A Geometric Distortion Solution Specifically for Historical Observations and its Implementation Post-print

Authors: F. R. Lin, Q. Y. Peng, Z. J. Zheng and B. F. Guo

Date: 2024-12-06T00:00:00+00:00

Abstract

Geometric distortion (GD) critically constrains the precision of astrometry. Using well-established methods to correct GD requires calibration observations, which can only be obtained using a special dithering strategy during the observation period. Unfortunately, this special observation mode is not often used, especially for historical observations before those GD correction methods were presented. As a result, some telescopes have no GD calibration observations for a long period, making it impossible to accurately determine the GD effect. This limits the value of the telescope observations in certain astrometric scenarios, such as using historical observations of moving targets in the solar system to improve their orbits. We investigated a method for handling GD that does not rely on the calibration observations. With this advantage, it can be used to solve the GD models of telescopes which were intractable in the past. The method was implemented in Python and released on GitHub. It was then applied to solve GD in the observations taken with the 1 m and 2.4 m telescopes at Yunnan Observatory. The resulting GD models were compared with those obtained using well-established methods to demonstrate the accuracy. Furthermore, the method was applied in the reduction of observations for two targets, the moon of Jupiter (Himalia) and binary GSC 2038-0293, to show its effectiveness. After GD correction, the astrometric results for both targets show improvements. Notably, the mean residual between the observed and computed position (O − C) for binary GSC 2038-0293 decreased from 36 to 5 mas.

Full Text

Preamble

Research in Astronomy and Astrophysics, 24:115008 (10pp), 2024 November

A Geometric Distortion Solution Specifically for Historical Observations and its Implementation

F. R. Lin[1],[2], Q. Y. Peng[2],[3], Z. J. Zheng[2],[4], and B. F. Guo[2],[3]

[1] School of Software, Jiangxi Normal University, Nanchang 330022, China; frlin@jxnu.edu.cn
[2] Sino-French Joint Laboratory for Astrometry, Dynamics and Space Science, Jinan University, Guangzhou 510632, China
[3] Guangdong Ocean University, Zhanjiang 524000, China

Received 2024 April 28; revised 2024 September 16; accepted 2024 September 27; published 2024 October 30

Abstract

Geometric distortion (GD) critically constrains the precision of astrometry. Conventional correction methods require calibration observations obtained through special dithering strategies during the observation period. Unfortunately, this special observation mode is rarely employed, particularly for historical observations predating the development of these GD correction methods. Consequently, some telescopes lack GD calibration observations for extended periods, making accurate determination of GD effects impossible and limiting the value of these observations for certain astrometric applications, such as using historical observations of solar system moving targets to improve their orbital solutions.

We investigated a method for handling GD that does not rely on calibration observations. This advantage enables solving GD models for telescopes that were previously intractable. The method was implemented in Python and released on GitHub. We applied it to solve GD in observations from the 1 m and 2.4 m telescopes at Yunnan Observatory, comparing the resulting GD models with those obtained using established methods to demonstrate accuracy. Furthermore, we applied the method to reduce observations of two targets: Jupiter's moon Himalia (J6) and the binary system GSC 2038-0293, demonstrating its effectiveness. After GD correction, astrometric results for both targets showed improvements, most notably with the mean residual between observed and computed positions (O − C) for GSC 2038-0293 decreasing from 36 to 5 mas.

Key words: astrometry – methods: data analysis – techniques: image processing

1. Introduction

Both space- and ground-based optical astrometric observations are inevitably affected by geometric distortion (GD). In most scenarios, GD correction represents the primary factor limiting astrometric precision \cite{Peng_{2017}, Wang_{2017}, McKay_{Kozhurina}-Platais_{2018}, Wang_{2019}, Casetti-Dinescu_{{{et}}*{{al}}}*{2021}}. For example, observations from the 2.4 m telescope at Yunnan Observatory showed nearly a factor of two improvement in astrometric precision after accurate GD correction \cite{Peng_{2017}}.

Beyond its impact on precision, GD correction is also necessary for certain applications to improve computational speed or meet fundamental requirements. For instance, in real-time applications involving substantial data processing for detecting and tracking near-Earth objects, \cite{Zhai_{{{et}}*{{al}}}*{2018}} employed GD correction when converting pixel coordinates to equatorial coordinates. After GD correction, the iterative mapping process from pixel to standard coordinates converges faster, accelerating the process while maintaining high-precision results.

