Research Status of Star Formation Laws (Post-print)

Authors: Zhang Chao

Date: 2024-12-31T00:00:00+00:00

Abstract

The physical environment of molecular clouds exerts a significant influence on star formation rates, and understanding star formation is of great importance for comprehending planet formation and galaxy evolution. This review primarily examines the relationship between star formation and gas in the Milky Way and its neighboring galaxies over the past two decades (i.e., the star formation law), with the aim of better understanding the nature of star formation, the relationship between star formation and gas, and whether stars form in a unified manner. It introduces the components of the interstellar medium and methods for measuring interstellar medium density and star formation rates. The focus is on reviewing the relationship between dense gas and star formation rates from galactic scales to molecular cloud scales, and providing a summary and outlook for research on star formation laws.

Full Text

Preamble

PROGRESS IN ASTRONOMY Vol. 42, No. 4 Dec., 2024
doi: 10.3969/j.issn.1000-8349.2024.04.02

A Review of the Laws of Star Formation

ZHANG Chao1;2;3
(1. Department of Physics, Taiyuan Normal University, Jinzhong 030619, China;
2. Institute of Computational and Applied Physics, Taiyuan Normal University, Jinzhong 030619, China;
3. Shanxi Key Laboratory for Intelligent Optimization Computing and Blockchain Technology, Taiyuan Normal University, Jinzhong 030619, China)

Abstract

The physical environment of molecular clouds significantly influences star formation rates, and understanding star formation provides crucial insights into planet formation and galaxy evolution. This review primarily examines the relationship between star formation and gas in the Milky Way and nearby galaxies over the past two decades—known as the star formation law—with the aim of better understanding the nature of star formation, the connection between star formation and gas, and whether stars form in a unified manner. We introduce the components of the interstellar medium and methods for measuring interstellar medium density and star formation rates, focusing particularly on the relationship between dense gas and star formation rates from galactic scales down to molecular cloud scales. We conclude with a summary and outlook for future research on star formation laws.

Keywords: star formation law; massive star formation; star formation efficiency
CLC number: P152
Document code: A

1 Introduction

Star formation research addresses two fundamental questions—the origin of stars and the origin of planets—while also linking to galaxy formation and evolution. Astronomers seek to understand star formation activities across multiple physical scales, from individual molecular clouds within the Milky Way to entire galaxies and even cosmic large-scale structures. Questions of when, where, and how stars form have long puzzled astronomers. Schmidt [**?**] first discovered in 1959, through quantitative measurements, a strong correlation between neutral hydrogen (HI) clouds in the interstellar medium and star formation rates, suggesting that stars likely form within gas. Subsequent research revealed that atomic gas traced by H atoms has relatively low density and high temperature, and is dominated by internal thermal pressure rather than being gravitationally bound on galactic scales [**?**]. As studies progressed, it became clear that atomic gas is not directly related to star formation; rather, molecular clouds—where H atoms cool and become H_2 molecules—serve as the true nurseries of stars. However, not all molecular clouds form stars; only extremely dense molecular cloud clumps are directly associated with star formation [**?**]. When local density in a molecular cloud increases, it exerts stronger gravitational attraction on surrounding gas, causing more gas to accumulate. Molecular clouds of 1–10 pc fragment into filamentary structures that experience continuous compression and collision from turbulence and magnetic fields, creating a delicate balance with the cloud' s self-gravity. When this equilibrium is disrupted—for example, when the cloud' s mass exceeds the Jeans mass—gravity dominates, triggering cloud collapse to form one or more stars. During collapse, the cloud' s gravitational energy converts to thermal energy, raising the temperature until it reaches the threshold for H-He nuclear fusion, and new stars are born. While

astronomers have identified some physical mechanisms of star formation, many questions remain unresolved: Do massive stars form through mergers or accretion? Before collapse, molecular clouds are supported not only by gravity but also by magnetic fields and turbulence—what are their distributions and generation mechanisms? Molecular cloud interiors are not uniform but composed of many filamentary structures—what are their characteristics and how do they relate to star formation? What is the specific relationship between gas density and star formation, and how does it connect to star formation rates?

The physical environment and geometric structure of molecular clouds affect the initial conditions for star formation. To investigate these issues, we must fully understand the physical properties of molecular clouds (such as density, temperature, elemental abundance, etc.). These properties are not static: from a macroscopic perspective, nuclear reactions in stellar interiors provide energy sources for galaxies, while stellar death supplies galaxies with heavy elements and dust. Supernova-driven winds can eject material into interstellar space. Violent star formation, supernova explosions, active galactic nuclei, and large-scale shocks all significantly impact the physical properties of molecular clouds. The Milky Way has experienced mergers [**?**] and starburst activities [**?**]; studying the physical environments of nearby galaxies helps us understand the Milky Way' s early history. Since high-redshift galaxies are too distant for easy detection, studying nearby galaxies provides ideal templates for understanding the early universe. This review examines the relationship between gas surface density and star formation rates from galactic to molecular cloud scales over the past 20 years—the K-S law—to understand the connection between star formation and gas surface density and whether stars form in a unified manner. Section 2 introduces interstellar medium classification and various probes for measuring gas surface density; Section 3 discusses star formation rates and their tracers; Section 4 examines the relationship between star formation rates and gas density in the Milky Way and nearby galaxies; and Section 5 provides a brief summary and outlook.

