A GPU Algorithm for Solving the Positions of New Pulsars (Postprint)

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Date: 2024-12-06T00:00:00+00:00

Abstract

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Full Text

Preamble

Research in Astronomy and Astrophysics, 24:115004 (7pp), 2024 November

https://doi.org/10.1088/1674-4527/ad8121

A GPU Algorithm for Solving the Positions of New Pulsars

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Received 2024 April 2; revised 2024 July 18; accepted 2024 July 30; published 2024 October 25

Abstract

Timing newly discovered pulsars requires gradually building up a timing model that connects observations taken days to months apart. This can be challenging when our initial knowledge of the pulsar's position is arcminutes off from its true position, as such position errors lead to significant arrival time shifts resulting from Earth's orbital motion. Traditional downhill-fitting timing algorithms become ineffective when the model predicts incorrect pulse rotations for subsequent observations. For some pulsars where the model prediction is not too far off, the correct rotation number can be found through trial-and-error methods. However, for the remaining challenging cases, a more generalized method is needed. This paper proposes a GPU-based algorithm that exhaustively searches a large area of trial positions to find probable timing solutions, enabling phase-connected timing solutions for new pulsars using brute force.

Key words: methods: data analysis – (stars:) pulsars: general – (stars:) pulsars: individual (J0706+2707, J2149+5643)

1. Introduction

Pulsars are powerful tools for studying fundamental physics, gravitational theories, and gravitational waves [**?**, **?**, **?**, **?**]. Measuring neutron star masses helps us understand the equation of state of super-nuclear matter and the mechanism of neutron star formation [**?**, **?**]. Pulsars can probe the Milky Way's large-scale structures in magnetic fields and ionized medium [**?**]. The extreme density, pressure, and magnetic field of neutron stars, as well as their internal superfluidity and super-conductivity, provide ideal laboratories for testing particle acceleration mechanisms in the magnetosphere, high-energy radiation, electromagnetism, and relativistic plasma physics [**?**]. Furthermore, pulsar timing arrays have detected evidence for nanohertz gravitational waves [**?**, **?**, **?**, **?**].

These scientific cases begin with discovering pulsars in surveys and then solving and timing the discovered pulsars. According to the ATNF catalog version 2.1.16 [**?**], more than 3534 pulsars have been discovered as of 2024 January, among which 574 are so-called millisecond pulsars with spin periods shorter than 30 ms.

Solving new pulsars means determining their properties through a series of initial timing observations by modeling the time of arrival (TOA) with pulse numbers and phase values predicted by a timing model. For an isolated normal pulsar, this modeling can typically be well constrained with several observations taken over a few years. However, a millisecond pulsar in a binary orbit often requires more effort because a binary system contains more parameters that must be determined accurately to phase-connect all observations. More importantly, a millisecond pulsar has a small rotational period that may be much smaller than the uncertainty of model predictions when projecting months into the future. Without predicting the correct rotation number, traditional downhill-fitting algorithms cease to improve the timing model. In such cases, one must conduct more observations in a short span of time, attempting to keep more TOA phases connected and improve model precision to eliminate pulse number ambiguity.

This process can sometimes take a long time and require many extra observations, even when there are already sufficient TOAs to provide strong constraints on the timing model. Freire & Ridolfi (2018) developed the Dracula program script that automatically tests possible pulse numbers for each TOA epoch to find the correct timing solution through trial and error. Phillips & Ransom (2022) developed APT for solving isolated pulsars, and Taylor et al. (2024) developed APTB for solving binary pulsars using similar trial-and-error methods. This approach can help solve some millisecond pulsars but becomes computationally prohibitive when the true pulse number solution is many rotations away from current best guesses. In these difficult cases, the offset in pulse numbers is often caused by errors in the pulsar's position.

This paper proposes a GPU-based algorithm that exhaustively searches a range of R.A. (α) and decl. (δ) values with necessary fine-gridding to brute-force solve a pulsar. The algorithm can help solve millisecond binary pulsars when the binary model is reasonably well constrained and R.A./decl. errors are the main cause of TOA offsets. The search in position space requires fine gridding—in our test cases, we must search over square arcminutes of parameter space, amounting to 10^9–10^{10} position trials with a milliarcsecond grid. Fortunately, this computation is highly parallelizable, making it ideal for GPU implementation. Our program complements existing pulsar-solving tools.

