Postprint: Deimos Dynamical Model and Precise Orbit Determination

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Abstract

The model comprehensively accounts for factors influencing the motion of Phobos, including the two-body motion model of Phobos and Mars, the Martian gravity field, three-body perturbations from major solar system bodies, general relativistic effects, Martian solid tides, and Phobos libration, thereby establishing a Phobos dynamical model, and extends the precision orbit determination methodology for artificial satellites to the natural celestial body Phobos, establishing an adjustment model for fitting the dynamical model to observational data. Three mainstream integration algorithms in orbital research—the 8th-order RKF (Runge-Kutta-Fehlberg), 12th-order ABM (Adams-Bashforth-Moulton), and Gauss-Radau—are employed to solve the Phobos orbit, computational efficiencies are compared, and the calculation results from the complete relativistic model used in orbit computation are contrasted with those from the simplified relativistic model. Numerical experimental results indicate that the established Phobos dynamical model and adjustment model are stable and reliable; under identical experimental conditions, the three integration algorithms achieve comparable computational accuracy, with the 12th-order ABM integration algorithm demonstrating the highest computational efficiency; and the two relativistic models yield comparable results.

Full Text

Preamble

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Research on the Dynamics Model and Precision Orbit Determination of Deimos

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Abstract

This paper comprehensively considers factors influencing the motion of Deimos, including the two-body motion model between Deimos and Mars, Mars' gravity field, third-body perturbations from major solar system bodies, general relativistic effects, Martian solid tides, and the libration of Deimos, to establish a dynamical model for Deimos. The methodology of precision orbit determination for artificial satellites is extended to the natural satellite Deimos, and an adjustment model for fitting dynamic model data is established. Three mainstream integration algorithms in orbital research—8th-order RKF (Runge-Kutta-Fehlberg), 12th-order ABM (Adams-Bashforth-Moulton), and Gauss-Radau—are employed to solve the orbit of Deimos, and their computational efficiency is compared. The calculation results using the complete relativistic model and the simplified relativistic model in orbit computation are also compared. Numerical experimental results demonstrate that the established dynamical model and adjustment model for Deimos are stable and reliable. Under equivalent experimental conditions, the computational accuracy of the three integration algorithms is comparable, with the 12th-order ABM integration algorithm exhibiting the highest computational efficiency. The calculation results from the two relativistic models are comparable.

Keywords: Deimos; dynamics model; adjustment model; numerical integration; ephemeris

Classification code: P134 Document code: A

1 Introduction

The Mars system in the solar system shares remarkably similar characteristics with the Earth-Moon system. On the one hand, Mars and Earth have comparable rotation periods and orbital inclinations; on the other hand, both Martian moons and the Moon are in tidally locked states with their central bodies. Based on these features, research on the Mars system contributes to humanity's in-depth study of terrestrial planetary systems. Consequently, the Mars system has consistently been a significant target for deep space exploration.

Human understanding of Martian moons began in 1877 when Asaph Hall of the U.S. Naval Observatory first discovered Mars' two natural satellites, naming them Phobos and Deimos. Phobos, the larger of the two Martian moons, has been extensively studied regarding its orbit and gravity field [1-3]. Deimos, the smaller Martian satellite, is known to orbit Mars at a high altitude of 23,458 km. Compared to Phobos, Deimos has received less public research attention, making it the primary focus of this study. Its basic physical parameters are listed in Table 1 [4]. Exploration of Phobos and Deimos not only facilitates deeper understanding of terrestrial planetary system formation and evolution but also provides crucial clues for studying solar system formation and evolution. To date, researchers have launched multiple Mars probes and conducted extensive observations of the two Martian moons. Approximately 90 years after their discovery, Mariner 4 flew by Mars in 1965. Subsequently, in 1969, Mariners 6 and 7 executed additional Mars flyby missions over the Martian equator and south polar region; however, the approach and flyby images were not archived, yielding no data on the Martian moons. Had this data been preserved, it would have extended the observational time span of the moons by two years, which is significant for determining their orbits. In 1971, the first Mars orbiter, Mariner 9, was launched, obtaining 214 grayscale images of Phobos and Deimos during close flybys—the earliest spacecraft observations of the Martian moons. These images revealed that both satellites have highly irregular shapes, while studies of their orbital velocities confirmed that Phobos is spiraling inward toward Mars with increasing speed, whereas Deimos is receding from Mars due to tidal forces. In the late 1970s, the Soviet Union launched two orbiters—Viking 1 and Viking 2—each carrying a lander. The landers observed Phobos and Deimos from the Martian surface. Additionally, the Viking program elevated the exploration of Phobos and Deimos to a major scientific objective during its extended mission phase, conducting close flybys of the Martian moons. The Viking orbiter and lander dataset remained the most comprehensive data on Martian moons until recently, when it was surpassed by ESA's Mars Express mission.