2.1 Components of the Interstellar Medium

The universe contains vast amounts of interstellar medium, comprising dust and gas with physical structures spanning enormous scales—from intergalactic medium (Mpc scale) to cold neutral medium (kpc scale), to molecular clouds (~10–100 pc scale), which fragment and collapse into denser clumps (~1 pc scale) and molecular cloud cores (~0.1 pc scale), eventually forming stars ($R_\odot$ scale) and planets (~AU scale). In the Milky Way, nearly half the volume is filled with hot ionized medium with density < 0.01 cm^{-3} and temperature $> 10^5$ K; the remaining half consists of warm ionized and warm neutral media with densities of ~0.1–1 cm^{-3} and temperatures of several thousand K [**?**]; only a small fraction is cold neutral medium with density > 10 cm^{-3} and temperature < 100 K [**?**]. Molecular clouds, as the coldest and densest component of cold neutral medium, are the true nurseries of stars [**?**]. The boundaries of molecular clouds

are typically defined by detecting CO low rotational transition emission above a certain threshold [**?**] or by using extinction of background stars to a certain degree. For example, the probability distribution function of surface density derived from extinction measurements can be fitted to a log-normal function for low extinction values ($A_V \approx 2$–5 mag) [**?**], while for higher extinction values, the tail follows a power-law distribution in at least some molecular clouds. In practice, active star formation occurs in molecular clouds with power-law tails [**?**]. In complex regions, radial velocity can distinguish molecular clouds at different distances along the line of sight. However, molecular clouds are surrounded by atomic layers with a transition region where H is primarily molecular while C is mainly atomic [**?**]. These regions are called PDRs, representing photodissociation regions or photon-dominated regions [**?**]. Molecular clouds contain many filamentary structures with dense cores. Relative to molecular clouds, cores have smaller masses but much higher densities, and these cores may form single stars, binaries, multiple stars, or even small clusters. Theoretically, besides defining the birthplace of individual stars as cores, a term is needed to denote the birthplace of individual clusters, for which Williams et al. [**?**] proposed using "clump." Table 1 lists some parameters of molecular clouds [**?**].

In the Milky Way, approximately 1% of the interstellar medium exists in solid form, primarily silicates and carbonaceous materials [**?**]. Dust particle sizes range from 0.35 nm to 1 μm. Using dust extinction and reddening, we can determine both dust and gas column densities and obtain molecular cloud structures. Thanks to improved telescope resolution and sensitivity, and expanded observational wavelengths from optical to radio bands, detectable dust extinction has increased from $A_V \approx 6$ mag to $A_V \approx 40$ mag, enabling more precise determination of core structures within molecular clouds. Larger dust grains absorb UV photons and re-emit the energy in infrared or millimeter bands. This characteristic allows using infrared radiation as a probe to measure star formation rates in dust-rich galaxies.

2.2 Atomic Probes

The universe contains abundant H atomic gas. Transitions between two hyperfine levels of H atoms (electron spin flipping from parallel to antiparallel) produce the well-known 21 cm line. Although the 21 cm line has a low spontaneous emission probability (2.9×10^{-15} s^{-1}), the enormous quantity of H atoms in the universe still yields substantial transition events (10^7 cm$^2 \cdot$ s^{-1}) [**?**], enabling measurement of hydrogen atomic gas column density through 21 cm line observations.

2.3.1 CO Molecular Probe

In molecular clouds, H_2 has the highest abundance, but lacks a permanent electric dipole moment and has high excitation energy, making its spectrum difficult to obtain. CO, the second most abundant molecule, is commonly used as

a molecular cloud tracer due to its strong spectral lines. CO(1-0) has a high dissociation energy (~11.1 eV) and relatively low critical density (~10^2 cm^{-3}) and excitation temperature (~5.5 K) [**?**], making it easily excited in both diffuse and dense molecular gas. CO rotational transition lines lie in the (sub)millimeter band, and combining multiple CO rotational lines can effectively trace the physical properties and excitation environments of molecular clouds. However, CO is not easily excited in very low-density molecular clouds, and its emission lines saturate in very high-density clouds, affecting information about high-density regions.

After obtaining integrated intensity from CO spectra ($I_{CO} = \int T d\nu$), the "X-factor" is typically used to relate it to H_2 column density: $N_{H_2} = X_{CO} I_{CO}$. For example, Bolatto et al. [**?**] summarized the X-factor for the Milky Way and nearby spiral galaxies as $X_{CO} = 2 \times 10^{20}$ $cm^{-2} \cdot K^{-1} \cdot km^{-1} \cdot s$. Molecular cloud temperature, density, metallicity, and other factors affect the X_{CO} value. Two common calibration methods exist: (1) using optical extinction and standard gas-to-dust ratios to obtain H_2 [**?**, **?**, **?**]; (2) comparing γ-ray column density with CO column density from isotopic measurements to calibrate X_{CO}, where γ-rays are produced by cosmic ray interactions with H_2 [**?**, **?**].

Maloney and Black [**?**] found that, ignoring metallicity, $X_{CO} \propto n^{0.5} T_k^{-1}$, where n is the average molecular cloud density and T_k is temperature. Shetty et al. [**?**] found a weak relationship: $X_{CO} \propto T_k^{-0.5}$. Leroy et al. [**?**] found that galaxies with lower metallicity have larger X_{CO} values. Note that these results were obtained under specific conditions and may not apply in other environments.

CO probes can also determine molecular cloud masses. When measuring CO luminosity, if the molecular cloud is optically thin, emission from within the cloud can be detected, allowing relatively accurate mass estimation from CO lines. However, CO is typically optically thick, and measurements only yield luminosity from the cloud' s outermost layer. Larson [**?**] discovered a relationship between CO line width and molecular cloud size: line width is proportional to the square root of cloud size (the Larson relation, which assumes giant molecular clouds are self-gravitating bound systems). If giant molecular clouds have constant surface density of 100 $M_\odot \cdot pc^{-2}$ [**?**], the Larson relation yields CO luminosity proportional to molecular gas virial mass, enabling total gas mass estimation in optically thick cases [**?**, **?**]. Like buildings behind a wall where only the wall is visible, if we assume all buildings are similar and one building' s mass is known, we can roughly estimate total mass from the wall' s apparent area. Based on this principle, CO(1-0) luminosity is often used to measure molecular cloud masses through the relation: $M = \alpha_{CO} L_{CO}$. The α_{CO} value is also affected by molecular cloud temperature, density, metallicity, etc. The $X_{CO} = 2 \times 10^{20}$ $cm^{-2} \cdot K^{-1} \cdot km^{-1} \cdot s$ corresponds to $\alpha_{CO} = 4$ $M_\odot \cdot K^{-1} \cdot km^{-1} \cdot s$ (excluding He) [**?**]. In nearby starburst galaxies and (ultra)luminous infrared galaxies (LIRGs, $L_{IR} \approx 10^{11}$ $L_\odot$; ULIRGs, $L_{IR} \approx 10^{12}$ $L_\odot$), using the Milky Way α_{CO} value yields masses greater than the dynamical masses of starburst galaxies [**?**]. Downes and Solomon [**?**] proposed $\alpha_{CO} = 0.8$ $M_\odot \cdot K^{-1} \cdot km^{-1} \cdot s$ for