2. Method

When a radio pulsar is discovered, we first learn its dispersion measure (DM), spin period, and rough position precise to a few arcminutes. We then conduct a series of observations to determine its nature. If the pulsar's spin period remains stable in subsequent observations, it is likely an isolated pulsar; otherwise, it may be a binary system.

Several scattered observations over a year often suffice to determine precise spin parameters and position for an isolated pulsar. For a binary system, dense observations are typically needed to identify the orbital period. In these close observations, we monitor how the pulsar's spin period varies over time and infer the binary system's initial parameters by fitting the spin periods and sometimes spin period derivatives using customized programs or existing tools like PRESTO [**?**].

The initial spin parameters (often only the spin period) and initial binary parameters constitute the binary pulsar's initial solution, which can be further refined by fitting the pulsar's TOAs in timing software such as TEMPO [**?**], TEMPO2 [**?**], and PINT [**?**].

For most pulsars, we can continue refining our initial solution by extending observations to once per month and fitting for position and spin period derivatives without losing phase connection, eventually obtaining a long-term stable timing solution that reflects the pulsar's true position and spin properties. However, for some exceptional pulsars—often millisecond pulsars—we may find it difficult to maintain phase connection after several months when residuals caused by position error become significantly larger than the pulsar period. This occurs because continuing our timing practice requires our best model to predict the next epoch's pulse number so that TOAs from the last epoch fall within one rotation of the correct model. If our model is inaccurate enough to predict the wrong rotation number for the next epoch, we often cannot immediately obtain a new solution that works with the new TOAs. This problem is called losing phase connection, which can sometimes be solved by adding or subtracting a few rotations but may require extensive experimentation.

For this reason, Freire & Ridolfi (2018) programmed the DRACULA script that automatically experiments by adding or subtracting rotations from observation epochs disconnected in phase from earlier ones. While effective for some challenging pulsars, this method is not guaranteed to work. Some pulsars require more experiments than a loop-based script can handle, with model predictions potentially off by dozens of rotations at several epochs, making trial-and-error methods impractical. An effective method is needed to exhaustively search for the correct solution under these circumstances.

We therefore developed a GPU-based code to find the correct solution by searching the most probable range of R.A. and decl. A fine grid is required because position error effects can be significant on TOAs. Assuming a position error of $\Delta\alpha \sim 1'$ ($ $0.0003 rad), the resulting TOA error can be roughly estimated as 0.0003×500 s, i.e., $ $0.15 s. For a millisecond pulsar with a spin period of 1.5 ms, the position-induced error could be nearly 100 rotations. To search and place our solution within one rotation of the correct model, we need to search position space with $ $0.1” precision. However, our algorithm must search to nearly the same level as TOA errors, which are often at the 10 s level, requiring us to search position space with 1–10 mas precision.

The pulsar rotation model is usually expressed in the reference frame co-moving with the pulsar. Since pulsars do not rotate at a constant pulse frequency, we typically use a Taylor expansion to describe the rotational phase as

$$N(t_{\rm PSR}) = N_0 + \nu_0(t_{\rm PSR} - t_0) + \frac{1}{2}\dot{\nu}_0(t_{\rm PSR} - t_0)^2 + \frac{1}{6}\ddot{\nu}_0(t_{\rm PSR} - t_0)^3,$$

where $N(t_{\rm PSR})$ is the phase/pulse number of the TOA $t_{\rm PSR}$ observed in the pulsar's co-moving reference frame, N_0 is the phase/pulse number at reference epoch t_0, ν_0 is the pulse frequency at t_0, and $\dot{\nu}_0$ and $\ddot{\nu}_0$ are the first and second derivatives of pulse frequency.

To compute a pulsar's timing residual, we perform a time transformation from a topocentric TOA ($t_{\rm topo}$) to the pulsar time ($t_{\rm PSR}$):

$$t_{\rm PSR} = t_{\rm topo} + t_{\rm corr} + \Delta_{\rm D} + \Delta_{\rm R} + \Delta_{\rm S} + \Delta_{\rm E} + \Delta_{\rm B} + \frac{d}{c}.$$