While adopting high-order plate constants for reduction can mitigate GD effects, this approach requires sufficient reference stars in the field of view (FOV), a condition not met in specific high-precision applications. For some ground-based observations of moving targets, instrument limitations and other factors may result in insufficient reference stars. Consequently, a GD solution derived from calibration observations becomes essential for obtaining high-precision positions. Additionally, in Hubble Space Telescope (HST) extremely deep field observations where stellar magnitudes reach up to 30 mag \cite{Illingworth_{{{et}}*{{al}}}*{2013}}, no star catalog provides accurate positions for reference stars, making GD correction necessary since high-order plate constants cannot be applied.

The most notable research on addressing GD was conducted by \cite{Anderson_{{{King}}*{{2003}}}}. Their self-calibration technique does not rely on star catalogs and has been applied to solve GD for multiple HST cameras \cite{Bellini*{{{Bedin}}*{{2009}}}} and several ground-based telescopes \cite{Anderson*{{{et}}*{{al}}}*{2006}, Bellini_{{{Bedin}}*{{2010}}}}, achieving high-precision astrometry. An alternative approach solves GD using known reference star positions unaffected by GD, such as HST positions with GD correction \cite{Service*{{{et}}*{{al}}}*{2016}} or positions from non-optical wavelengths \cite{Reid_{{{Menten}}*{{2007}}}}. This method requires fewer observations than self-calibration but may suffer from errors in the external reference system \cite{Bernard*{{{et}}*{{al}}}*{2018}}. \cite{Peng_{{{et}}*{{al}}}*{2012}} proposed a novel method to mitigate catalog error influence on GD solutions, reducing accuracy requirements for external reference catalogs. This approach was subsequently improved by \cite{Wang_{{{et}}*{{al}}}*{2019}} to handle GD in observations from the 2.3 m Bok telescope at Kitt Peak \cite{Peng_{{{et}}*{{al}}}*{2023}}. These methods effectively solve GD models, achieving positional measurements with precision up to the 0.01 pixel level and substantially improving astrometric precision for natural satellites \cite{Peng_{{{et}}*{{al}}}*{2015}, Peng_{{{et}}*{{al}}}*{2017}, Wang_{{{et}}*{{al}}}*{2017}}.

Nonetheless, these GD correction methods require well-planned observational strategies with optimally dithered and overlapping frames \cite{Anderson_{{{King}}{{2003}}}, Peng{{{et}}{{al}}}{2012}} to offset GD and catalog error effects \cite{Zheng_{{{et}}{{al}}}{2021}}. Obtaining such calibration observations requires additional telescope time, which is sometimes difficult to secure due to observational constraints. More importantly, historical observations predating these methods lacked any plan to obtain such calibration data, making these methods incapable of addressing GD in many historical datasets.

To address this issue, we propose a method to derive an analytical GD model without requiring calibration observations. We demonstrate its performance by comparing it with established GD correction methods \cite{Peng_{{{et}}{{al}}}{2012}, Wang_{{{et}}{{al}}}{2019}}. Additionally, we reduce observations of Himalia (J6) from the 2.4 m telescope at Yunnan Observatory and 60 cm telescope observations of the binary system GSC 02038-00293.

Himalia is the largest member of the Jovian irregular satellites \cite{Grav_{{{et}}{{al}}}{2015}}. We have observed and measured positions of irregular satellites for the past decade \cite{Peng_{{{et}}{{al}}}{2017}, Shang_{{{et}}{{al}}}{2022}}, dedicated to improving astrometric methods for high-precision results. High-precision astrometric observations of irregular satellites can improve ephemerides and advance understanding of early solar system formation. GSC 2038-00293 is a close binary system with high magnetic activity; studying its nature is significant for understanding stellar evolution \cite{Dal_{{{et}}{{al}}}{2012}}. Although extensive research has been conducted on this system, including light-curve and out-of-eclipse analyses, its nature remains unclear. A new study aims to combine previous work with the binary system's positional changes to reveal its unknown characteristics, prompting us to perform astrometry on its historical observations.

These observations were initially reduced using plate constants without GD correction, but the results were unsatisfactory. Like many telescopes primarily employed for photometry, the 60 cm telescope at Yunnan Observatory has never performed GD calibration observations. However, the novel research focus on binary GSC 02038-00293 highlights the importance of astrometry. Since our method does not require GD calibration observations, it can be applied to these historical observations.