these galaxy types. Note that when using L_{CO} to derive gas mass, the assumption is that all star formation in giant molecular clouds is similar—an assumption that is problematic in starburst and (ultra)luminous infrared galaxies.

2.3.2 Dense Gas Probes

Dense gas generally refers to gas where molecular cloud surface density (or volume density) exceeds a certain threshold. Only when molecular cloud density is sufficiently high to exceed the critical density of "dense molecular probes" can spectral lines be excited. However, the threshold value remains controversial. One view suggests the threshold is the typical average density of molecular clouds, i.e., $N_{H_2} > 10^2$-10^3 cm^{-3}, which happens to be above CO' s critical density. The second view proposes the threshold is the actual core density, about two orders of magnitude higher than the average density, i.e., $N_{H_2} > 10^4$ cm^{-3}, at which point molecular clouds become gravitationally bound and begin efficiently forming stars [**?**, **?**, **?**]. We favor the second threshold. Note that besides density thresholds, surface density ($\Sigma_{mol} > 116$-129 M$_\odot \cdot$ pc^{-2}) [**?**, **?**, **?**], column density ($N_{H_2} > 6 \times 10^{21}$ cm^{-2}), or extinction ($A_V \approx 6$ mag) can also define thresholds [**?**].

Common dense gas probes include high rotational level CO transitions and lines from molecules like CS, HCN, and HCO$^+$. The excitation of dense molecular probes, like CO, depends on physical environment. In fact, dense molecular probes are more sensitive to UV radiation than CO, making them more dependent on metallicity [**?**]. Caution is needed when analyzing dense molecular probes, as they cannot simply replace CO. Each dense gas probe has advantages and should be selected based on specific circumstances. Compared to other probes, HCN and HCO$^+$ have higher abundances and stronger emissions, making them most widely used in galaxy studies. To date, HCN and HCO$^+$ have been systematically surveyed in moderate-mass galaxies [**?**, **?**, **?**, **?**, **?**, **?**]. The CS probe is less affected by infrared pumping [**?**] and shocks [**?**, **?**], and its energy levels require minimum excitation temperatures mostly below 70 K, making it ideal for probing dense, cold molecular gas.

2.4 Dust

Dust can also trace molecular gas. Assuming dust and gas are well mixed and knowing the gas-to-dust ratio (~100 in the Milky Way), gas mass can be derived from dust mass [**?**, **?**]. Note that this method yields total gas mass, including both molecular and atomic gas. Besides assuming dust-gas mixing, assumptions about metallicity and dust models are required to ultimately obtain molecular gas mass.

3.1 Initial Mass Function

The initial mass function (IMF) is an empirical function describing the distribution of initial masses among a group of stars, expressed as $dN/dM \propto M^{-\alpha}$ or $dN/d\lg M \propto M^{-\gamma}$ ($\alpha = \gamma + 1$), where N is the number of stars in a given mass interval (from M to $M + dM$) and M is stellar mass. In 1955, Salpeter [**?**] fitted the stellar initial mass function as a power-law function of $\lg M$, obtaining a power-law index $\alpha = 2.35$ for the range (0.4–10) $M_\odot$, known as the Salpeter IMF. The Kroupa IMF is another initial mass distribution function [**?**] that considers variations in stellar mass distribution across different mass ranges. It typically adopts a piecewise power-law distribution with two breaks at approximately 0.5 $M_\odot$ and 1 $M_\odot$. For stars of (0.08–0.5) $M_\odot$, the slope $\alpha \approx 0.3$; for (0.5–1) $M_\odot$, $\alpha \approx 1.3$; and for stars > 1 $M_\odot$, $\alpha \approx 2.3$, consistent with the Salpeter IMF.

3.2 Star Formation Rate and Its Probes

The star formation rate (SFR) refers to the rate of star formation, with units of $M_\odot \cdot a^{-1}$ or $M_\odot \cdot Ma^{-1}$. The most direct method for measuring SFR is counting different types of stars. Ideally, SFR can be obtained from the formula: $SFR = N(M_*, t_*)M_*/t_*$, where $N(M_*, t_*)$ represents the number of stars per unit mass $M_* = M_t$ and per unit stellar lifetime, and t_* is the longest lifetime among all stars. In practice, complete stellar information is unavailable, though in a few nearby galaxies (like the Large and Small Magellanic Clouds), counting young massive stars is relatively easy. Since young stellar objects (YSOs), particularly massive young stars, dominate luminosity, counting newly formed stars can trace SFR. With a given IMF, the total SFR can be derived from YSO counts: $SFR_{SO} = N_{YSO}\langle M_*\rangle/t$, where $\langle M_*\rangle$ is the average mass of newly formed stars and t is the average YSO lifetime.