Currently, the Mars Express mission launched by ESA has acquired the most extensive observational data on the Martian moons, profoundly influencing our understanding of their orbits and the development of their ephemerides. Pathfinder lander data has played a crucial role in studies of Mars' rotation and gravity field, and its generated gravity field and rotation model MRO120F will be used in this research [5]. China successfully launched its first Mars probe, "Tianwen-1," in July 2020, achieving the three-step milestone of "orbiting, landing, and roving" for the first time [6]; its observational data will be utilized for research on Mars' internal mass distribution and gravity field [7]. NASA held three international conferences in 2006, 2011, and 2016 to discuss scientific and exploration strategies for Phobos and Deimos [8]. Japan has also formulated exploration plans for Mars and its two moons [9].

In various exploration activities of extraterrestrial bodies, studying their orbital motion constitutes important research content, holding significant implications for understanding their internal structure and physical parameters. Regarding planetary orbit computation methods, celestial mechanics offers both analytical and numerical approaches. With the continuous development of deep space exploration, particularly the increase in observational data and computational power, numerical methods have gradually replaced analytical methods as the primary approach in planetary orbit computation due to their high accuracy and simple formulation. In this context, multiple international institutions have released several versions of numerical ephemeris data products for Martian moons based on existing Mars probe orbital data. Currently, mainstream ephemeris products for Martian moons include the JPL Mars satellite ephemeris Mars097 [1], the Russian EPM (Ephemeris of Planets and the Moon) series [10], and the French NOE series ephemerides [11].

This paper presents preliminary exploration for establishing China's independently developed high-precision ephemeris for Martian moons: it comprehensively considers factors influencing Deimos' motion to establish a high-precision dynamical model for Deimos; it employs the least squares method to establish an adjustment model for data fitting; it explores and studies the selection of numerical solution methods for the dynamics model; and it compares the computational efficiency of three integration algorithms—8th-order RKF (Runge-Kutta-Fehlberg) [12], 12th-order ABM (Adams-Bashforth-Moulton) [13], and Gauss-Radau [14]—to provide reference for method selection in solving dynamical equations for celestial bodies like Martian moons.

2 Dynamics Model of Deimos

Compared to the central body Mars, Deimos has an extremely small mass. Therefore, in the model establishment process, Deimos can be treated as a point mass. In modeling the motion of planetary natural satellites, the gravitational force from the central body constitutes the primary source of their dynamics, with natural satellites moving primarily under the control of the central body's gravity. The equation of motion can be expressed as: dt2 = (cid:0)µ r3 + F, where r is the position vector of the natural satellite relative to the central body; r is the distance between the natural satellite and the central body; µ = G(M + m), with G being the universal gravitational constant, and M and m representing the masses of the central body and natural satellite, respectively; the first term on the right side of the equation represents the central body's gravity; and F represents the sum of all other perturbing accelerations, mainly including the central body's non-spherical perturbation, third-body perturbations from major solar system bodies, general relativistic effects, etc. At the J2000 epoch, the perturbation magnitudes of each force model (relative to the ratio of Mars' gravitational force on Deimos) are shown in Table 2 . This study aims to establish a high-precision dynamical model for Deimos in the ICRF (International Celestial Reference Frame).

