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Full Text

Preamble

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Prospect of Detecting Magnetic Fields from Strongly-magnetized Binary Neutron Stars

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Abstract

Binary neutron star mergers are unique sources of gravitational waves in multi-messenger astronomy. The inspiral phase of binary neutron stars can emit gravitational waves as chirp signals. The present waveform models of gravitational waves only consider gravitational interaction. In this paper, we derive the waveform of the gravitational wave signal taking into account the presence of magnetic fields. We find that electromagnetic interaction and radiation can introduce different frequency-dependent power laws for both the amplitude and frequency of the gravitational wave. We show from the results of the Fisher information matrix that third-generation observatories may detect magnetic dipole moments if the magnetic field is 10^{17} G.

Key words: stars: neutron – magnetic fields – gravitational waves

1. Introduction

In the 20th century, the observation of the first binary pulsar PSR B1913+16 by Russell A. Hulse and Joseph H. Taylor indicated an energy loss due to gravitational radiation. Later, the GW150914 event marked the first direct detection of a gravitational-wave signal from the coalescence of two black holes and opened the era of GW astronomy. Then GW170817 and a gamma-ray burst announced the first direct observation of GWs from the merger of two neutron stars and subsequent electromagnetic radiation. Moreover, due to the accompanying electromagnetic counterparts, this event offers an independent standard siren measurement. GW170817 can also provide constraints on physics such as the nuclear coupling of light axion fields and the emitting region of the gamma-rays.

The merger of a binary neutron star system can be divided into distinct phases: an inspiral phase where the objects gradually approach, and a merger phase marked by rapid coalescence. The GW signal emitted during the inspiral phase is a source for detectors such as the Advanced LIGO/Virgo detectors, KAGRA (LVK), and future Einstein Telescope (ET), Taiji, and DECIGO. The Advanced LIGO/Virgo are expected to give a merger rate of binary neutron stars (BNSs) ranging from 0.4 to $400\ \mathrm{yr}^{-1}$, with an upper limit of 12,600 $\mathrm{Gpc}^{-3}\ \mathrm{yr}^{-1}$ since there is a considerable amount of the Hubble distribution in our Milky Way. Lately, the study of systems that emit electromagnetic radiation when they coalesce has attracted interest due to the rich information that can be extracted from this scenario.

A neutron star gains its strong magnetic field due to the conservation of magnetic flux after the collapse. Previous observations provided that the magnetic field carried by neutron stars can reach up to 10^{15} G. In some relativistic simulations, the magnetic fields can reach values as strong as 10^{18} G. A neutron star that contains a magnetic field can be treated as a magnetic dipole. The motion of a magnetic dipole and its precession around the axis of rotation can give rise to electromagnetic radiation.

Some research shows that a constant magnetic dipole moving arbitrarily can emit electromagnetic radiation, since the moving magnetic dipole moment is equivalent to a current density vector and the radiated potential can be derived by solving Maxwell equations. On the other hand, when the magnetic dipole moment of a neutron star is misaligned with its spin axis, spin-down will take place due to energy loss. Such neutron stars with changing magnetic dipole moments, observed as pulsars, can support rich observational effects in multi-messenger astronomy.

In the past, most research focused on GW emissions by simulating large sets of BNS systems before, during, and after mergers. Many of these simulations aimed to measure the equations of state, explore the effects of magnetohydrodynamics on GWs during the process, or study the remnants following the merger. Data analyses have been conducted to test general relativity, particularly after the detection of GW170817. The detection of GWs from the inspiral phase demands accurate waveform templates owing to the weak amplitude of the wave.

For the detection of BNSs, most present gravitational waveform models consider only gravitational interaction containing the effects of tidal deformation and spin. Few studies have focused on electromagnetic interaction in detail, which we will introduce and examine. Due to the presence of strong magnetic fields, both the electromagnetic dipolar interaction and energy loss can affect the evolution of orbital separation distance so that the time-evolution of orbital quadrupole moments differs from that in a purely gravitational dominant system. Energy loss of a magnetized neutron star originates not only from GW emission but also from electromagnetic waves. It is certain that the presence of magnetic dipole moments in compact objects can affect the amplitude and changing rate of the angular frequency of GWs. Additionally, relativistic simulations provide a possibility for the presence of strong magnetic fields, starting from which we deduce the frequency-domain waveform model containing the effect of magnetic dipole moments and evaluate the possibility of detection.