For more distant galaxies where current instruments lack sufficient resolution for stellar counting, reasonable assumptions allow deriving SFR from massive star luminosities combined with the IMF: (1) Assume SFR is constant within the probe' s effective timescale. This holds for most disk and elliptical galaxies without recent starburst activity. For starburst galaxies, assume roughly constant SFR within the probe' s effective timescale. (2) Assume star formation follows some IMF. This is the fundamental assumption for most SFR derivations, as only through the IMF can we extrapolate from massive star masses (or luminosities) to total stellar mass. Unfortunately, this assumption remains highly controversial. (3) Assume observational data fully sample the IMF, meaning at least one star exists at the maximum mass sampling point. For actively star-forming galaxies, this is mostly satisfied. Under these assumptions, simple linear relationships can derive masses from massive young star luminosities and extrapolate to total formed stellar mass via the IMF [**?**].

SFR probes fall into two categories: (1) Global SFR probes, primarily for studying average galaxy-scale properties. These don' t require high resolution or sensitivity, facilitating statistical studies of large samples of distant galaxies. (2)

Local SFR probes, suitable for smaller scales (sub-kpc). These require higher resolution and sensitivity. Since different molecular clouds have vastly different physical properties, the scale cannot be too small, otherwise statistical deviations from individual uniqueness become large [**?**].

Common SFR probes include: (1) UV-infrared radiation. UV radiation primarily comes from massive young stellar photospheres, directly correlating with SFR. Since UV photons are easily affected by dust extinction, relying solely on UV radiation for SFR yields large errors. If all UV radiation is effectively absorbed by dust, which re-emits the energy in infrared bands, UV luminosity can be linked to infrared luminosity: $L_{UV,\mathrm{corr}} = L_{UV,\mathrm{observed}} + \eta L_{IR}$ (coefficient η depends on the passbands for UV and IR measurements), allowing SFR derivation. For dust-rich nuclear starburst galaxies, where dust extinction in infrared bands is minimal, total infrared radiation may be the best SFR probe: $\mathrm{SFR} = 4.5 \times 10^{-51} L_{TIR}$, where L_{TIR} is the integrated mid-to-far infrared luminosity [**?**] in $\mathrm{J \cdot s^{-1}}$ and SFR is in $\mathrm{M_\odot \cdot a^{-1}}$. (2) Emission line probes. The most widely used emission line probes are optical and near-infrared lines from ionized gas around massive young stars. Gas around massive young stars becomes ionized by their radiation, emitting lines from visible to infrared bands. For a standard IMF, these emission line probes trace stars with masses $\gtrsim 15\ \mathrm{M_\odot}$, with peak contributions from (30–40) $\mathrm{M_\odot}$ stars. Thus, these lines (and free-free radio continuum) represent nearly instantaneous SFR measurements, tracing stellar lifetimes of $\sim 3 \times 10^6$–10^7 a [**?**]. Common emission line probes include Hα lines (for nearby and distant galaxies), [O II] lines (for moderate-redshift galaxies), and Lyα lines (for high-SFR galaxies). (3) Radio radiation. Centimeter-wave radio radiation includes thermal and synchrotron components, with low-frequency radiation primarily produced by supernova-accelerated charged particles gyrating in magnetic fields. Gyro radiation shows strong correlation with far-infrared radiation, enabling SFR derivation (with limitations, applicable to certain galaxies). Thanks to improved radio instrument resolution and sensitivity, radio radiation is now mainly used to calculate SFRs for high-redshift galaxies with ongoing star formation.

Table 2 lists commonly used SFR probes and their application ranges. After obtaining luminosity from SFR probes, SFR can be calculated using $\lg M_* = \lg L_x - \lg C_x$, where C_x is the conversion factor between luminosity and SFR (see Table 2, column 4).

4 K-S Law

Schmidt [**?**] proposed a proportional relationship between SFR and gas density, first quantitatively discovering a strong correlation between neutral hydrogen (HI) atomic clouds in the interstellar medium and SFR, indicating that stars likely form in gas. Kennicutt [**?**] studied a series of galaxies and found a relationship between disk-averaged SFR surface density and gas surface density: $\Sigma_{SFR} \propto \Sigma_{gas}^{1.4}$. Since then, increasing research has investigated the relationship between gas and star formation, hoping to identify the specific physical

processes of gas evolution into stars. Such relationships are called Kennicutt-Schmidt (K-S) laws: $\Sigma_{SFR} = A\Sigma_{gas}^{N}$.

4.1 Galaxy-scale K-S Law

Figure 1 [Figure 1: see original paper] summarizes observational results for the K-S law. Each point represents a galaxy, extending from 61 normal disk galaxies and 36 starburst galaxies [**?**] to metal-poor starburst galaxies ($Z < 0.3Z_{\odot}$) and nearby low surface brightness galaxies [**?**]. SFR measurements used Hα, Paα, and infrared radiation as probes. Gas surface density used CO as a probe, with all galaxies adopting $X_{CO} = 2.3 \times 10^{20}$ cm$^{-2} \cdot$ K$^{-1} \cdot$ km$^{-1} \cdot$ s (excluding He) for simplicity.

Figure 1 shows a strong correlation between SFR surface density and gas surface density, with slope $N \approx 1.4$–1.5, indicating that star formation increases exponentially with gas surface density. This means gas-to-star conversion efficiency (star formation efficiency SFE = SFR/M_{gas}) differs across different star formation systems. For example, ultraluminous infrared galaxies have star formation efficiencies nearly two orders of magnitude (~100×) higher than nearby normal disk galaxies. Why does the same gas mass produce such different star formation efficiencies?

One existing model [**?**] explains this by assuming star formation follows: $\rho_{SFR} = f_{H_2}\epsilon_{ff}\rho_{gas}/t_{ff}$, where f_{H_2} is the molecular gas fraction in total gas, ϵ_{ff} is star formation efficiency per free-fall time, ρ_{gas} is gas density, and t_{ff} is the free-fall time satisfying $t_{ff} \propto \rho_{gas}^{-0.5}$, yielding $\rho_{SFR} \propto \rho_{gas}/t_{ff} \propto \rho_{gas}^{1.5}$, i.e., $N = 1.5$. Note this model assumes the star formation timescale is the free-fall timescale, but besides gravity, molecular clouds also contain turbulence and magnetic fields—whether star formation timescales are truly free-fall timescales remains to be verified.