2.1 Two-Body and Three-Body Models

Natural satellites move primarily under the gravitational control of their central body. If perturbing forces are neglected and only the central body's gravity is considered, the equation of motion can be written as: dt2 = (cid:0)µ. In celestial mechanics, the motion described by this equation is called the two-body problem, for which analytical solutions can be directly obtained. When Deimos moves, in addition to the gravitational force from the central body Mars, it also experiences gravitational perturbations from other solar system bodies such as the Sun, Jupiter, Saturn, and Earth. These perturbations are generally referred to as third-body perturbations. In the solar system barycentric reference frame, the position vectors of Mars' center, Mars to Deimos, and perturbing body j are rm, r, and rj, respectively. In this inertial frame, the acceleration of Deimos is as follows: a = (cid:0) GMmars (cid:0) GMj ∆j = r (cid:0) rj. Equation (3) describes the acceleration of Deimos relative to the solar system barycenter. Converting this acceleration to the Mars-centered inertial frame yields: a = (cid:0) GMmars (cid:0) GMj. In Equation (5), the first term represents the two-body acceleration between Mars and Deimos, while the second term represents the acceleration exerted on Deimos by the perturbing body.

The planetary positions and main physical parameters used in this model are derived from ephemeris DE430 [15], and the gravitational constants of major solar system bodies are listed in Table 3 .

This study analyzes the perturbation magnitudes of solar system bodies on Deimos' gravity, selecting several bodies with larger perturbation magnitudes (Sun, Earth, Jupiter, Saturn). Starting from the J2000 epoch, the closest distances between Deimos and these bodies are calculated over one Martian orbital period (687 days), with results shown in Table 4 . Subsequently, using the times of closest approach between each perturbing body and Deimos as integration starting points, the gravitational influence on the dynamical model is calculated, with results shown in Figure 1 [Figure 1: see original paper]. The results demonstrate that the influence of major solar system bodies cannot be neglected as integration time increases.

2.2 Mars Gravity Field Perturbation Model

Due to Mars' irregular shape and non-uniform mass distribution, it cannot be treated as an ideal uniform sphere when calculating its gravitational effect on Deimos, and non-spherical perturbations must be considered. The gravitational potential function can be expanded using spherical harmonics as follows [13]: ¯Pnm(sin L) ¯Cnm cos mλ + ¯Snm sin mλ. Here, GM is Mars' gravitational constant, Rm is Mars' equatorial radius, ¯Pnm are fully normalized Legendre functions, ¯Cnm and ¯Snm are fully normalized spherical harmonic coefficients of the Mars gravity field model, and λ, L, r are the longitude, latitude, and distance to Mars' center of Deimos in the Mars-fixed coordinate system. The primary model adopted in this study is the Mars rotation and gravity field model MRO120F generated by Konopliv using Pathfinder lander data [5]. When applying formula (6) in practice, the coordinates of Deimos in the inertial frame must first be transformed to the Mars-fixed coordinate system, then transformed back to the inertial frame for integration. The transformation method is [5]:

RIN = RZ((cid:0)N)RX((cid:0)J)RZ((cid:0))RX((cid:0)I)RZ((cid:0))RBF,

RX( ) = RZ( ) = cos 0 (cid:0) sin sin cos cos (cid:0) sin sin cos 0

where RIN and RBF represent the coordinates of Deimos in the inertial and Mars-fixed frames, respectively; N is the angle between the vernal equinox and the intersection of the J2000 mean Earth equator plane (Earth-mean-equator of J2000, EME2000) and the J2000 mean Mars equator plane; J is the inclination of the mean Mars orbital plane relative to the EME2000 plane; is the angle from the intersection of EME2000 and the mean Mars orbital plane to the intersection of the mean Mars orbital plane and the true Mars equatorial plane; I is the inclination of the true Mars equator relative to the mean Mars orbital plane; and is the angle from the intersection of the mean Mars orbital plane and the true Mars equator to Mars' prime meridian.

2.3 Relativistic Effects Perturbation Model

Satellites are affected by general relativistic effects during their motion. Since Deimos' orbital eccentricity is extremely small, its orbit can be considered near-circular. Under this condition, the acceleration produced by Mars' general relativistic effects on Deimos is [16]: arel = (cid:0) GM jrj3 r. Here, GM is Mars' gravitational constant, v is Deimos' velocity in the Mars inertial frame, r is Deimos' position vector relative to Mars, and c is the speed of light. This model is a simplified relativistic model, which offers computational simplicity compared to the complete relativistic model. This simplified model is the relativistic effects model used in establishing the dynamics model of this paper. The difference in data fitting between the simplified model and the complete relativistic model will be discussed in Section 3.2. Using the J2000 epoch as the integration starting point, the influence of general relativistic effects on orbit computation results is shown in Figure 2 [Figure 2: see original paper]. The results indicate that the error caused by general relativistic effects increases with integration time.