This article is organized as follows. In Section 2 we derive the equation of motion for two magnetized neutron stars for the circular-orbit case using the Euler–Lagrange equation. We adopt the post-Newtonian method including dipolar interaction. In Section 3 we calculate the total energy loss rate contributed by both gravitational radiation and electromagnetic radiation averaged over a period in the adiabatic approximation. We use this result to compute the time derivative of orbital radius and further derive the waveform governed by gravitational and electromagnetic interaction by performing the Fourier transform and stationary phase approximation in Section 4. We compare the waveform containing the influence of magnetic field with that predicted for a system dominated by pure gravity using matched-filtering techniques for LIGO and ET in Section 5. In addition, we report in this section the results of evaluating parameter estimation obtained from the Fisher information matrix.

2. Equation of Motion for a Bound Binary System

Throughout this paper, we choose Gaussian units and keep gravitational constant G and the speed of light c within the expressions. For a binary system formed with a long initial separation, it spends most of its lifetime in the inspiral phase during which the lowest-order post-Newtonian approximation is employed to describe it. Thus we consider Keplerian orbits of two magnetized neutron stars with masses m_1 and m_2 and magnetic dipole moments d_1 and d_2, respectively. For simplicity, we assume that the magnetic dipole moments of these two neutron stars are aligned with their respective spin axes and with the angular momenta of orbits. Note that we consider neither higher-order post-Newtonian corrections nor full general relativity since in the long-distance inspiral phase the interactions between electromagnetic and gravitational fields enter the 2.5 post-Newtonian order, which is negligible. In addition, the internal currents that generate the magnetic field of the primary are not distorted significantly by the external field of the secondary and vice-versa since they are well-separated. Furthermore, the contribution of electromagnetic fields to spacetime is much less than the masses and can be neglected.

It is necessary to introduce the relationship between the magnetic dipole moment and the surface magnetic field B at the pole: d = B R_N^3, where R_N is the radius of a neutron star, and we set the same radius for the two neutron stars throughout this paper. Moreover, we neglect the influence of spin on the orbital motions. Due to electromagnetic interaction, the orbits will differ from those of a binary system dominated solely by gravity.

Given a dipole d_2 immersed in the field generated by a dipole d_1, the interaction potential energy is, according to field theory: U_{em} = [$d_1 \cdot d_2$ - 3($d_1 \cdot \hat{n}$)($d_2 \cdot \hat{n}$)]/R^3, where we have defined $\hat{n}$ = R and R = |R|. In our case, this expression simplifies as d_1 d_2 L, giving U_{em} = -$d_1 d_2/R^3$.

The Lagrangian for the bound system is given by L = T - U_g - U_{em}, where T is kinetic energy, U_g = -Gm_1m_2/R is gravitational potential energy, and the dot denotes derivation with respect to time t. It is convenient to choose the center of mass as the origin of coordinates, i.e., $m_1r_1 + m_2r_2 = 0$, where M = $m_1 + m_2$ is the total mass of the system. Therefore, $r_1 = [m_2/(m_1 + m_2)]R$ and $r_2 = -[m_1/(m_1 + m_2)]R$. By introducing the reduced mass = m_1m_2/M and transforming to polar coordinates (r, ϕ), we can rewrite the Lagrangian as L = ½ ($\dot{R}^2 + R^2\dot{\phi}^2$) + $Gm_1m_2/R + d_1d_2/R^3$.

According to this Lagrangian, we find that the respective kinetic and potential energies are: T = ½ ($\dot{R}^2 + R^2\dot{\phi}^2$) and U = U_g + U_{em} = -Gm_1m_2/R + d_1d_2/R^3. The average total kinematic energy and potential energy is T = -½ U , where n < -1 for BNS systems such that the potential reaches zero asymptotically at infinity. Thus the kinematic energy in our case can be written as T = -½U.