This paper is structured as follows: Section 2 provides detailed information about the observations and corresponding instruments, Section 3 describes the GD correction method based on Gaia DR3 \cite{Gaia_{{{Collaboration}}{{et}}}{{{al}}_{{2023}}}}, Section 4 presents performance comparisons with established methods and demonstrates advantages in reducing observations of J6 and GSC 02038-00293, and Section 5 concludes with closing remarks.

2. Observations

We used observations from multiple telescopes: the 60 cm telescope \cite{Zang_{{{et}}*{{al}}}*{2022}}, the 1 m telescope (IAU code 286, longitude—E102°47 18 , latitude—N25°1 46 , height—2000 m above sea level) at Yunnan Observatory, and the 2.4 m telescope (IAU code O44, longitude—E100°1 51 , latitude—N26°42 32 , height—3193 m above sea level) at Yunnan Observatory (YNO 60 cm, YNO 1 m, and YNO 2.4 m). Additional instrumental details for the reflectors and CCD detectors are listed in Table 1 . The patterns and magnitudes of GD differ among these instruments.

Observation sets 1 and 2 were captured using a dithering strategy, taking multiple dithered exposures of the same sky field with different offsets \cite{Peng_{{{et}}*{{al}}}*{2012}}. These sets demonstrate that our proposed method achieves the same accuracy as other established GD correction methods. Observation sets 3 and 4 are significantly affected by higher-order GD, but only about a dozen bright stars can be used to solve plate constants, making GD solution crucial for high-precision astrometry of the targets. Detailed observation information is provided in Table 2 .

In this paper, “bright stars” refers to stars brighter than a certain magnitude threshold where astrometric precision becomes magnitude-limited. For our observations, the signal-to-noise ratio (SNR) corresponding to this threshold is about 100, serving as a criterion for identifying bright stars. This threshold may vary due to factors such as atmospheric turbulence, so the specific criterion depends on each observation set. Figure 1 [Figure 1: see original paper] presents a frame of GSC 02038-00293 observations from the YNO 60 cm telescope.

3. Methods

Our method derives an analytical GD model characterized by a high-order polynomial using distortion-free stellar positions from the Gaia catalog. This analytical GD model effectively describes GD effects in ground-based observations because most GD components can be characterized by polynomials. Remaining components, typically described using lookup tables \cite{Wang_{{{et}}*{{al}}}*{2019}}, generally account for only a minor portion of GD, as confirmed experimentally in the subsequent section.

The principle is to extract the GD effect present in each observation frame and derive the GD model from multiple frames. In essence, the method uses the weighted average of plate constants to derive the GD model. Since fitting errors are eliminated by averaging coefficients from multiple frames, the final GD solution avoids overfitting issues even when only a dozen bright stars are available for solving high-order polynomials.

The implementation proceeds as follows. Two-dimensional Gaussian fitting determines the pixel positions of observed stars, which are then cross-matched with the Gaia catalog \cite{Gaia_{{{Collaboration}}*{{et}}}*{{{al}}*{{2023}}}} to obtain reference positions. Specifically, reference positions are topocentric astrometric positions calculated from catalog data \cite{Kaplan*{{{et}}*{{al}}}*{1989}}. To ensure GD solution accuracy, we account for factors that may degrade precision, including differential color refraction and charge transfer efficiency issues, which can be effectively addressed using the method presented in \cite{Lin_{{{et}}*{{al}}}*{2020}}. Consequently, we obtain pixel coordinates (x_i, y_i) and equatorial coordinates (α_i, δ_i) for each star i. Standard coordinates (ξ_i, η_i) are converted from equatorial coordinates via central projection \cite{Green_{1985}}.

To extract GD effects on pixel positions, we solve a six-parameter linear transformation to obtain approximate pixel positions of reference stars. The linear transformation is expressed as:

$$x_i^L = \tilde{a} + \tilde{b}\xi_i + \tilde{c}\eta_i \tag{1}$$

$$y_i^L = \tilde{d} + \tilde{e}\xi_i + \tilde{f}\eta_i \tag{2}$$

where coefficients $\tilde{a}$ through $\tilde{f}$ (denoted as C_{std}) are estimated through least-squares fitting. Using this linear transformation, standard coordinates (ξ_i, η_i) are converted to approximate pixel positions (x_i^L, y_i^L). The linear transformation coefficients C_{std} are initially inaccurate because they are affected by GD. During iterative GD solving, pixel positions (x_i, y_i) in Equation (1) are replaced by GD-corrected positions in each iteration, causing approximate pixel positions (x_i^L, y_i^L) to converge toward distortion-free pixel positions.

