Influence of neutron-pair condensation on the nuclear symmetry energy slope

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Abstract

Highly linear correlation between the charge radii difference of mirror-pair nuclei and the slope parameter of symmetry energy has been built in the existing literatures. In this work, the impact of neutron-proton correlation deduced from the neutron and proton pairs condensation around the Fermi surface on determining the slope parameter of nuclear symmetry energy is investigated based on the Skyrme density functionals. The differential charge radii of Ni isotopes are employed to inspect the validity of this recently developed model. The calculated results suggest that the modified model can reproduce the shell quenching of charge radii at the neutron number $N = 28$ along Ni isotopic chain. The shell closure effect of the charge radii can also be predicted at the neutron number $N = 50$. The correlations between the charge radii differences of mirror partner nuclei ^{32}Ar-^{32}Si and ^{54}Ni-^{54}Fe and the slope parameters of symmetry energy are also analyzed. It is shown that the covered range of the symmetry energy slope is influenced by the neutron-pairs condensation around the Fermi surface. Moreover, a relatively stiff equation of state can be inferred from the mirror pairs ^{32}Ar-^{32}Si and ^{54}Ni-^{54}Fe when the influence coming from the neutron-pairs condensation is taken into account.

Full Text

Preamble

Influence of neutron-pair condensation on the nuclear symmetry energy slope

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A highly linear correlation between the charge radii difference of mirror-pair nuclei and the slope parameter of symmetry energy has been established in the existing literature. In this work, the impact of neutron-proton correlation deduced from neutron and proton pair condensation around the Fermi surface on determining the slope parameter of nuclear symmetry energy is investigated based on Skyrme density functionals. The differential charge radii of Ni isotopes are employed to inspect the validity of this recently developed model. The calculated results suggest that the modified model can reproduce the shell quenching of charge radii at neutron number N = 28 along the Ni isotopic chain. The shell closure effect of the charge radii can also be predicted at neutron number N = 50. The correlations between the charge radii differences of mirror partner nuclei ^{32}Ar-^{32}Si and ^{54}Ni-^{54}Fe and the slope parameters of symmetry energy are also analyzed. It is shown that the covered range of the symmetry energy slope is influenced by neutron-pair condensation around the Fermi surface. Moreover, a relatively stiff equation of state can be inferred from the mirror pairs ^{32}Ar-^{32}Si and ^{54}Ni-^{54}Fe when the influence coming from neutron-pair condensation is taken into account.

Keywords: charge radii, shell closure effect, mirror nuclei, slope parameter of symmetry energy, equation of state

Introduction

As accurately measured quantities in terrestrial laboratories, charge radii are generally used to encode nuclear structure phenomena such as shape-phase transitions [1–6], shell quenching phenomena [7–13], and odd-even staggering (OES) effects [14–19]. Precise knowledge of nuclear size plays an indispensable role in the course of nuclear physics and astrophysics [20]. In particular, the charge radii difference in mirror-paired nuclei—naturally defined with neutron number N and proton number Z exchanged but with the same mass number A = N + Z—is intimately associated with our understanding of fundamental interactions. The highly linear correlation between the difference of charge radii of mirror partner nuclei (ΔR_{ch}) and the symmetry energy slope (L) was first proposed in Ref. [21], and subsequent studies have verified that the charge radii differences of mirror-pair nuclei are linearly correlated with the slope parameter of symmetry energy [22–31].

Owing to advanced experimental techniques, much more data on charge radii of nuclei far from the β-stability line have been accumulated [32, 33]. Generally, a nucleus can be regarded as an incompressible liquid drop, and its size can be described by the A^(1/3) or Z^(1/3) law [34–36]. However, microscopic aspects cannot yet be featured, such as information about proton density distributions and single-particle levels. The relativistic mean field theory [37, 38] and non-relativistic Skyrme Hartree-Fock-Bogoliubov (HFB) approach [39, 40] can also be used to describe the systematic evolution of nuclear charge radii, but the local variations of nuclear charge radii cannot be captured adequately along a long isotopic chain.