Several details in Figure 1 warrant attention: metal-poor galaxies and low surface brightness galaxies deviate from the main body, likely because the same X_{CO} was adopted for different galaxies. If these metal-poor galaxies used larger X_{CO} values, they would lie closer to the main body [**?**]. Different X_{CO} values also significantly affect the slope: using X_{CO} values five times lower in starburst galaxies increases the overall K-S law slope from 1.4–1.5 to 1.7–1.9 [**?**]. Recent galaxy-scale K-S law studies include: Liu et al. [**?**] used radio radiation as an SFR probe to analyze 181 galaxies (115 normal galaxies, 66 (ultra)luminous infrared galaxies), finding slope N is sensitive to X_{CO} values, which can increase N from 1.14 to 1.6; Ballantyne et al. [**?**] numerically simulated 132 Eddington-limited star-forming disks with radii of 10–100 pc, showing that under Eddington limits, slope N strongly depends on optical depth, and that using CO as a probe to derive gas surface density is unsuitable for warm, dense disks, requiring higher-order dense molecular probes.

4.2 Spatially Resolved K-S Law

Galaxy-scale K-S laws show average properties of SFR and gas mass (density) on galactic scales, averaging the physical properties of star formation activities and gas probes. However, to further understand the specific physical mechanisms of gas evolution into stars, observations of gas-SFR relationships on smaller scales (sub-galactic and giant molecular cloud scales) are necessary. Improved telescope resolution and sensitivity have enabled such studies. Bigiel et al. [**?**] and Schruba et al. [**?**] used the VLA (Very Large Array) and IRAM 30 m telescope to map CO(2-1) and HI in nearby normal disk galaxies, comparing SFR with atomic gas (THINGS survey), molecular gas (HERACLES survey, ^{12}CO(2-1)), and total gas. They found atomic gas traced by HI shows little relation to star formation, while H_2-traced molecular gas correlates linearly with star formation on sub-galactic scales. However, on smaller scales (tens of pc), the H_2-star formation relationship breaks down. For even smaller structures like dense cores (pc scale), H_2 and star formation show no clear correlation.

In fact, not all giant molecular clouds have active star formation. Star formation is influenced not only by gravity but also supported by turbulence and magnetic fields, which vary significantly across different molecular clouds. On giant molecular cloud scales, the densities of H_2 gas traced by ^{12}CO(1-0) and ^{12}CO(2-1) are insufficient for star formation, making giant molecular clouds suboptimal units for studying star formation.

4.3 K-S Law for Dense Gas Probes

Studies show that within the Milky Way, star formation efficiency in dense gas is much higher than in giant molecular clouds. This dense gas has densities of $\sim 10^4$-10^6 cm^{-3}, 2-4 orders of magnitude higher than average giant molecular cloud densities, and its star formation efficiency is also much higher than that of total molecular gas traced by CO [**?**, **?**]. This evidence suggests star formation is physically more closely related to dense molecular gas. Gao and Solomon [**?**] observed HCN(1-0) lines in 65 nearby galaxies, finding a strong linear relationship ($N = 1$) between HCN-traced dense gas and SFR (see Figure 2 [Figure 2: see original paper]). This relationship extends to dense molecular clumps within the Milky Way (see Figure 3 [Figure 3: see original paper]) and possibly to high-redshift quasars (see Figure 3b) [**?**]. Liu et al. [**?**] used radio radiation as an SFR probe to analyze 181 galaxies (115 normal galaxies, 66 (ultra)luminous infrared galaxies), also finding a strong linear relationship between dense gas surface density and SFR: $\dot{M}_\odot \approx 1.2 \times 10^{-8} M_{dense}$, where $\dot{M}_\odot$ is in $\mathrm{M}_\odot \cdot \mathrm{a}^{-1}$ and M_{dense} is in $\mathrm{M}_\odot$. This linear relationship is interpreted as follows: assuming gas mass, SFR, and star formation efficiency are similar across different dense clumps—the "basic unit" of star formation [**?**]—the collective behavior of these basic units is identical across physical scales. Thus, SFR and dense gas mass can be considered as simply accumulating by adding more basic units. Note that for extragalactic systems (especially starburst and ultraluminous infrared galaxies) with extreme physical environments, this hypothesis remains to be verified.

Lada et al. [?] mapped 11 nearby molecular clouds using HCN as a dense gas probe (see Figure 4 [Figure 4: see original paper]), combining with Gao and Solomon' s galaxy data [?] to obtain: $\dot{M}_{\odot} = 4.6 \times 10^{-8} f_{dense} M_{mol}$, where f_{dense} is the dense gas fraction in molecular clouds and M_{mol} is molecular cloud mass. Figure 4a shows the correlation between molecular cloud SFR and molecular cloud mass. Solid black dots represent high-density gas, while open circles represent low-density gas; three dashed lines from top to bottom indicate dense gas fractions of 100%, 10%, and 1%. Figure 4b is similar but includes Gao and Solomon' s galaxy data [?]. Figure 4 demonstrates that, whether for individual molecular clouds or entire galaxies, star formation follows a fundamental principle: the rate at which molecular gas converts to stars depends on the dense gas mass fraction in molecular clouds. Moreover, if the dense gas fraction is the same, the correlation between SFR and dense gas is linear in both molecular clouds and galaxies. Additionally, Evans et al. [?] used radio radiation (free-free emission) and HCN as probes to study massive dense clumps, finding a good linear relationship between dense gas mass and SFR. Zhang et al. [?] used dense molecular probes (CS, HCN, HCO^+) to study star formation laws in 20 nearby galaxies, finding that as long as molecular density exceeds 10^4 cm^{-3} (dense gas), SFR linearly correlates with surface density traced by dense gas probes.