It is clear that the system has 2 degrees of freedom and the orbital motion takes place in a two-dimensional space. We can express the Euler–Lagrange equation in terms of the ϕ dimension as: d/dt(L/ $\dot{\phi}$) - L/ ϕ = 0, so the total energy has the form E = P_R $\dot{R}$ + P_ϕ $\dot{\phi}$ - L.

Furthermore, the canonical momentum P_ϕ is given by P_ϕ = L/ $\dot{\phi}$ = $R^2\dot{\phi}$. As a result, the canonical momentum P_ϕ, commonly referred to as the orbital angular momentum l, is conserved throughout the process. This gives the relationship between angular velocity and angular momentum: $\dot{\phi}$ = l/(R^2).

Subsequently, if we examine the Euler–Lagrange equation for R, we obtain: $\ddot{R}$ = l^2/(R^3) - Gm_1m_2/R^2 + $3d_1d_2/R^4$. Multiplying by a factor $\dot{R}$ on both sides we find dE/dt = 0, so the total energy of this system is conserved, as expected. Under the circular condition, both radial velocity and acceleration vanish, leading to a simplified equation of motion: l^2/(R^3) - Gm_1m_2/R^2 + $3d_1d_2/R^4 = 0$, where we have defined ω_s^2 $\dot{\phi}^2$, which is obviously a constant.

3. Evolution of Orbital Radius

According to the Virial theorem, for an N-body system bounded by a potential, if the potential energy U is the sum of power functions of r: U = Σa r^n, the relationship between kinetic and potential energies is 2 T = r · U/ r . For our system, this gives the orbital frequency relation.

3.1. Electromagnetic Radiation

Due to the presence of both gravitational and electromagnetic radiation, conservation of total energy is no longer maintained. We begin by investigating energy emission resulting from electromagnetic radiation. We note that magnetic reconnection will contribute to energy loss and bring additional phenomena; however, this is very complicated and will be considered in future research.

As the neutron star moves, it is necessary to consider the transformation of electromagnetic fields between reference frames. Denoting the rest frame of the neutron star by S' and the observer frame by S, the magnetic dipole moment d' in S' is static. When S' moves with arbitrary velocity v, it induces an electric dipole moment d_e according to the Lorentz transformation of the 4-potential A_ . In the low-velocity limit, the electric dipole moment can be expressed in frame S as: d_e = (v/c) × d'.

On the other hand, the radiated field B_R at field point r of a magnetic dipole in arbitrary motion is calculated by Heras (1994): B_R = [n̂ × [(n̂ - v/c) × $\ddot{d}$]]/(c^2R), where n̂ is the unit vector from source to observer. This expression is complicated. Under the low-velocity limit and if we consider the source small compared to the distance (i.e., |r'| R), the calculation can be performed to leading order in v/c and |r'|/R, following Ioka & Taniguchi (2000): B_R [n̂ × (n̂ × d̈)]/(c^2R).

Then we can calculate the total power of electromagnetic wave by integrating over the solid angle: P_{em} = (c/4π)|B_R|^{2}R^{2}dΩ. The only well-defined energy emission power is given by the time average over one period of the wave: P_{em} = (1/T) $_0$ P_{em} dt.

Due to the Lorentz boost of the inspiraling neutron star being different at two different times because of the changing direction of velocity, the total Lorentz transformation can be treated as a Lorentz boost plus a pure rotation. This leads to an additional evolution called Thomas precession. We consider two different cases for the evolution of magnetic dipole moments due to Thomas precession.

First, for circular orbits with both dipoles aligned with the orbital angular momentum. The acceleration and velocity are perpendicular to each other and both lie in the orbital plane, thus the cross-product points to the normal of the orbital plane. We set the direction of angular momentum to be the z-axis, making the angular velocity vector along the z-axis and parallel to the two dipoles. This gives zero changing rate of dipoles: ḋ = 0, so there is no dipole radiation due to Thomas precession.