Fine structures of nuclear charge radii can be influenced by various underlying mechanisms [41]. With density gradient terms included in its pairing interaction part, the discontinuous behavior of charge radii can be well described [42]. As shown in Refs. [27, 43], the correlations between the charge radii differences of mirror-pair nuclei and the slope parameter of symmetry energy are reduced by pairing effects. Meanwhile, shape deformation can also influence the determination of local variations of nuclear charge radii [1, 6]. As mentioned in Ref. [30], quadrupole deformation correction has been taken into account in describing the charge radii difference of ^{32}Ar-^{32}Si, but the influence from shape deformation is almost negligible. Besides, recent studies suggest that the compression modulus of finite nuclei cannot be ignored in ascertaining the slope parameter of symmetry energy [43, 44].

Short-range correlations between neutrons and protons influence charge density distributions around the Fermi surface [45–47]. This means that neutron-proton correlations should be considered appropriately in describing the bulk properties of finite nuclei [48, 49]. Based on the relativistic mean-field model, a novel ansatz derived from neutron-pair condensation around the Fermi surface has been incorporated into the root-mean-square (rms) charge radii formula [50]. This recently developed approach can reproduce the systematic evolution of nuclear charge radii, including the corresponding OES phenomena and shell closure effects [51–53]. This appears to provide an available method to depict the systematic trends of nuclear charge radii changes.

As demonstrated above, proton density distributions in mirror nuclei can provide an alternative access to ascertain the equation of state of isospin asymmetric nuclear matter. It is also worth noting that the influence of neutron-pair condensation on rms charge radii has been extended to Skyrme EDFs [54], but further discussions on ascertaining the symmetry energy slope are hardly found in the existing literature. Therefore, the influences of neutron-pair condensation on the determination of charge radii differences should be emphasized in this work. In our calculations, the spherical Hartree-Fock-Bogoliubov (HFB) framework is employed, and the differential mean-square charge radii of Ni isotopes are further used to test the validity of this theoretical model. Moreover, the charge radii differences of mirror-pair nuclei ^{32}Ar-^{32}Si and ^{54}Ni-^{54}Fe are also applied to analyze the correlation between the charge radii difference of mirror partner nuclei and the nuclear symmetry energy slope.

The outline of the present paper is as follows. In Section 2, the theoretical framework is briefly presented. In Section 3, the numerical results and discussion are provided. Finally, a summary together with some perspectives is given in Section 4.

II. Theoretical Framework

The Skyrme density functional has achieved considerable success in describing various physical phenomena [39, 40, 55–57]. In our calculations, the Skyrme-like effective interaction is recalled as follows [58, 59]:

$$V(r_1,r_2) = t_0(1+x_{0P_\sigma})\delta(r)+\frac{1}{2}t_1(1+x_{1P_\sigma})[P'^2\delta(r)+\delta(r)P^2]+t_2(1+x_{2P_\sigma})P'\cdot\delta(r)P+\frac{1}{6}t_3(1+x_{3P_\sigma})[\rho(R)]^\alpha\delta(r)+i$$

Here, $r = r_1 - r_2$ and $R = (r_1 + r_2)/2$ are naturally related to the positions of two nucleons r_1 and r_2, $P = (\nabla_1 - \nabla_2)/2i$ represents the relative momentum operator acting on the right and the corresponding counterpart $P' = -(\nabla'_1 - \nabla'_2)/2i$ characterizes its complex conjugate acting on the left, and $P_\sigma = (1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)/2$ represents the spin exchange operator that dominates the relative strength of the $S = 0$ and $S = 1$ channels for a given term in the two-body interactions, with $\vec{\sigma}_{1(2)}$ being the Pauli matrices. The last term features the spin-orbit force, in which $\sigma = \vec{\sigma}_1 + \vec{\sigma}_2$. The quantities α, t_i, x_i (i = 0-3), and W_0 represent the parameters of the effective forces used in this work.