Vutisalchavakul et al. [?] found that SFR also linearly correlates with surface density traced by dense gas probes in more distant and denser cores within the Galactic plane. They further demonstrated that dense gas star formation efficiency is constant from nearby molecular clouds to Galactic plane molecular clouds, and from nearby galaxies to distant (U)LIRGs (as shown in Figure 5a [Figure 5: see original paper]). Conversely, molecular gas star formation efficiency shows greater dispersion (as shown in Figure 5b). This indicates that dense gas is more closely connected to star formation than molecular gas. The overall star formation efficiency in molecular clouds is lower than in dense gas, likely because most molecular clouds are not gravitationally bound while dense gas is gravitationally bound [?]. This also explains the tight connections between infrared luminosity in molecular cloud clumps, dense gas probe luminosity (i.e., the dense gas star formation law), and gas luminosity with virial mass of dense clumps [?]. Additionally, feedback from early star formation affects gas structure in molecular cloud clumps, reducing SFR and thus affecting star formation efficiency [?].

However, recent studies have questioned the dense gas star formation law. Stephens et al. [?] proposed that most HCN(1-0) brightness in the Milky Way comes from subthermal emission in diffuse gas rather than dense gas. When observing galaxies as whole systems, HCN luminosity inevitably includes contributions from diffuse gas, requiring deeper interpretation of the linear relationship found by Wu et al. [?] for massive gas clumps in the Milky Way and extended to other galaxies. Recent work indicates HCN emission may originate from more diffuse regions, with studies focusing on clouds in the solar neighborhood (extending to Orion) [?, ?]. Luong et al. [?] extended these conclusions to the M17 nebula. Evans et al. [?] subsequently found

HCN(1-0) and HCO^+(1-0) emission at lower densities (50–100 cm^{-3}). Since low-density regions dominate most molecular clouds, they can dominate total probe luminosity. These studies question whether dense gas probe luminosity primarily originates from dense gas, challenging the effectiveness of commonly used dense gas probes like HCN and HCO^+ for tracing dense gas mass—an issue requiring further investigation.

Additionally, Mills and Battersby [**?**] used 12 m Mopra telescope and Herschel Hi-GAL data to study the correlation between HCN(1-0) and dense gas mass in the Galactic center, finding good correlation on ~10 pc scales of individual molecular cloud cores. However, some clouds show deviations of about 0.75 dex, possibly due to infrared pumping exciting HCN emission and HCN self-absorption affecting the IR/HCN(1-0) ratio. Therefore, multi-line observations with different molecular probes will be effective for demonstrating consistency among dense gas tracers in galaxies. Li et al. [**?**] observed the relationship between dense gas probe luminosity and infrared luminosity using multiple dense gas probes (HCN(1-0), HCO^+(1-0), HNC(1-0), and CS(3-2)). They used the IRAM 30 m telescope to observe 70 nearby galaxy sources with different infrared luminosities at 3 mm and 2 mm bands, finding tight linear correlations between dense gas probe brightness and infrared luminosity, with no systematic differences among probes in tracing molecular gas.

Compared to HCN(1-0), HCO^+(1-0), and other lines, their isotopologue lines typically have lower optical depths and can trace denser gas due to lower isotopologue abundances and higher excitation densities. Zhang et al. [**?**] used the APEX 12 m submillimeter telescope to observe HCN(4-3), HCO^+(4-3), and CS(7-6) lines (tracing gas densities up to 10^6 cm^{-3}) in 20 nearby star-forming galaxies, finding linear correlations between luminosities of these three molecular lines and infrared luminosity, consistent with HCN(1-0) and CS(5-4) results. Liu et al. [**?**] conducted large-sample CO multi-transition ($J = 4-3$-$12-11$) observations of Milky Way molecular clouds, 167 nearby galaxies, and high-redshift star-forming galaxies, finding tight linear correlations between luminosities of all nine detected high-J ($J > 4$) CO lines and far-infrared luminosity. Liu et al. [**?**] used molecular probes $H^{13}CO^+$ and $H^{13}CN$ to trace dense gas mass in 146 star-forming regions in the Milky Way, finding that dense gas traced by $H^{13}CO^+$ and $H^{13}CN$ still shows linear relationships with SFR.

5 Summary and Outlook

The K-S law provides the most classic and significant statistical characterization of the physical properties of star-forming clumps, describing the empirical relationship between star formation rate surface density Σ_{SFR} and total gas surface density Σ_{gas} ($\Sigma_{gas} = \Sigma_{HI} + \Sigma_{H_2}$). The K-S law averages disk galaxies as a whole, providing a direct quantitative method for linking gas emission to star formation activity. This law can be extended to distant, hard-to-observe high-redshift galaxies, laying the foundation for theoretical simulations and calculations of galaxy evolution and large-scale structure evolution. However, cur-

rent research has not yet provided a clear description of the nature of the linear relationship in the dense gas probe star formation law. Key areas for further exploration include:

(1) The definition of "dense" gas traced by dense gas probes—i.e., the actual spatial distribution of dense gas. Studies suggest that commonly used dense gas probes (e.g., HCN(1-0)) may derive most of their luminosity from subthermal emission in diffuse gas rather than dense gas, contradicting their assumed role as dense gas tracers [**?**, **?**]. Stephens et al. [**?**] proposed that infrared luminosity from massive star-forming clumps and dense gas probe HCN luminosity may represent only a small fraction of the entire galaxy, with HCN emission primarily originating from low-mass star-forming clumps or subthermal emission. Evans et al. [**?**] used molecular probes HCN(1-0) and HCO^+(1-0) to conduct high-resolution mapping of six distant ($d \approx 3.5$–10.4 kpc) molecular clouds, finding that gas below the $A_V \approx 8$ mag threshold contributes a large fraction of total luminosity –i.e., HCN and HCO^+ emission exists at lower densities $n = 50$–100 cm^{-3}. Since low-density regions dominate most molecular clouds, they could dominate total probe luminosity. Whether different dense gas probes trace the same gas structures in molecular clouds, what fraction of total gas luminosity originates from the smallest units of clustered star formation (dense cores) versus subthermal emission from diffuse gas—these questions remain at a superficial, phenomenological level in current research. Although limited, such studies have raised doubts about whether dense gas probes effectively trace dense gas.