2.4 Solid Tide Perturbation Model

Due to the gravitational influence of other solar system bodies, Mars' gravity field varies, and these variations directly affect Deimos. Therefore, a solid tide model must be considered in the Deimos dynamics model. For a body with mass Mj that raises solid tides on Mars, the gravitational potential function affecting Deimos is [17]: 3 (ˆrjd (cid:1) ˆr)2 (cid:0) 1 Vtide = jrjdj3. Here, rjd is the position vector from the perturbing body to Deimos, r is the position vector from Mars to Deimos, Rm represents Mars' equatorial radius, and ˆrjd and ˆr are normalized position vectors. The k2 value for Mars is 0.169 [18]. This study primarily considers solid tide effects caused by the Sun and Phobos. As shown in Figure 3 [Figure 3: see original paper], using the J2000 epoch as the integration starting point, the maximum influence of the solid tide model on Deimos' motion is approximately 0.3 m for an integration duration of 50 days. This difference increases further with longer integration times. Therefore, the Martian solid tide effect cannot be neglected when establishing a high-precision dynamics model for Deimos.

2.5 Deimos Libration Model

Based on research regarding satellite libration in reference [1], this paper incorporates the satellite's libration effect into the Deimos dynamics model. Libration describes the spatial oscillation of a satellite's motion. According to Cassini's laws, tidally locked satellites have identical orbital and rotational periods. Under ideal conditions, Deimos always presents the same face toward Mars, with its minimum inertia axis constantly pointing toward Mars' center. However, due to the combined gravitational effects of surrounding bodies, Deimos' rotation is not uniform, and this deviation from uniform rotation is termed libration. The acceleration produced by the libration effect on Deimos can be expressed as [1]:

F = (cid:0) 3 (J2 + 6C22 cos 2) ˆr + 4C22 sin 2 ˆt

= f (cid:0) M + 1 (cid:0) 3(B (cid:0) A) sin M,

where µ represents Mars' gravitational constant, R is Deimos' radius, r is the distance from Mars to Deimos, J2 is Deimos' second-degree zonal harmonic coefficient, C22 is Deimos' second-degree tesseral harmonic coefficient, f and M are the true anomaly and mean anomaly of Deimos' orbit, e is the orbital eccentricity, A, B, and C are Deimos' three principal moments of inertia, ˆr is the unit vector from Mars to Deimos, and ˆt is the unit vector in Deimos' orbital plane perpendicular to ˆr and aligned with Deimos' motion velocity. Using the J2000 epoch as the integration starting point with a time span of 50 days, the results are shown in Figure 4 [Figure 4: see original paper]. The libration effect can introduce errors of approximately 300 m and therefore must be considered in establishing the Deimos dynamics model.

2.6 Model Results

Thus far, we have considered the factors influencing Deimos' motion and established a Deimos dynamics model. In this section, the integration results of the dynamics model are compared with the MARS097 numerical ephemeris for Martian moons released by NASA. It should be noted that the solar system body positions in the MARS097 ephemeris are sourced from DE421 ephemeris, while the masses of Mars and its moons are from the work of Alex et al. [19]. This study uses the same masses for Mars and Deimos as MARS097, but the positions of solar system bodies are sourced from the newer generation DE430 ephemeris.

Using the position and velocity of Deimos from the MARS097 ephemeris at the J2000 epoch as initial conditions, the established dynamics model is numerically integrated and compared with the MARS097 numerical ephemeris, with results shown in Figure 5 [Figure 5: see original paper]. Experimental results demonstrate that the difference between the model calculations and the MARS097 numerical ephemeris is 500 m over a 10-year time span, with the difference growing linearly. This discrepancy may be related to various factors such as the choice of integration step size during integration. In further research, using observational data to fit the model is essential to improve the accuracy of the dynamics model calculations.