Based on the pattern and magnitude of GD in each telescope's optical system, we select an appropriate polynomial order N to characterize its analytical GD model. The general polynomial formula is:

$$U = \sum_{m=0}^{N} \sum_{n=0}^{N-m} k_{mn} X^m Y^n \tag{3}$$

$$V = \sum_{m=0}^{N} \sum_{n=0}^{N-m} j_{mn} X^m Y^n \tag{4}$$

where k_{mn} and j_{mn} are parameters to be fitted. Setting (X, Y) as coordinates (x_i^L, y_i^L) and (U, V) as (x_i, y_i), an Nth-order polynomial characterizing GD effects can be fitted. We denote these polynomial coefficients as C_{pix}. By solving for C_{pix} for each frame in an observation set and applying a weighted average based on image quality, we obtain an average GD solution $\bar{C}_{\text{pix}}$. Most random errors are offset in this weighted averaging, leaving primarily the GD effect.

To determine the GD effect at any pixel position using polynomial coefficients $\bar{C}_{\text{pix}}$, we must handle cases where GD changes dramatically within small image ranges. We construct a 16×16 grid uniformly distributed across the image pixel coordinates (Figure 3 [Figure 3: see original paper]). Grid positions (x_g, y_g) are transformed via the polynomial using coefficients $\bar{C}_{\text{pix}}$, producing distorted positions (U_g, V_g). Inverse transformation coefficients C_{inv} are then determined by fitting from (U_g, V_g) back to (x_g, y_g). GD-corrected pixel positions are calculated by setting (X, Y) as star coordinates (x_i, y_i) and using C_{inv} in Equation (2). Figure 2 [Figure 2: see original paper] illustrates the coefficient solving process for these transformations between different coordinate levels.

Since stellar pixel positions are contaminated by different random noise, weights are introduced into all fitting procedures involving (x_i, y_i). The GD model is derived iteratively, with weights initially uniform. After the first GD solution iteration, each star's weight is determined as the inverse of its positional measurement variance, calculated by fitting a sigmoidal function to magnitude-SD data (Figure 4 [Figure 4: see original paper]). The sigmoidal function is expressed as:

$$\sigma(m) = A_2 + \frac{A_1 - A_2}{1 + \exp\left(\frac{m - m_0}{dm}\right)}$$

where m is stellar magnitude, $\sigma(m)$ is positional standard deviation from the previous iteration, and A_1, A_2, m_0, and dm are free parameters. Detailed weight calculation procedures are described in \cite{Lin_{{{et}}*{{al}}*}{2019}}. Weights and coefficients C_{std} are updated each iteration to solve a more accurate GD model. The final analytical GD model is obtained after two to four iterations. For community convenience, a Python implementation is available on GitHub. For comparison, we also apply classical plate constant reduction, which solves a polynomial transformation from reference star pixel positions to standard positions, then uses this transformation to calculate target astrometric positions. Using an Nth-order polynomial for reduction can handle GD up to Nth-order if sufficient reference stars are available \cite{Green_{1985}, Peng_{{{Fan}}_{{2010}}}}.

4. Results

We verified the accuracy of our GD model by comparing it with models determined by established methods \cite{Peng_{{{et}}*{{al}}*}{2012}, Wang_{{{et}}*{{al}}*}{2019}}. We then applied our method to reduce observations of Himalia (J6) and GSC 02038-00293, demonstrating its advantages. Computed positions for J6 and GSC 02038-00293 were retrieved from the Institute de Méchanique Céleste et de Calcul des Éphémérides (IMCCE) and Gaia DR3, respectively.

4.1. Comparison with the Well-established Method

Since observation sets 1 and 2 were acquired using dithering strategies, established methods could also solve GD for these sets. We solved GD models using both our method and the method of \cite{Wang_{{{et}}*{{al}}*}{2019}}. Figure 3 [Figure 3: see original paper] presents GD models for the YNO 1 m and 2.4 m telescopes solved by each method, with differences shown in the right panels. The analytical GD model for the YNO 1 m telescope uses a 4th-order polynomial, while the YNO 2.4 m telescope model uses a 5th-order polynomial.

After GD correction, six-parameter plate constants reduce observations to obtain astrometric results. We use results corrected by \cite{Wang_{{{et}}*{{al}}*}{2019}} as reference hereafter. Figure 4 [Figure 4: see original paper] shows each star's positional SD, while Figure 5 [Figure 5: see original paper] shows differences in mean (O − C) between our results and reference results. Figures 4 and 5 demonstrate that astrometric results corrected using our GD solution are consistent with reference results. The SDs from both methods are equivalent. For YNO 1 m telescope observations (less affected by GD), the mean (O − C) difference is merely 1 mas; for YNO 2.4 m telescope observations, the difference is only 2 mas. Thus, our method efficiently corrects GD and produces reliable astrometric results.