(2) The impact of feedback from young star formation on gas spatial distribution. Gas expansion and radiation pressure from active objects like forming stars significantly affect gas distribution. These factors vary across different molecular cloud regions, creating complexity in gas spatial distribution and star formation efficiency [**?**]. High CCH/$H^{13}CO^+$ and $H^{13}CN/H^{13}CO^+$ ratios are good probes of photodissociation regions [**?**, **?**]. Figure 6 [Figure 6: see original paper] shows these ratios in molecular cloud G9.62+0.19 [**?**]. The ionized hydrogen region within the circular contour in the lower right compresses the approximately spherical molecular cloud into a hemispherical shape, ionizing or dispersing the other half. The photodissociation region in the ionized hydrogen region is clearly connected to the remaining molecular cloud and filamentary structures, indicating their interaction and demonstrating that feedback from young stars affects gas distribution. Zhang et al. [**?**] analyzed dense gas probe CS(2-1), classifying regions by the presence or absence of radio recombination line H40α emission (tracing HII regions, i.e., young stars) to study whether early star formation affects SFR. Star-forming regions with H40α emission show sublinear star formation laws (lower SFR), as shown in Figure 7a [Figure 7: see original paper]. Figure 7b shows L_{bol}/L_{CS} versus galactocentric distance. Regions with HII regions have higher L_{bol}/L_{CS} ratios than those without, and more evolved star-forming regions may have consumed more

gas, leading to higher L_{bol}/L_{CS} values that don' t change with Galactic environment. This demonstrates that feedback from young star formation affects gas distribution, but different factors have varying impacts in different molecular cloud regions, creating complexity that requires more high-resolution data for further study.

Current observations are generating more data: (1) Herschel Hi-GAL is an infrared survey project specifically targeting the Galactic plane, using the Herschel Space Observatory' s powerful capabilities for unprecedented detailed observations of star-forming regions in the Milky Way. By observing dust continuum emission and molecular lines in the Galactic plane, Herschel Hi-GAL obtains detailed information on physical parameters (temperature, density, and chemical composition) of star-forming regions, providing important data for validating and developing star formation theories, including star formation laws. (2) The MALATANG survey [**?**] uses the East Asian Observatory' s JCMT 15 m submillimeter telescope to map HCN(4-3) and HCO^+(4-3) in a sample of nearby star-forming galaxies, covering approximately 50% of the galaxies' optical radii. MALATANG is the first international project to systematically map high-J dense molecules in a large galaxy sample, providing a unique observational database for star formation research, including dense molecular gas excitation, star formation and its feedback, and especially dense gas and star formation law studies, when combined with multi-wavelength data from molecular and atomic gas, radio continuum, infrared spectroscopy, and X-ray radiation. (3) The ATOMS project [**?**] uses the world's largest millimeter-wave array ALMA to obtain high-resolution observations of different molecular probes (HCO^+, HCN, $H^{13}CO^+$, CS, $H^{13}CN$, etc.) in 146 star-forming regions in the Milky Way, plus radio recombination line H40α and 3 mm continuum observations. ATOMS sources form a diverse sample with wide spatial distribution in the Milky Way, suitable for statistical studies of star formation under different physical conditions. The ATOMS project provides large-sample, high-resolution gas mapping for direct study of dense gas probes. These observational projects will play important roles in further research on the definition of dense gas traced by dense gas probes, star formation efficiency, feedback from young star formation, and explanations for linear phenomena in dense gas star formation laws.

References

[1] Schmidt M. ApJ, 1959, 129: 243
[2] Meidt S E, Schinnerer E, García-Burillo S, et al. ApJ, 2013, 779: 45
[3] Gao Y, Solomon P M. ApJ, 2004, 606: 271
[4] Gilmore G, Wyse R F G, Norris J E. ApJ, 2002, 574: L39
[5] Gratton R G, Carretta E, Bragaglia A. A&A Rev, 2012, 20: 50
[6] Cox D P. ARA&A, 2005, 43: 337
[7] Field G B, Goldsmith D W, Habing H J. ApJ, 1969, 155: L149
[8] Robert C, Kennicutt Jr, Evans N J. ARAA, 2012, 50: 531
[9] Lombardi M, Lada C J, Alves J. A&A, 2010, 512: A67