2.7 Integrator Selection

In orbit integration, commonly used integration methods internationally include the single-step RKF (Runge-Kutta-Fehlberg), multi-step ABM (Adams-Bashforth-Moulton), and Gauss-Radau methods. To investigate the accuracy and efficiency of different integrators when integrating the dynamics model and to provide guidance for integrator selection in subsequent research on Martian moon numerical ephemerides, we selected these three integrators to perform integration operations on the Deimos dynamics model established in the previous section, obtaining Deimos' motion state at arbitrary times, and conducted comparative analysis of integration results and computational efficiency to select the most suitable integrator.

The Runge-Kutta (RK) method improves upon the traditional Euler integration method and largely eliminates integration errors caused by significant differences in secant slopes. Fourth-order RK uses a weighted average of four slopes of the function to calculate its increment function, achieving computational accuracy comparable to fourth-order Taylor polynomials while offering the advantage of not requiring higher-order derivatives, making it well-suited for application when spacecraft dynamics equations are highly complex. Since conventional RK error control is relatively complicated, Fienga et al. [11] proposed a new RKF algorithm for estimating integration errors. Under conditions requiring higher accuracy, we can employ the more accurate 8th-order RKF algorithm, whose basic principle is the same as the 4th-order RKF algorithm.

ABM is a commonly used multi-step integration method that stores previous integration results to minimize function evaluations. Based on the accuracy requirements of the dynamics model, this study selected the more accurate 12th-order ABM integration algorithm. Its basic principle is as follows: first, the 12th-order Adams-Bashforth method performs integration operations, with its increment function determined by state vectors at 12 given moments and corresponding integration coefficients. These 12 initial state vectors can generally be obtained using fourth-order or higher-precision RKF integration methods. The calculation results are then predicted and corrected by the Adams-Moulton method, with accuracy improved through iterative processes. In actual integration, due to the complexity of the Martian moon dynamics equations, the 12th-order ABM multi-step integration method significantly improves integration efficiency compared to single-step RKF. Additionally, we selected the multi-step Gauss-Radau algorithm for comparison. Using the J2000 epoch as the starting point, the errors in Deimos orbit calculations using the three integration algorithms are shown in Figure 6 [Figure 6: see original paper].

The computational efficiency and error growth rates of the three integrators compared with the MARS097 ephemeris are listed in Table 5 . The results show that for integrating the Deimos dynamics model over 50 days, the error growth rates of the three integration algorithms are comparable, and within the time scale studied in this paper, the computational accuracy of all three algorithms is sufficient. However, with the same integration step size, the 12th-order ABM computation time is approximately half that of Gauss-Radau and one-third that of the RKF algorithm, demonstrating the highest computational efficiency. Therefore, the 12th-order ABM integration algorithm is recommended for subsequent development of Martian moon numerical ephemerides.

3 Adjustment Model

In Chapter 2, we modeled the factors influencing Deimos' motion, including the two-body motion model between Deimos and Mars, Mars' gravity field, third-body perturbations from major solar system bodies, general relativistic effects, Martian solid tides, and Deimos libration, to establish a Deimos dynamics model. In this chapter, we utilize the methodology of precision orbit determination for artificial satellites, using the positions of Deimos from the American MARS097 ephemeris as observational data to fit the Deimos dynamics model. The fundamental principle is to use the least squares method to determine the initial states and parameters of the dynamics model to best fit the observational data.

3.1 Variational Equations

In the dynamics model, the relationship between the orbit obtained by integrating Deimos' acceleration and the state vector at the initial epoch can be described by the state transition matrix Φ, while the relationship with the dynamics model parameters is described by the sensitivity matrix S. The differential equations for the state transition matrix and sensitivity matrix combine to form the variational equations, whose basic form is: d(Φ, S) 03(cid:2)3 13(cid:2)3 6(cid:2)6 (cid:1) (Φ, S) + 03(cid:2)6 03(cid:2)np 03(cid:2)6 6(cid:2)(6+np). Solving the variational equations yields the six motion state quantities of Deimos at the initial epoch. Solving Equation (14) requires calculating the partial derivatives of each perturbing force in the dynamics model with respect to position and velocity. For the two-body problem, the partial derivative of acceleration with respect to position is basic.