On the left-hand side, $t_{\rm PSR}$ represents the TOA in the pulsar's reference frame. On the right-hand side, $t_{\rm topo}$ is the topocentric arrival time observed with the observatory clock, and $t_{\rm corr}$ represents clock corrections to the topocentric TOAs. $\Delta_{\rm D}/f^2$ is the time delay due to dispersion in the interstellar medium, where $\Delta_{\rm D} = D \times {\rm DM}$, D is a dispersion constant, DM is expressed in units of pc cm^{-3}, and frequency f is in MHz. d is the distance to the pulsar, and c is the speed of light in vacuum; since d/c is constant, it can be omitted from our calculation. $\Delta_{\rm R}$ is the solar system Römer Delay—the time delay from light travel between the telescope phase center and the solar system barycenter (SSB) [?]. The Römer Delay varies significantly with pulsar position and is the main effect we consider. $\Delta_{\rm S}$ is the solar system Shapiro Delay due to gravitational perturbation of the light path [?] from solar system bodies; in the initial solving process, we only consider delays caused by the Sun and major planets (such as Jupiter). $\Delta_{\rm E}$ is the solar system Einstein Delay comprising gravitational redshift and time dilation [?]. $\Delta_{\rm B}$ represents time delays from the pulsar's binary system, such as the Römer Delay, which is a function of orbital parameters: projected semimajor axis x, orbital period P_b, eccentricity e, periastron time T_0, and the angle between periastron and ascending node ω. Since $\Delta_{\rm B}$ does not depend on pulsar position, we do not describe it in detail here.

After deducting all solar system, propagation, and pulsar binary effects, the resulting $t_{\rm PSR}$ differs from the pulsar's pulse emission time only by an unknown constant Doppler factor. Therefore, this $t_{\rm PSR}$ can be modeled by the simple rotational phase model in Equation (1).

Among all solar system effects, only the Röemer Delay varies significantly with pulsar position, so the TOA offset $\Delta t_{\rm topo}$ due to position error ($\Delta\alpha$ and $\Delta\delta$) can be expressed as:

$$\Delta t_{\rm topo} = \frac{\hat{s}_{\rm new} - \hat{s}_{\rm initial}}{c} \cdot \mathbf{r}_{\rm earth},$$

where $\hat{s}_{\rm initial}$ and $\hat{s}_{\rm new}$ are the initial and new unit vectors pointing from the solar system barycenter to the pulsar system barycenter, $\mathbf{r}_{\rm earth}$ is the vector from the SSB to the telescope phase center, and c is the speed of light.

Our program uses the GPU to conduct a grid search over $\Delta\alpha$ and $\Delta\delta$ using a fine grid of milliarcsecond order to find the desired solution. Suppose we move to a new position $\Delta\alpha, \Delta\delta$ away from our initial position; we obtain a new set of TOA residuals $R_{\rm new}$ offset from our initial residuals $R_{\rm initial}$:

$$R_{\rm new} = R_{\rm initial} + \Delta t_{\rm topo}.$$

If the new position is exactly the pulsar's true position, the new residuals will likely form a linear trend (or a linear trend with some TOAs offset by integral rotation numbers). We assume the trend would be linear because only spin period error manifests in the residual, while spin derivative is not yet prominent. If the new residuals form a linear trend, we can solve for the trend's slope k and intercept b using least squares:

$$k = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}, \quad b = \bar{y} - k\bar{x},$$

where x_i, y_i are the TOA ($t_{\rm topo}$) and new residual $R_{\rm new}$ of each pulse, and $\bar{x}, \bar{y}$ are the mean values of TOAs and new residuals. We can revise the spin period offset according to the least-χ^2 solution:

$$P_{\rm new} = P_{\rm initial} \times (1 - kP_{\rm initial}),$$

where k is the trend's slope, $P_{\rm initial}$ is the spin period of the initial model, and $P_{\rm new}$ is the updated period for the new position.

We illustrate the steps for solving the normal pulsar J0706+2707 in Figure 1 [Figure 1: see original paper]. First, we use an initial model containing only spin frequency (ν_0) and initial orbital parameters (x, P_b, T_0, etc.), omitting $\dot{\nu}_0$, and attempt to solve the pulsar step by step until we cannot connect the last TOA. The resulting residuals are shown in the top panel. Second, we search for possible positions within the error box of the initial observed position. For this normal pulsar, the spin period is $ $70.3 ms, which is larger than the residual offset due to position error in most cases. When our grid search finds the correct position, we obtain new residuals forming a linear structure, as shown in the middle panel of Figure 1. Finally, we use the least-square fit of the linear trend to update the pulsar's period parameter and recalculate the residual with the GPU for the new position, with results shown in the bottom panel of Figure 1.

Before solving a binary pulsar's position, we must first estimate the binary system's orbital parameters based on the first few observations. We fit the pulsar period measured in the initial observations with a circular orbit model using `fit_{{{circular}}_{{orbit}}}.py` [?] to estimate Keplerian parameters such as orbital period (P_b), epoch of periastron passage (T_0), and projected semimajor axis (x).