Second, we consider that one of the dipoles is not parallel to the angular momentum, e.g., d_2. Suppose there is an inclination angle α, that is, $d_2 \cdot L = d_2L$ cosα. Note that since total angular momentum is conserved, the orbital angular momentum will change with time, so the motion is generally three-dimensional. However, we can still require the orbit to be circular for a rough estimation of radiation power magnitude. Due to Thomas precession, the changing rate of d_2 is given by: $\dot{d}_2$ = Ω_T × d_2, where Ω_T is the Thomas precession frequency.

The energy emission power averaged over a period is then given by P_{em} (2/3c^3)|$\ddot{d}_2$|^2. Taking the time derivative, we obtain the radiation power from the misaligned dipole. However, the magnitude of this additional radiation power from Thomas precession is much less than the main electromagnetic radiation term, so it can be neglected in our analysis.

3.2. Gravitational Radiation

To continue, we calculate energy emission from gravitational radiation. Following Maggiore (2007), the total radiated power in the quadrupole approximation integrated over all directions is: P_{gw} = (G/5c^5) Q$\{ijQ\}${ij} , where we employ the Einstein summation convention and Q_{ij} is the mass quadrupole moment.

For a circular orbit with sufficiently slow GW emission, the radius of the binary and the time derivative of ϕ remain constant instantaneously. According to the relation between Cartesian and polar coordinates, we derive the specific expressions for non-zero components of mass quadrupole moments: $Q_{xx} = R^2\cos^2\phi$, $Q_{yy} = R^2\sin^2\phi$, and $Q_{xy} = Q_{yx} = R^2\sin\phi\cos\phi$.

To continue our analysis, it is convenient to introduce dimensionless expressions. We define a characteristic radius $R_* = Gm/c^2$ and dimensionless time $\hat{t} = t/t_c$, where t_c is a characteristic timescale. We can therefore rewrite quantities such as t, R, M as dimensionless expressions: $\tilde{R} = R/R_$, $\tilde{M} = M/M_$, etc. Note that [R] = length, so the dimensionless expression is $\tilde{R} = R/R_*$.

Then the energy emission power of GWs averaged over one period is $P_{gw} = (32/5)(G^4/c^5)(m_1{}^2 m_2{}^2 M/R^5)$. Finally, the rate of total energy emission is $dE/dt = - P_{gw} - P_{em}$.

3.3. Evolution of Orbital Separation

Due to energy loss, the separation distance of the binary is reduced and the two neutron stars merge. We neglect higher-order post-Newtonian or general relativistic corrections to the flux since they start at 2.5PN order. By performing time derivatives on both sides of the energy equation, we obtain the relation between the time evolution of R and the energy loss rate.

Throughout this paper, we regard the electromagnetic interaction as a perturbation of orbits, i.e., $d_1 d_2/(Gm_1 m_2 R)$ 1. Furthermore, from the orbital dynamics we have: $dR/dt = -(64/5)(G^3 m_1 m_2 M)/(c^5 R^3)\ [1 + \gamma]$, where $\gamma = \beta/\alpha$ represents the ratio of electromagnetic to gravitational effects.

Setting the initial time to be 0, this gives the solution: $R(t) = R_0[1 - (t/t_c)]^{1/4}\ [1 - (\gamma/4)\ln(1 - t/t_c)]$. According to an estimation of magnitude, the logarithmic term is much smaller than the power-law parts, so we neglect its contribution, thus approximately having the first-order solution: $R(t)$ $R_0[1 - (t/t_c)]^{1/4}$.

To solve the differential equation completely, we define the time to coalescence as t_0 when the two neutron stars merge. So the equation becomes: $R(t) = (GM/c^3)^{1/4}\ [\tau(t)]^{1/4}\ [1 + O(\gamma)]$, where $\tau(t) = t_c - t$ is the time to coalescence.