4.2. Application of the GD Solution

As noted in Section 1, our method is particularly useful when only a limited number of bright reference stars (typically about a dozen) are available for reduction, commonly occurring in sparse FOVs where moving targets pass through. We processed observations of two such targets, which were not taken with dithered FOVs, making established GD solutions inapplicable.

Figure 6 [Figure 6: see original paper] shows astrometric results for J6 observations from the YNO 2.4 m telescope. The left panel shows obviously greater SD for the target than for other bright stars because insufficient reference stars lead to overfitting of 3rd-order plate constants. The higher precision for reference stars in the left panel is illusory, as overfitting absorbs residuals in reduction. More severe overfitting yields poorer calibration and lower target precision. To address this, we corrected GD corresponding to the 3rd-order polynomial using our method, then applied six-parameter plate constants for reduction. After GD correction, J6's astrometric precision improved, with positional SD decreasing from 20 to 17 mas (right panel of Figure 6).

The improvement is more significant for YNO 60 cm telescope observations. Figure 7 [Figure 7: see original paper] shows the GD model solved with observation set 4, characterized by a 3rd-order polynomial. We applied this GD solution in reducing observation set 4. For comparison, Figure 8 [Figure 8: see original paper] presents results using different-order plate constants. The left panel shows positional SD, while the right panel shows corresponding mean (O − C) calculated by:

$$\langle O-C\rangle = \langle O-C\rangle_{\text{systematic}} + \langle O-C\rangle_{\text{random}}$$

Since observation set 4 points to a fixed FOV, low-order plate constants can achieve precise positional measurement for the target (see 25.0 mas in panel (a) of Figure 8). However, despite good plate constant fits, all stars show large mean (O − C) values in right panel (b), indicating significant GD effects and rendering these astrometric results unreliable. As plate constant order increased, target positional SD in panels (c) and (e) became greater than that of other bright stars. Even 2nd-order plate constants cause overfitting that worsens with increasing order, consistent with J6 results. Panels (d) and (f) show decreasing mean (O − C) for reference stars due to overfitting absorbing residuals.

The bottom two panels in Figure 8 show results from applying our GD correction method first, followed by six-parameter plate constant reduction. These results show significant advantages: target astrometric precision is comparable to low-order plate constant results, while panel (h) reveals mean (O − C) values for target and reference stars are substantially smaller than values obtained using 1st- and 2nd-order plate constants. This demonstrates that systematic errors from GD are significantly reduced after applying our GD correction.

For observations lacking sufficient reference stars to solve high-order plate constants, the GD solution substantially improves astrometric results. Empirically, when using weighted least-squares to determine plate constants, the number of bright reference stars should be approximately 1.5 times the number of fitting parameters, which is crucial for determining when a GD solution is necessary.

5. Conclusions and Discussions

We investigated a GD correction method based on the high-precision Gaia catalog. This effective and easily implemented method was evaluated through reduction of open cluster observations from the 1 m and 2.4 m telescopes at Yunnan Observatory. Our method and an established method \cite{Peng_{{{et}}{{al}}}{2012}, Wang_{{{et}}{{al}}}{2019}} produced results with equivalent precision. The mean (O − C) difference between our results and reference results was only 1 mas for YNO 1 m telescope observations and 2 mas for YNO 2.4 m telescope observations.

Additional reductions investigated conditions necessary for our method. We found that no more than 15 frames are sufficient to derive an effective GD solution. These frames may have different or identical FOVs, provided each contains a sufficient number of approximately evenly distributed reference stars. Typically, the number of bright reference stars (with SNR ≥ 100) should exceed half the number of GD model parameters. Since fitting errors are eliminated by averaging coefficients from multiple frames, the final GD solution avoids overfitting even when only a dozen bright stars are available.

The major advantage of this method is its independence from special calibration observations, which is valuable for historical observations where GD correction was previously unattainable due to absent calibration data. Even with Gaia DR3, limited observation equipment performance often yields insufficient reference stars in historical observations for solving high-order plate constants. Reduction using 1st-order plate constants introduces significant systematic errors when GD effects are severe. The YNO 60 cm telescope observations of binary GSC 02038-00293 exemplify this situation. Applying our GD solution significantly improved astrometric results, reducing mean $(O - C)$ from 36 to 5 mas. Additionally, J6 observations in a sparse FOV from the YNO 2.4 m telescope were successfully corrected, demonstrating great potential for improving astrometric precision in historical observations.