The partial derivative matrix of Mars' gravity field with respect to position has symmetry, and the sum of diagonal elements is zero, reducing the independent elements to five. The calculation method is:

Cunningham's derived nth m-order expressions are employed:

[The long set of equations for partial derivatives]

Simultaneously, the partial derivatives of the remaining model components—solid tides caused by the satellite and Sun, general relativity, and satellite libration—with respect to position are solved. These are then substituted into the variational equations for simultaneous integration with the state vector. Finally, the Deimos state data from the MARS097 ephemeris is fitted using the least squares method. Using the J2000 epoch as the starting point, the fitting results over a 10-year span are shown in Figures 7 [Figure 7: see original paper] and 8 [Figure 8: see original paper].

The fitting results show that differences in all three coordinate axes are below 10 m. These minor differences may stem from updates to the Mars gravity field model and different dynamics model parameters. Currently, differences among the three mainstream Martian moon numerical ephemerides have reached the kilometer level, while the integration results from the dynamics model and adjustment model established in this paper differ from the MARS097 ephemeris by less than 10 m in all three coordinate axes over a 10-year span, fully demonstrating the engineering applicability of our results.

3.2 General Relativity Model Analysis

In this study, due to Deimos' small orbital eccentricity, its orbit is approximated as circular. Under this approximation, the acceleration produced by general relativistic effects on Deimos can be calculated using the simplified formula (10). Without this approximation, the acceleration produced by Mars' general relativistic effects on Deimos is [16]: arel = (cid:0) GM (cid:0) v2 r + 4(r (cid:1) v)v. Here, GM is Mars' gravitational constant, r is the distance from Mars to Deimos, r is Deimos' position vector relative to Mars, and v is Deimos' velocity in the Mars inertial frame. During computation, the dynamics model calculates multiple times based on integration time and step size; therefore, compared to the simplified formula (10), Equation (22) increases computational time. Replacing the simplified model in the dynamics model with the complete relativistic model and using the orbit fitted to the MARS097 ephemeris by the original dynamics model as the fitting data for the new model ensures consistency in parameters between the two dynamics models, with integration results reflecting only the differences caused by the two relativistic models. Figure 9 [Figure 9: see original paper] shows the data fitting results using the complete relativistic dynamics model.

Using the J2000 epoch as the integration starting point, the influence of the complete relativistic dynamics model on the integration results over a 10-year span is approximately 5 cm in all three coordinate directions, representing a very small difference from the simplified relativistic dynamics model results. Therefore, when computing Deimos' orbit, it can be treated as a circular orbit for computational efficiency considerations.

4 Summary and Outlook

This paper analyzes the primary factors influencing Deimos' motion, establishes a Deimos dynamics model, and analyzes the magnitudes of various perturbing forces in the model. Comparison with the MARS097 Martian moon numerical ephemeris shows that the integration results of this dynamics model differ from the ephemeris by approximately 500 m over 10 years, with the difference growing linearly. This discrepancy may result from different dynamics model parameters, and the numerical simulation results fully demonstrate that the dynamics model is stable and reliable.

In the subsequent data fitting process, we utilize the methodology of precision orbit determination for artificial satellites, calculate analytical expressions for the partial derivatives of each perturbing force with respect to Deimos' motion state, and establish an adjustment model for Deimos data fitting. The fitting results differ from the MARS097 ephemeris by 10 m, demonstrating the engineering applicability of these results.

Furthermore, in studying the computational efficiency of different integration algorithms, we employed the RKF algorithm, 12th-order ABM, and Gauss-Radau algorithm to integrate the established model. With comparable integration accuracy, the 12th-order ABM algorithm requires the shortest computation time—half that of the Gauss-Radau algorithm used in the French NOE ephemeris and only one-third that of the RKF algorithm. This conclusion can provide reference for subsequent research on dynamical equation integration and numerical ephemeris development for Martian moon-type bodies.

Finally, we used the established model to numerically simulate the calculation results of both complete and simplified relativistic models. The results show that the difference between the simplified model, which treats Deimos' orbit as circular, and the complete relativistic model is approximately 5 cm, with very small differences between the two models. Therefore, the simplified model can be directly used when considering computational efficiency.

The dynamics model in this study is the model currently used in ephemerides and does not include complete rotation like the lunar dynamics model. Therefore, in subsequent research, it is necessary to further improve and develop the dynamics model by considering the complete rotation of Martian moons. The conclusions and methods of this paper lay the foundation for this subsequent work.

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Note: Figure translations are in progress. See original paper for figures.