We then fit the TOAs from these first observations in TEMPO2 using the estimated Keplerian parameters (P_b, x, T_0) in an ELL1 timing model [?] while fixing the pulsar position to the discovery position. This yields an initial timing solution including a binary model that works well until position errors cause significant residuals. Note that this method likely only works for pulsar-white dwarf systems with very low eccentricities. For systems with non-negligible eccentricities, a more comprehensive method such as a non-circular orbit fitter is required. Taking PSR J2149+5643 as an example, its $x \sim 8.2$ ls and $e \sim 10^{-5}$, meaning the maximum residual offset caused by neglecting eccentricity is about $ex/c \sim 0.1$ ms, or \$ \$0.007 in rotation phase. This relatively small offset does not significantly affect our procedure and can be corrected later by adding more timing parameters.

Initial timing solutions obtained this way often keep TOAs phase-connected for the earliest several months but fail to connect after residuals from position error become significant. At that point, we begin fitting or searching for a better pulsar position using our GPU program.

For millisecond pulsars in binary systems, the spin period is often much smaller than residuals caused by position errors. In this case, the rotation number of the TOAs—especially the last disconnected one—may be offset from the true value. The steps for solving such pulsars using the GPU are shown in Figure 2 [Figure 2: see original paper].

In the first step, we calculate the pre-fit residuals of the initial model, shown in panel (a) of Figure 2. The last TOA residuals already deviate from zero by a significant fraction of phase. What is not visible in this plot is that the phase residual of the TOAs relative to the pulse numbers from panel (a) could be many rotations away from the true value for the last TOA. In the second step, we update the pulsar position in the timing model and use the GPU to compute residuals for the updated model, shown as yellow dots in panel (b) of Figure 2. These yellow dots exhibit a sinusoidal trend expected from a position shift and are far from the linear trend seen for normal pulsars in Figure 1's middle panel. However, if our new position is close to the true position, the sinusoidal trend should converge back to a linear line when we also correct the rotation period. The linear trend differs from the yellow dots only by an integral number of pulse phases. We employ a small trick to bring residuals to roughly the correct rotation: the TOAs of the two most adjacent residual groups (marked as green crosses) are often close to phase-connected and clearly reside in the same rotation phase. We use them to determine a likely correct linear trend, drawing a straight line across these two points to obtain the purple line in panel (b). We then shift the new residuals (yellow dots) to the same rotation as determined by the purple line by adding or subtracting integral phases, marking them as green dots.

After moving all residuals to their respective rotations, we remove the linear trend. In some cases, such as our test on PSR J2149+5643, the rotation-phase-corrected residuals (green dots) did not all reside in one rotation phase; the corrected residual gradually grew to more than one rotation and wrapped around the purple line. However, this behavior is predictable, so we implemented an additional step that checks for sudden jumps in residual phase and corrects phase wrapping by adding or subtracting one rotation from all subsequent residuals, resulting in a nearly continuous final residual curve. If the assumed new position is correct, the final residuals should all be close to phase zero after linear trend removal. We test this by calculating the χ^2 of the final residuals. In the third step, we identify the best-fit position producing the smallest reduced χ^2 in the second step. The TOA residuals from this new position, calculated by the GPU program, are shown as green dots in panel (c) of Figure 2. The corresponding $\Delta\alpha, \Delta\delta$, and new ν_0 are read out and used to update our timing ephemeris. In the fourth step, we apply the new ephemeris to the TOAs in PINT/TEMPO2, enable fitting for binary parameters, add eccentricity parameters if needed, and update the timing residuals. With the correct position found, this updated ephemeris produces flat residuals (bottom plot of Figure 2).

It is worth noting that since this residual calculation method differs from timing software like TEMPO, TEMPO2, and PINT, the χ^2 values may differ slightly, but the differences are negligible. As shown in Figure 3 [Figure 3: see original paper], χ^2 is unacceptably high over the entire search region except within a few milliarcseconds around the correct solution.

3. Result

We tested our algorithm on solving two new pulsars discovered in the FAST CRAFTS survey—an isolated pulsar PSR J0706+2707 and a binary pulsar PSR J2149+5643—from scratch. For these tests, we used an Intel Xeon Silver 4314 CPU and one NVIDIA A40 GPU to parallel compute 368,640,000 solution sets per sub-grid using 4.4 GB of memory. For pulsar J0706+2707, we used eight observations spanning one year to find the phase-connected timing solution in a quick search taking about two minutes. For binary pulsar J2149+5643, we used a step size of 1 mas and a range of 3 (in both R.A. and decl.), with the complete search taking 7.4 hours.