[10] Kainulainen J, Beuther H, Henning T, et al. A&A, 2009, 508: L35
[11] van Dishoeck E F, Black J H. ApJ, 1988, 334: 771
[12] Hollenbach D J, Tielens A. ARA&A, 1997, 35: 179
[13] Williams J P, Blitz L, McKee C F. Protostars and Planets IV, Arizona: The University of Arizona Press, 2000: 97
[14] Draine B T. ARA&A, 2003, 41: 241
[15] Pritchard J, Loeb A. Reports on Progress in Physics, 2012, 75: 8
[16] Papadopoulos P, Van der Werf, Xilouris E M, et al. MNRAS, 2012, 426: 2601
[17] Bolatto A D, Wolfire M, Leroy A K. ARA&A, 2013, 51: 207B
[18] Dickman R L. ApJ, 1975, 202: 50
[19] Dickman R L. ApJS, 1978, 37: 407
[20] Dame T M, Hartmann D, Thaddeus P. ApJ, 2001, 547: 792
[21] Strong A W, Mattox J R. A&A, 1996, 308: L21
[22] Bolemen H. ARA&A, 1989, 27: 469
[23] Maloney P, Black J H. ApJ, 1988, 325: 389
[24] Shetty R, Glover S C, Dullemond C P, et al. MNRAS, 2011, 412: 1686
[25] Leroy A K, Bolatto A, Gordon K, et al. ApJ, 2011, 737: 12
[26] Larson R B. MNRAS, 1981, 194: 809
[27] Solomon P M, Rivolo A R, Barrett J, et al. ApJ, 1987, 319: 730
[28] Dickman R L, Snell R L, Schloerb F R. ApJ, 1986, 309: 326
[29] Bryant P M, Scoville N Z. ApJ, 1999, 117: 2632
[30] Downes D, Solomon P M. ApJ, 1998, 507: 615
[31] Gao Y, Solomon P M. ApJS, 2004, 152: 63
[32] Gao Y, Solomon P M. ApJ, 2004, 606: 271
[33] Lada E A. ApJ, 1992, 393: L25
[34] Goldsmith P F, Heyer M, Narayanan G. ApJ, 2008, 680: 428
[35] Heiderman A, Evans N J, Allen L E, et al. ApJ, 2010, 723: 1019
[36] Lada C J, Lombardi M, Alves J F. ApJ, 2010, 724: 687
[37] Rosolowsky E, Pineda J E, Gao Y. MNRAS, 2011, 415: 1977
[38] Bussmann R S, Narayanan D, Shirley Y L, et al. ApJ, 2008, 681: L73
[39] Garcia-Burillo S, Usero A, Alonso-Herrero A, et al. A&A, 2012, 539: A8
[40] Gracia-Carpio J, Garcia-Burillo S, Planseas P, et al. A&A, 2008, 479: 703
[41] Juneau S, Narayanan D T, Moustakas J, et al. ApJ, 2009, 707: 1217
[42] Sakamoto K, Aalto S, Evans A S, et al. ApJ, 2010, 725: L228
[43] Leen T M, Graff M M. ApJ, 1988, 325: 411
[44] Van der Tak, Boonman A, Braakman R, et al. A&A, 2003, 412: 133
[45] Sandstrom K M, Leroy A K, Walter F. ApJ, 2013, 777: 617
[46] Draine B T. Physics of the Interstellar and Intergalactic Medium, Princeton: Princeton University Press, 2011: 45
[47] Salpeter E. ApJ, 1955, 121: 161
[48] Kroupa P, Weidner C, Pflamm-Altenburg J, et al. arXiv:1112.3340, 2011
[49] Zhang Z Y. PhD Thesis. Nanjing: Purple Mountain Observatory, Chinese Academy of Sciences, 2014: 43
[50] Kennicutt Jr. ApJ, 1998, 498: 5411
[51] Kennicutt R C, Evans N J. ARA&A, 2012, 50: 531

[52] Kennicutt R. ARA&A, 1998, 36, 189
[53] Kennicutt R, Ii N. ARA&A, 2012, 50: 531
[54] Wyder T, Martin D, Barlow T, et al. ApJ, 2009, 696: 1834
[55] Narayanan D, Krumholz M, Ostriker E, et al. MNRAS, 2012, 421: 3127
[56] Narayanan D, Krumholz M R, Ostriker E C, et al. MNRAS, 2011, 418: 664
[57] Liu L, Gao Y, Greve. ApJ, 2015, 805: 31
[58] Ballantyne D, Armour J, Indergaard J. ApJ, 2013, 765: 1035
[59] Bigiel F, Leroy A, Walter F, et al. AJ, 2008, 136: 2846
[60] Schruba A, Leroy A K, Walter F, et al. AJ, 2012, 143: 138
[61] Jingwen W, Ii N, Gao Y, et al. ApJ, 2005, 635: 173
[62] Wu J, Evans N J, Shirley Y L, et al. ApJS, 2010, 188: 313
[63] Gao Y, Solomon P M. ApJS, 2004, 152: 63
[64] Gao Y, Carilli C L, Solomon P M, et al. ApJ, 2007, 660: L93
[65] Beuther H, Linz H, Henning T, et al, eds. Massive Star Formation: Observations Confront Theory, San Francisco: Sheridan Books, 2008: 401
[66] Lada C, Forbrich J, Lombardi M, et al. ApJ, 2011, 745:190
[67] Evans N J, Heiderman A, Vutisalchavakul N. ApJ, 2014, 782: 114
[68] Zhang Z, Gao Y, Henkel C, et al. ApJ, 2014, 784: 31
[69] Vutisalchavakul N, Evans I, Heyer M. ApJ, 2016, 831: 73
[70] Dobbs C, Burkert A, Pringle J. MNRAS, 2011, 413: 2935
[71] Barnes P, Hernandez A, O' Dougherty S, et al. ApJ, 2016, 831: 67
[72] Stephens I, Jackson J, Whitaker J, et al. ApJ, 2016, 824: 29
[73] Pety J, Guzmán V, Orkisz J, et al. A&A, 2017, 599: A98
[74] Kauffmann J, Goldsmith P, Melnick G, et al. A&A, 2017, 605: L5
[75] Luong N, Nakamura Q, Sugitani F, et al. ApJ, 2020, 891: 66
[76] Evans N J, Kim K T, Wu J, et al. ApJ, 2020, 894: 103
[77] Mills E, Battersby C. ApJ, 2017, 835: 76
[78] Li F, Wang J, Gao F, et al. MNRAS, 2021, 503: 4508
[79] Zhang Z Y, Henkel C, Gao Y, et al. A&A, 2014, 568: A122
[80] Liu D, Gao Y, Isaak K, et al. ApJ, 2015, 810: L14
[81] Liu T, Evans N J, Kim K T, et al. MNRAS, 2020, 496: 2821
[82] Krumholz M R, Bate M R, Arce H G, et al. Protostars and Planets VI, Arizona: The University of Arizona Press, 2014: 243
[83] Liu T, Evans N J, Kim K-T, et al. MNRAS, 2020, 496: 2790
[84] Zhang C, Evans N J, Liu T, et al. MNRAS, 2022, 510: 4998
[85] Jiang X-J, Greve T R, Gao Y, et al. MNRAS, 2020, 494: 1276

Note: Figure translations are in progress. See original paper for figures.