Pairing correlations can be generally treated either by the BCS method or by the Bogoliubov transformation [37, 39, 40, 60–62]. In this work, the Bogoliubov transformation is used to treat pairing correlations. The density-dependent zero-range pairing force is employed as follows [63]:

$$V_{\text{pair}}(r_1,r_2) = V_0\left[1-\eta\left(\frac{\rho(r)}{\rho_0}\right)^\gamma\right]\delta(r_1-r_2).$$

Here, $\rho(r)$ is the baryon density distribution in coordinate space and $\rho_0 = 0.16$ fm^{-3} represents the nuclear saturation density. Generally, the values of η are taken as 0.0, 0.5, or 1.0 for volume-, mixed-, or surface-type pairing interactions, respectively. As mentioned in Refs. [27, 43], pairing correlations influence the determination of the correlation between the charge radii difference of mirror-pair nuclei and the slope parameter of symmetry energy. Therefore, the mixed-type pairing force is chosen in our calculations. The quantity V_0 is adjusted by calibrating the empirical energy gaps with the three-point formula [63, 64]. The single-particle energy levels and wave functions of the constituent nucleons can be obtained by solving the HFB equations with the self-consistent iteration method [65].

The range of proton matter distributions can be deduced from the wave functions of the constituent protons. The quantity of nuclear charge radius (R_{ch}) can be defined as the root-mean-square radius of its proton distribution, which can be calculated through the following expression (in units of fm^2) [52]:

$$\langle r_{\text{ch}}^2\rangle = \langle r_p^2\rangle + 0.7056 + a_0\Delta D.$$

The first term $\langle r_p^2 \rangle$ represents the charge density distributions of point-like protons, and the second term is attributed to the finite size of protons [66]. For the third term, the expression ΔD is defined as $\Delta D = |D_n - D_p|$. The quantity $D_{n(p)}$ is recalled as follows:

$$D_{n;p} = \sum_k u_k v_k,$$

where $v_{n;p}$ is the amplitude of the occupation probability of the kth quasi-particle orbital for neutron or proton in the canonical basis, and $u_k^2 = 1 - v_k^2$. It is also mentioned that the quantity $D_{n;p}$ can be used to measure Cooper pair condensation around the Fermi surface [67, 68]. The expression ΔD is used to measure neutron-proton correlations around the Fermi surface [17, 50]. The parameter set $a_0 = 0.561$ is adjusted by reproducing the parabolic-like shape and odd-even oscillation behaviors in the charge radii of K and Ca isotopes [52]. The last term considers the correlation between simultaneously unpaired neutron and proton. For mirror pair nuclei, the difference in charge radii (ΔR_{ch}) can be obtained through Eq. (3). Actually, the last term is invalid due to time-reversal symmetry being assumed in this work, namely restricting ourselves to even-even nuclei.

III. Results and Discussion

The influence of neutron-proton correlation deduced from neutron and proton pair condensation around the Fermi surface has been incorporated into the rms charge radii formula based on Skyrme EDFs [54]. First, the differential mean-square charge radii of Ni isotopes are employed to further review the validity of this theoretical model. The mixed-type pairing interaction is used, and the pairing strength is chosen as $V_0 = 452.4$ MeV fm^3 by adjusting the empirical energy gap along the Ni isotopes. Two pairs of mirror nuclei, ^{32}Ar-^{32}Si and ^{54}Ni-^{54}Fe, are used to inspect the influence of neutron-proton correlations on determining the charge radii difference of the corresponding mirror-pair nuclei. For convenience, results obtained by Eq. (3) are labeled as HFB*, while results obtained by the approach without considering the influence from neutron pair condensation are marked as HFB.

The parameter sets of the effective forces used in this work and the corresponding values of the bulk properties of symmetric nuclear matter are shown explicitly in Table 1. Under specific incompressibility coefficients K, the slope parameter L and symmetry energy E_{sys} at saturation density ρ_0 cover a long range. It should be mentioned that the correlation between the charge radii differences of mirror partner nuclei and the slope parameter of symmetry energy can be influenced by the incompressibility coefficients of symmetric nuclear matter [43, 44]. Meanwhile, giant monopole resonances provide the value of the isoscalar incompressibility $K = 230 \pm 10$ MeV [70, 71]. Thus, the parameter sets s3028 and s3036, classified by almost equivalent incompressibility coefficients $K \approx 230$ MeV, are chosen to describe the differential charge radii of Ni isotopes.

