics, Astrophysics and Cosmology for financial support. We also thank Prof. X. D. Ji for discussions.

References

[1] C. F. v. Weizsacker, Z. Physik 96, 431 (1935).
[2] J. D. Jackson, *Classical Electrodynamics*, John Wiley & Sons, Inc. (1999), pp. 198.
[3] http://www.nndc.bnl.gov/ensdf/.
[4] I. Angeli and K. P. Marinova, At. Data Nucl. Data Tables 99, 69 (2013).