In our analysis we consider the assumption that γ 1. Actually, the terms involving $\tilde{d}_i$ in α are much less than unity even when we choose large values of B, which can be seen by substituting actual values into the dimensionless expressions. Finally, we have the approximate solution to first order in $1/\gamma$: $\omega_s(t) = (c^3/GM)^{3/8}\ \tau^{-3/8}\ [1 + (3\gamma/8)\tau^{-5/8}]$.

By inserting R(t) into the frequency expression and expanding to leading order, we get (noting the magnitude of $\tilde{d}_1\tilde{d}_2$ and defining the chirp mass as $M_c = {}^{\{3/5\}M}\{2/5\}$): $\omega_{GW}(t) = 2\omega_s(t) = 2(c^3/GM_c)^{3/8}\ \tau^{-3/8}\ [1 + (3\gamma/8)\tau^{-5/8}]$.

4. Waveform of Gravitational Waves

In the theoretical analysis of waveforms, we calculate in harmonic coordinates, thus there are only two independent components: the plus polarization h and the cross polarization h_×. The time-domain waveform is: h (t) = (4GM_c/c^2D) (πGM_c f_{GW}/c^3)^{2/3} (1 + $\cos^2$)/2 cosΦ(t), and h_×(t) = (4GM_c/c^2D) (πGM_c f_{GW}/c^3)^{2/3} cos sinΦ(t), where is the angle between the orbital angular momentum and the line of sight of the observer and D is the dimensionless luminosity distance. We select = $\pi/2$ so that the observed GW has only plus polarization: h (t) = A(t)cosΦ(t), with amplitude A(t) = (4GM_c/c^2D) (πGM_c f_{GW}/c^3)^{2/3}.

The phase is defined as: Φ(t) = 2π f_{GW}(t')dt' + Φ_0. From the orbital dynamics we see that ω_{GW} increases as the binary approaches coalescence. To observe the effects of magnetic dipole moments, we plot the waveforms of magnetized and non-magnetized neutron stars in Figure 1 [Figure 1: see original paper]. Note that we set the magnetic field to be of order 10^{17} G, following the relativistic simulation results of Cardall et al. (2001). The equations show that magnetic dipole moments contribute additional τ terms to both the amplitude and frequency of the wave. These additional frequency terms induce a considerable phase shift compared with the case without magnetic dipole moments.

Also, both amplitude and frequency increase gradually as coalescence is approached, a property referred to as "chirp." Each detector is sensitive to signals within a specific frequency range. For instance, LIGO operates within 10 Hz–1000 Hz, while LISA and Taiji operate in 10^{-4} Hz–10^{-1} Hz. To know the frequency distribution of a signal, we take the Fourier transform and rewrite the signal waveform in the frequency domain: h̃(f) = h(t)e^{-2πift}dt.

Note that GW propagates at speed c, so when a wavefront is emitted from a source, it takes time to reach the observer. The integrand must be evaluated at retarded time t_r = t - D/c. Taking into account that dt_r = dt, we can rewrite the expression as: h̃(f) = A(t_r)$e^{\{i\Phi(t\_r)\}e}${-2πift_r}dt_r.

Following Maggiore (2007), we use the stationary phase approximation, where only the term e^{i(-Φ+2πft)} has a stationary point, while the term e^{i(Φ+2πft)} oscillates rapidly and integrates to a negligibly small value. Thus the expression reduces to: h̃(f) A(t*)e^{iΨ(t)} √(2π/|Φ(t*)|), where Ψ(t)* = 2πf(t* + D/c) - Φ(t*) - π/4.

At the stationary point t* we have dΦ/dt = 2πf. This indicates that the largest contribution to Fourier components is obtained when the GW frequency equals the Fourier frequency. Taking into account that A(t_r) varies much more slowly than phase, we expand around t* to second order in (t - t*) and ultimately obtain the frequency-domain expression: h̃(f) = A(t*)e^{iΨ(t*)} √(2π/|Φ(t*)|).