PSR J0706+2707 is an isolated pulsar with a spin period of 70.3 ms. We easily found its phase-coherent solution in quick search mode, as illustrated in Figure 1. This was possible because the initial position was close to the correct position, and the deviation did not cause a significant pulse number offset. Comparing pulse numbers between pre-fit and post-fit ephemerides shows that all pulses were arranged in the correct global rotation count by the initial ephemeris, allowing timing residuals to show a clear linear structure from which we could easily find the correct position. Note that this same pulsar could also be solved using TEMPO/TEMPO2 through downhill methods; our GPU program shows no advantage for this particular case. Nevertheless, the demonstration confirms that the method functions as intended and leads to a correct solution.

PSR J2149+5643 is a millisecond pulsar in a binary system with a spin period of 15.3 ms and orbital period of 2.06 days. We discovered it in September 2021 and observed it 17 times until December 2023 (Miao et al. 2024, in preparation). The timing residuals of the initial model are shown in panel (a) of Figure 2. We commenced our GPU program when the observation time span reached one year. The last TOA falls far from the model prediction and has lost phase solution (reduced $\chi^2 = 8344.083$). After exhaustively testing a $6'' \times 6''$ grid with 1 mas resolution using the procedure described in Section 2, we found the lowest reduced $\chi^2 = 246.803$ residuals (green dots in panel (c)). After adding $\dot{\nu}_0$ and eccentricity-related parameters to the model and refitting the TOAs using the GPU-computed new position, the new best-fit residuals show excellent phase connection (panel (d)) (Miao et al. 2024, in preparation).

Thus, we have demonstrated that our GPU program correctly found the position, period, and pulse numbers to phase-connect all TOAs for both an isolated pulsar and a millisecond binary pulsar. One caveat is that the new parameters found by our GPU program do not include $\dot{P}$. Once we fit the TOAs with the new parameters plus $\dot{P}$, we obtain a new set of position parameters offset by about 20 mas from the initial GPU solution. These new position parameters belong to the phase-coherent solution that represents the most accurate position from existing observation data. This final solution also successfully connected TOAs from subsequent observations not included in our test.

4. Conclusions

Our results show that the GPU program presented in this paper can easily solve isolated pulsars and also help solve binary systems when conventional methods struggle. As with most previous binary pulsar-solving practices, our method requires conducting a dense orbital observation campaign first to obtain a good initial orbit model, ensuring that errors in the orbital model do not accumulate enough to compromise the fitting. One common source of error is orbital eccentricity, which is often difficult to measure in the first few observations. For most MSP-white dwarf systems, eccentricity is rather low, and the maximum offset from neglecting it is about ex/c, which is often smaller than a millisecond and does not accumulate over time. Inaccuracies in other orbital parameters, such as P_b, can accumulate but can be mitigated by obtaining more than one TOA per epoch and ensuring the orbital parameters correctly predict pulsar periods for each observing epoch. Our GPU program exhaustively searches only in position space; for this to work, the binary solution must be sufficiently precise to avoid introducing large TOA errors in subsequent observations. Thus, planning the initial orbital campaign with adequate dense observations remains important.

One caveat of our method is that the program does not yet search for spin period derivatives and thus does not work for observations spanning years where spin derivative effects become non-negligible.

A second caveat is that while the method finds a phase-connected solution with correct pulse numbers, it does not guarantee physical correctness since the period derivative is omitted, causing output position parameters to be offset from true values, especially for short observation spans. A physically trustworthy pulsar spin solution still requires over one year of phase-connected observations.

Nevertheless, we provide a new tool for solving difficult pulsars, and this work could enable further development of brute-force pulsar-solving tools.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (12041303), the National SKA Program of China (2020SKA0120200), the CAS Project for Young Scientists in Basic Research YSBR-063, the National Natural Science Foundation of China (NSFC grant Nos. 12203070 and 12203072), and the CAS-MPG LEGACY project. This work made use of data from FAST (Five-hundred-meter Aperture Spherical Radio Telescope). FAST is a Chinese national mega-science facility operated by the National Astronomical Observatories, Chinese Academy of Sciences.

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Note: Figure translations are in progress. See original paper for figures.