Charge radii of Ni isotopes have been measured accurately through collinear laser spectroscopy [24, 69, 72]. The extracted results suggest that shell quenching phenomena in charge radii can be observed significantly around the fully filled $N = 28$ shell. Moreover, across neutron number $N = 28$, the trend of changes in charge radii is similar to the Ca isotopic chain. Actually, the systematic evolution of nuclear charge radii across $N = 28$ shows a similar trend from K to Zn isotopic chains, meaning it is almost independent of the atomic number [52, 73].

As shown in Fig. 1, the differential charge radii of Ni isotopes relative to the reference nucleus ^{58}Ni are depicted by the HFB and HFB* methods with the effective forces s3028 and s3036, respectively. It can be seen that the HFB* model can reproduce the trend of charge radii changes across the $N = 28$ shell closure along the Ni isotopic chain, as shown in Fig. 1(a). Meanwhile, the decreasing trend of charge radii from ^{54}Ni to ^{56}Ni can also be reproduced. This leads to the significantly observed kink phenomenon at neutron number $N = 28$. Here, it can be found that the values of charge radii for isotopes ^{60},^{62}Ni are slightly underestimated.

In contrast to the HFB* model, the differential charge radii of Ni isotopes cannot be reproduced by the HFB model. Particularly, the rapid increase of charge radii cannot be reproduced well across the fully filled $N = 28$ shell. The same scenario is also encountered around $N = 50$, where the HFB model cannot represent the shrinking trend of charge radii. By contrast, the shell quenching phenomenon in charge radii of Ni isotopes could be predicted at neutron number $N = 50$ through the HFB* model.

As is well known, an inverted parabolic-like shape of charge radii can be evidently observed between neutron numbers $N = 20$ and $N = 28$ along K and Ca isotopes [32, 33]. The inverted parabolic-like shape of charge radii along the Ni isotopic chain can be reproduced by the HFB* model calculations between neutron numbers $N = 20$ and $N = 28$. However, this phenomenon cannot be represented in the HFB model. This can be found obviously in that the trend of charge radius change for the ^{54}Ni isotope cannot be reproduced well by the HFB model. Moreover, the inverted parabolic-like shape of charge radii obtained by the HFB* model can also be found between neutron numbers $N = 28$ and $N = 50$, though the amplitude is apparently weakened.

In Fig. 1(b), the results obtained by the HFB and HFB* models are drawn with the effective force s3036. The shell quenching effects of charge radii at neutron numbers $N = 28$ and $N = 50$ can also emerge from the HFB* model. Here, it should be mentioned that the HFB model still underestimates the systematic trend of charge radii changes in Ni isotopes. Significant deviations can be found between the absolute values obtained by effective force s3036 and experimental ones compared to those calculated by the s3028 force. This is due to the fact that the symmetry energy is different for these two effective forces. The stability properties of finite nuclei are mostly determined by the Coulomb force and symmetry energy. Larger symmetry energy means stronger isospin interactions can be captured in parameter set s3036, which leads to a larger range of charge density distribution radii [74]. This can be understood from the charge radius of ^{56}Ni, in which the value obtained through the s3036 force is larger than that calculated by s3028 by about 0.02 fm.

Accurate description of nuclear charge radii provides a sensitive indicator of our understanding of fundamental interactions in exotic nuclei. As mentioned in Refs. [46, 49, 75], isospin interactions from neutron-proton correlations should be considered properly in evaluating nuclear size. The greater ability of the HFB* model to reproduce the differential charge radii of Ni isotopes is attributed to neutron-proton correlations deduced from neutron and proton pair condensation around the Fermi surface. This is actually in accord with those in Refs. [76–78] where neutron-proton interactions derived from valence neutrons and protons can describe local variations of nuclear charge radii along a long isotopic chain. This further suggests that neutron-proton correlations around the Fermi surface play an indispensable role in determining the discontinuous variations of nuclear charge radii.

shows the charge radii (R_{ch}) and charge radii difference (ΔR_{ch}) databases for the $A = 32$ and 54 mirror pair nuclei. The parentheses on the values of charge radii and charge radii differences indicate systematic uncertainties [24, 30, 32, 33].