From the orbital dynamics we can immediately get, keeping in mind that ω_{GW} is twice the angular velocity, the full expression in frequency domain for further analysis: h̃ (f) = (GM_c/c^3)^{5/6} / (π^{2/3}D) f^{-7/6} [1 +

$(5\gamma/8)(\pi GM_c f/c^3)^{-5/3}]$ $e^{i\Psi(f)}$, where the phase is given by $\Psi(f) = 2\pi f t_c - \Phi_0 - \pi/4 + (3/128)(\pi GM_c f/c^3)^{-5/3} [1 + (\gamma/2)(\pi GM_c f/c^3)^{-5/3}]$.

To get the relation between t* and f, we solve the equation $\omega_{GW}(t) = 2\pi f$. *Since electromagnetic interaction is much weaker than gravity, we neglect the magnetic dipole contribution and obtain the relation:* $\tau = (5GM_c/c^3)(\pi GM_c f/c^3)^{-8/3}$.

Substituting this into the expressions for $A(t)$ *and* $\Phi(t)$ and restoring G, c, we finally obtain the frequency-domain waveform: $\tilde{h}(f) = (GM_c/c^3)^{5/6} / (\pi^{2/3}D) f^{-7/6} [1 + \delta_A f^{-5/3}] e^{i[\Psi_0 + (3/128)(\pi GM_c f/c^3)^{-5/3} + \delta_\Phi f^{-10/3}]}$, where δ_A and δ_Φ represent magnetic field corrections to amplitude and phase.

Clearly, the alteration of magnetic dipole moments introduces different dependencies on frequency in both amplitude and phase, reducing to the pure gravity scenario when $d_1 = d_2 = 0$.

5. Data Analysis

Using the waveform, we analyze the dependence of $\tilde{h}(f)$ on the physical properties of neutron stars, especially the surface magnetic field B. We also estimate how accurately parameters will be identified in future observations. In our analysis, we choose $d_2 = d_1/2000$ where the approximation γ 1 is satisfied.

5.1. Matched Filtering

The matched filtering technique is used to search for deviations between waveforms. We introduce the inner product between two time series a(t) and b(t): $(a|b) = 4\mathrm{Re} \int_0^\infty [\tilde{a}(f)\tilde{b}^*(f)/S_n(f)]df$, where tildes denote Fourier transforms and stars denote complex conjugation. The quantity $S_n(f)$ is the power spectral density (PSD) of noise for a particular detector.

We use this method to quantify differences between our modified waveform and the initial waveform, calculating the maximized fitting factor or match between two signals to quantify similarity: $\mathrm{match} = \max_{t_s,f_s} (h_1|h_2)/\sqrt{[(h_1|h_1)(h_2|h_2)]}$, where maximization is taken after proper shifts of time and phase.

As the post-Newtonian approximation is applicable in the long-distance condition, computation must be cut off well before a neutron star reaches the innermost stable circular orbit. Using the relationship between magnetic dipole moment and surface magnetic field, we compute different matches for various surface magnetic fields in the frequency domain. To provide comparison, we calculate the match for two GW detectors and illustrate matched-filtering results using fixed masses and radii for neutron stars.

As depicted in Figures 2 and 3 [Figure 2: see original paper][Figure 3: see original paper], we computed matches in the 10 Hz–1000 Hz frequency range for LIGO and ET, and 10^{-4} Hz–0.1 Hz for Taiji and DECIGO. We observe that the match diminishes as masses decrease. This results from GWs being generated by changing mass quadrupole moments. As we treat electromagnetic interaction as a perturbation, the deviation between waveforms remains small for magnetic fields $<10^{17}$ G. Note that we focus on whether the frequency-domain waveform can include information about magnetic dipole moments in the lowest-order Newtonian approximation. We choose magnetic fields of 8×10^{17} G as a special case or upper limit, not based on current observations. Since tidal deformation contributes fifth-order post-Newtonian corrections which are much smaller than our approximation and independent of dipolar effects, tidal deformation does not affect the dipole moment information in the waveform.