We note that we used the simpler, more easily calculated topocentric astrometric positions of reference stars to drive GD and found no precision loss for our observations. However, when establishing GD models, adopting observed positions of reference stars is more appropriate, as this eliminates differential atmospheric refraction and light aberration effects. We will adopt observed positions in future work to obtain more accurate GD solutions.

Acknowledgments

This work was supported by the National Key R&D Program of China (grant No. 2022YFE0116800), the National Natural Science Foundation of China (NSFC, grant No. 12203019), the Natural Science Foundation of Jiangxi Province (grant No. 20242BAB20033), the NSFC (grant Nos. 11873026 and 11273014), the China Manned Space Project (grant No. CMS-CSST-2021-B08), and the Joint Research Fund in Astronomy (grant No. U1431227). The authors thank chief scientist Qian S.B. of the 1 m telescope and his working group for their support and for sharing observations of binary GSC 02038-00293. This work has made use of data from the European Space Agency (ESA) mission Gaia (https://www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC, https://www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for DPAC has been provided by national institutions participating in the Gaia Multilateral Agreement.

References

Bellini, A., & Bedin, L. R. 2009, PASP, 121, 1419
Bellini, A., & Bedin, L. R. 2010, A&A, 517, A34
Bernard, A., Neichel, B., Mugnier, L. M., & Fusco, T. 2018, MNRAS, 473, 2590
Casetti-Dinescu, D. I., Girard, T. M., Kozhurina-Platais, V., et al. 2021, PASP, 133, 064505
Dal, H. A., Sipahi, E., & Özdarcan, O. 2012, PASA, 29, 150
Gaia Collaboration, Vallenari, A., Brown, A. G. A., et al. 2023, A&A, 674, A1

Grav, T., Bauer, J. M., Mainzer, A. K., et al. 2015, ApJ, 809, 3
Green, R. M. 1985, Spherical Astronomy (Cambridge: Cambridge Univ. Press)
Illingworth, G. D., Magee, D., Oesch, P. A., et al. 2013, ApJS, 209, 6
Kaplan, G. H., Hughes, J. A., Seidelmann, P. K., Smith, C. A., & Yallop, B. D. 1989, AJ, 97, 1197
Lin, F. R., Peng, J. H., Zheng, Z. J., & Peng, Q. Y. 2019, MNRAS, 490, 4382
Lin, F. R., Peng, Q. Y., & Zheng, Z. J. 2020, MNRAS, 498, 258
McKay, M., & Kozhurina-Platais, V. 2018, WFC3/IR: Time Dependency of Linear Geometric Distortion, Instrument Science Report WFC3 2018-09, Space Telescope Science Institute
Peng, H. W., Peng, Q. Y., & Wang, N. 2017, MNRAS, 467, 2266
Peng, Q. Y., & Fan, L. Y. 2010, ChSBu, 55, 791
Peng, Q. Y., Vienne, A., Zhang, Q. F., et al. 2012, AJ, 144, 170
Peng, Q. Y., Wang, N., Vienne, A., et al. 2015, MNRAS, 449, 2638
Peng, X. Y., Qi, Z. X., Zhang, T. M., et al. 2023, AJ, 165, 172
Reid, M. J., & Menten, K. M. 2007, ApJ, 671, 2068
Service, M., Lu, J. R., Campbell, R., et al. 2016, PASP, 128, 095004
Shang, Y. J., Peng, Q. Y., Zheng, Z. J., et al. 2022, AJ, 163, 210
Wang, N., Peng, Q. Y., Peng, H. W., et al. 2017, MNRAS, 468, 1415
Wang, N., Peng, Q. Y., Zhou, X., Peng, X. Y., & Peng, H. W. 2019, MNRAS, 485, 1626
Anderson, J., Bedin, L. R., Piotto, G., Yadav, R. S., & Bellini, A. 2006, A&A, 454, 1029
Anderson, J., & King, I. R. 2003, PASP, 115, 113
Zang, L., Qian, S. B., Zhu, L. Y., & Liu, L. 2022, MNRAS, 511, 553
Zhai, C. X., Shao, M., Saini, N. S., et al. 2018, AJ, 156, 65
Zheng, Z. J., Peng, Q. Y., & Lin, F. R. 2021, MNRAS, 502, 6216

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv — Machine translation. Verify with original.