Highly linear correlation between the charge radii difference (ΔR_{ch}) of mirror-pair nuclei and the slope parameter (L) of symmetry energy has been utilized to ascertain the isospin components in the equation of state of asymmetric nuclear matter [21–31]. Therefore, it is essential to further review the influence of neutron-proton correlations on determining the values of ΔR_{ch} in a given pair of mirror nuclei. In our calculations, mirror nuclei with mass numbers $A = 32$ and 54 are employed to illustrate the influence of neutron-proton correlations on the charge radii difference. The corresponding experimental data are shown in .

The theoretical results obtained by HFB and HFB* models are used to access the correlation between ΔR_{ch} and L. As shown in [Figure 2: see original paper], the results of ΔR_{ch} for mirror partner nuclei ^{54}Ni-^{54}Fe and ^{32}Ar-^{32}Si are presented as a function of slope parameter L. The shadowed planes indicate the systematic uncertainties of the charge radii difference between the corresponding mirror nuclei, which cover the ranges of 0.049(4) fm (^{54}Ni-^{54}Fe) and 0.194(14) fm (^{32}Ar-^{32}Si), respectively.

The results obtained using the HFB and HFB* models display an approximately linear correlation between ΔR_{ch} and L for both pairs of mirror nuclei. Generally, the uncertainty range of ΔR_{ch} covered by the fitted line is used to constrain the slope parameter L. It should be mentioned that the covered range of L is changed obviously by the HFB* model. Here, the lower limit of L is constrained at 0.0 MeV in our discussion. As shown in Figure 2: see original paper, the covered range of L obtained by the HFB model falls into $0.0 \leq L \leq 38.11(\pm5.07)$ MeV. By contrast, the HFB* model gives the range of $33.18 \leq L \leq 78.64(\pm3.58)$ MeV. This is in accord with Ref. [24] where the deduced L value falls into the range $21 \leq L \leq 88$ MeV. This means that neutron-proton correlations around the Fermi surface can influence the determination of the values of ΔR_{ch}. Moreover, larger deviation can also be encountered in the ^{32}Ar-^{32}Si mirror pair. The range of $51.22 \leq L \leq 97.87(\pm6.17)$ MeV can be extracted from ^{32}Ar-^{32}Si with the HFB method. The HFB* method gives the range of $62.21 \leq L \leq 111.59(\pm5.19)$ MeV. As demonstrated in Ref. [30], a rather soft equation of state (EoS) has been obtained, namely $L \leq 60$ MeV. However, a stiffer EoS can be extracted from the charge radii difference of mirror pair nuclei ^{32}Ar-^{32}Si in our calculations. Our results suggest that the HFB* model seems to give a rather stiff EoS compared to those obtained by the HFB model. Furthermore, it appears to suggest that neutron-proton correlations should be considered properly in constraining the slope parameter L.

As mentioned in Refs. [43, 44, 79], quantification uncertainty suffering from nuclear matter incompressibility is inevitable in evaluating the isospin components. To facilitate quantitative comparison, the ΔR_{ch} of ^{54}Ni-^{54}Fe obtained by the HFB and HFB* models is depicted as a function of slope parameter L in [Figure 3: see original paper]. The effective forces classified by various incompressibility coefficients K are used to distinguish the covered range of slope parameters. Here it should be mentioned that the covered ranges of the slope parameter L obtained by the HFB method are distinguished evidently from those obtained by the HFB* model for the mirror-pair nuclei ^{54}Ni-^{54}Fe. For specific incompressibility coefficients, the covered lower and upper ranges of L are simultaneously enlarged when neutron-proton correlations around the Fermi surface are considered. However, for $K = 230$ MeV and $K = 240$ MeV, the covered ranges of L are almost similar for the HFB* model. In contrast to $K = 230$ MeV and $K = 240$ MeV, the HFB* model gives a relatively soft range of L when effective forces classified by $K = 250$ MeV are used.