5.2. Evaluation of Parameter Estimation

The Fisher information matrix (FIM) is employed to characterize parameter estimation performance for detectors. When considering parameter influence, the waveform is a function of parameters λi. *Once we have the waveform, we can write the strain amplitude detected by a detector in the frequency domain:* h{det}(f) = F h̃ (f) + F__×h̃×*(f), where the antenna pattern functions F and* $F\times$ are given by: F = (1 + $\cos^2$)cos2ϕcos - 2cos sin2ϕcos , and F__× = (1 + $\cos^2$)sin2ϕcos + 2cos cos2ϕcos , with and ϕ denoting sky location.

We are especially interested in the influence of magnetic dipole moments on the waveform. We treat the magnetic dipole moment of the first neutron star as an independent parameter since surface magnetic fields and radii are completely coupled in magnetic dipole moments. The FIM for a given waveform h is: Γ__{ij} = (h__,i|h__,j), where commas denote partial derivatives with respect to parameters. The square root of diagonal elements of the inverse FIM provides parameter errors: σ__i = √[(Γ^{-1})__{ii}].

For comparison, we computed relative errors of main parameters for LIGO and ET. In calculations, we set m_1 = 1.65 M , m_2 = 1.55 M , D = 100 Mpc, B = 6.0×10^{17} G, and R__N = 13.8 km, giving $d_1 = 7.88 \times 10^{35}$ $G \cdot cm^3$. Table 1 shows relative errors for d_1, D, and M__c.

We also estimated relative errors for surface magnetic field B of different m_1 values using the ET detector for sources in our Milky Way (D = 20 kpc). Table 2 shows that for $B = 7.0 \times 10^{14}$ G, corresponding to magnetic dipole moment 9.20×10^{32} $G \cdot cm^3$, the relative error can reach within 30%.

The errors derived from FIM show promising evaluation for parameter estimation with high signal-to-noise ratio, especially for magnetic dipole moment estimation limited to a few percent for ET. Figure 4 [Figure 4: see original paper] plots likelihood contours for parameters derived from FIM, showing 1σ, 2σ, and 3σ levels. This method characterizes parameter estimation performance, giving more optimistic estimation than practice.

6. Conclusions and Discussions

The mergers of binary neutron stars are promising electromagnetic counterparts of GW sources in multi-messenger astronomy. In this paper, we derived the equation of motion for a binary neutron star system taking into account magnetic dipolar interaction. We calculated the total energy emission rate and time evolution of orbital radius for circular orbits using the post-Newtonian method and lowest-order multipole radiation for gravitational and electromagnetic waves. Both magnetic dipolar interaction and electromagnetic radiation modify orbital motion and evolution. Considering these modifications, we calculated the gravitational waveform in the frequency domain including terms related to magnetic dipole moments. The magnetic dipole moment introduces a significant phase shift proportional to f^{-1/3} and an additional term proportional to f^{1/6} in the amplitude.

After obtaining waveforms containing magnetic dipole moments, we employed matched filtering and FIM to evaluate parameter estimation for LVK detectors and future ET detectors. In the strong magnetic field regime, the match between signals with and without magnetic dipoles decreases as magnetic fields increase. This suggests potential for detecting extremely strong magnetic fields or providing constraints on neutron star magnetic fields in future observations. ET could measure or constrain magnetic fields with higher accuracy. FIM analysis provides optimistic evaluation for parameter estimation with high SNR, especially limiting magnetic dipole moment estimation to a few percent for ET. Furthermore, it can estimate magnetic fields at 10^{14} G to within 30% error in our Milky Way.

Since neutron stars usually carry strong magnetic fields which can affect GW waveforms, this provides possibility to detect strong magnetic fields in future GW observations. We can also constrain magnetic field magnitudes and further constrain equations of state of neutron stars. It is important to acknowledge that neutron star spin might not align with its magnetic axis, resulting in different dynamical modifications. Thus future work will generalize to magnetic dipole moments with arbitrary orientation. Further, the modifications introduced here can be extended to black hole-neutron star binaries, where a rotating black hole in magnetic fields produced by the neutron star can accrete charges and form a magnetic dipole moment, allowing investigation of black hole-neutron star mergers and methods to search for charged black holes.

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Note: Figure translations are in progress. See original paper for figures.