As mentioned above, the incompressibility coefficient influences the determination of the symmetry energy slope. As shown in [Figure 4: see original paper], ΔR_{ch} of ^{32}Ar-^{32}Si as a function of L is also presented with the HFB and HFB* models. For parameter sets classified by various incompressibility coefficients, the upper and lower limit ranges of the slope parameter L obtained by the HFB* model are systematically enlarged with respect to those obtained by the HFB method. For $K = 240$ MeV and $K = 250$ MeV, the covered ranges of L are almost similar for the HFB* model. However, for the $K = 230$ MeV case, the upper range of L is lower than the $K = 240$ MeV and $K = 250$ MeV cases by about 20 MeV. This further suggests that the correlation between the charge radii difference of mirror partner nuclei and the slope parameter of symmetry energy can be influenced by effective forces classified by various incompressibility coefficients.

Generally, neutron skin thickness of a heavy nucleus provides superior access to constrain the equation of state of isospin asymmetric nuclear matter. The difference of proton density distributions in mirror nuclei is intimately related to neutron skin thickness [25, 80]. This indicates that information about neutron skin thickness can be extracted from directly measured charge radii data. As demonstrated in Refs. [81, 82], precise data on mirror charge radii cannot make a rigorous constraint on the slope parameter L, and an even worse correlation can be obtained between the mirror charge radii difference ΔR_{ch} and the slope parameter L. The bulk properties of finite nuclei cannot be captured adequately by effective forces deduced from infinite nuclear matter. This can be understood from specific aspects that characterize the radii range of proton density distributions, such as shell closure effects [11, 83] and the influence from isospin symmetry breaking [84–86]. Actually, proton and neutron matter distributions mutually influence each other. As discussed in this paper, neutron-proton correlations deduced from neutron and proton pair condensation around the Fermi surface and the compression modulus influence the determination of the charge radii difference of mirror nuclei. Therefore, the influence derived from neutron-proton correlations around the Fermi surface and the compression modulus of symmetric nuclear matter cannot be negligible in describing nuclear charge radii [43, 52, 54].

IV. Summary

In this work, the influence of neutron-pair condensation around the Fermi surface on the determination of symmetry energy slope is investigated for the first time based on Skyrme EDFs. The validity of this theoretical model has been reviewed by reproducing the trend of changes in differential charge radii along Ni isotopes. The differential charge radii of Ni isotopes can be well described by this modified model. In particular, the kink phenomenon in charge radii can be significantly reproduced at neutron number $N = 28$ through this modified model. Meanwhile, the shell closure effect at $N = 50$ can also be expected in the charge radii of Ni isotopes. Intriguingly, the inverted parabolic-like shape in charge radii can be predicted between neutron numbers $N = 20$ and $N = 28$ as well. The same scenario can also be observed between $N = 28$ and $N = 50$, but the amplitude is apparently weakened.

As is well known, nuclear symmetry energy plays an essential role in various simulation codes [87–98]. Therefore, an available density dependence of symmetry energy is required from multimessenger constraints. The correlations between the charge radii differences of mirror partner nuclei ^{32}Ar-^{32}Si and ^{54}Ni-^{54}Fe and the slope parameters of symmetry energy can be affected by neutron-proton correlations around the Fermi surface. In particular, a rather stiff equation of state can be obtained by the HFB* model in comparison to those deduced from the HFB method. The mean-square charge radius of a nucleus is naturally extracted from charge density distributions. Recent studies suggest that charge radii can be derived from charge-changing cross-section measurements of exotic nuclei [99, 100]. Moreover, the difference in charge-changing cross sections of mirror nuclei can also provide an alternative approach to evaluate the equation of state of nuclear matter [101]. Thus, a unified model is required for ascertaining the slope parameter of symmetry energy through the charge radii difference of mirror-pair nuclei.

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