The Progenitors of Be stars Paired with O subdwarfs: The Spin-up of a Be Star at the Stage of Conservative Mass Exchange (Postprint)

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Abstract

The spinning-up of the accreting component in the process of conservative mass exchange is considered in binary systems—progenitors of systems consisting of a main sequence Be-star and an O-subdwarf. During the mass exchange, the meridional circulation transfers 80%–85% of the angular momentum that entered the accretor together with the accreted matter to the accretor surface. This angular momentum is removed from the accretor by the disk. When the mass exchange finishes, the accretor has a rotation typical of classical Be-type stars.

Full Text

Preamble

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The Progenitors of Be-stars Paired with O-subdwarfs: The Spin-up of a Be Star at the Stage of Conservative Mass Exchange

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Abstract

The spinning-up of the accreting component during conservative mass exchange is considered in binary systems that are progenitors of systems consisting of a main-sequence Be-star and an O-subdwarf. During mass exchange, meridional circulation transfers 80%–85% of the angular momentum that entered the accretor together with the accreted matter to the accretor's surface. This angular momentum is then removed from the accretor by the disk. When mass exchange finishes, the accretor has a rotation typical of classical Be-type stars.

Key words: (stars:) binaries (including multiple): close – stars: emission-line – Be – (stars:) binaries: general

1. Introduction

Classical Be-stars exhibit rapid rotation (Porter & Rivinius 2003). The rotational velocities at the equator, V_e, of Be-stars in early spectral subclasses (B0–B3) span a wide range: between $0.4 \lesssim V_e/V_c \lesssim 0.6$ at the lower limit and $0.9 \lesssim V_e/V_c \lesssim 1.0$ at the upper limit (Cranmer 2005), where V_c is the Keplerian velocity at the star's equator. Such rapid rotation of Be-stars could be achieved as a result of mass exchange in binary systems (Pols et al. 1991; Portegies Zwart 1995; Shao & Li 2014; Hastings et al. 2021).

To date, numerous Be-stars have been found in binary systems paired with O-subdwarfs (Chojnowski et al. 2018; Wang et al. 2023). These binary systems have already passed the stage of mass exchange. The orbital periods exceed 30 days, and the masses of the Be-stars range from 6–12 $M_\odot$. Most Be-stars have early spectral subclasses B0–B3 (Wang et al. 2018). Accreting components in long-period Algols also exhibit enhanced rotation: $0.15 \lesssim V_e/V_c \lesssim 0.3$ (Dervisoglu et al. 2010). The periods of these systems range from 5–15 days, and the masses of the accreting components are 2–7 $M_\odot$. The mass range of Be-stars in systems with O-subdwarfs partially overlaps with the mass range of accreting components in long-period Algols. The periods of Be-star binaries with subdwarfs are longer than those of long-period Algols, and the axial rotation of the Be-stars is also greater than that of the accreting components in long-period Algols.

The binary system ϕ Per is one of the most well-known and extensively studied systems comprising a Be-star and an O-subdwarf (Gies et al. 1998; Schootemeijer et al. 2018). The evolution of this system's progenitor has been thoroughly investigated, and ϕ Per formed as a result of conservative mass transfer in the Hertzsprung gap.

Galactic Be-binaries with helium-star companions have also been analyzed through population synthesis (Shao & Li 2021). The orbital periods and component masses of these binaries can be produced through conservative mass exchange in the Hertzsprung gap and/or non-conservative mass exchange during main-sequence evolution. The observed characteristics of long-period

Algol systems can also be reproduced (except for rotation) through simulations of conservative mass exchange in the Hertzsprung gap (Van Rensbergen & De Greve 2020). In the case of conservative mass accretion, the accreting component can increase its mass by up to approximately two times.

Interacting binary stars exchange both mass and angular momentum. The axial rotation of a star receiving mass depends on the amount of angular momentum in the accreted mass, the transfer of angular momentum in the stellar interior, and the mechanisms of angular momentum loss from the star. If angular momentum is instantly redistributed throughout the stellar interior to achieve solid-body rotation and no mechanisms exist for removing angular momentum from the star, the star can reach critical rotation after a mass increase of only 5%–10%, provided the added mass had Keplerian rotation (Packet 1981). Further increase in the star's mass and angular momentum is possible because part of the angular momentum of matter falling onto the star is removed by an accretion disk (Paczynski 1991; Bisnovatyi-Kogan 1993).

In a single star, the outer region containing half the star's mass holds more than half of the star's angular momentum (Staritsin 2007, 2009). In the case of an accreting component in a binary system undergoing conservative mass exchange, such an outer region consists of accreted matter. This matter brings with it Keplerian angular momentum and cannot become part of the star without decreasing its angular momentum. This decrease occurs because meridional circulation transfers part of the angular momentum from the accreted layers to the accretor's surface (Staritsin 2022, 2023a, 2024). This portion of angular momentum is removed from the accretor by the disk (Paczynski 1991; Bisnovatyi-Kogan 1993), while a small fraction is transferred by meridional circulation to the inner layers of the star.

We studied the accretor spin-up during conservative mass exchange in binary systems as a function of the system mass and component separation. The masses and separations have values typical of progenitors of Be-star/O-subdwarf pairs and long-period Algol-type binaries. Angular momentum transfer in the accretor's interior is carried out by meridional circulation and shear turbulence.

2.1. The Meridional Circulation Role in Accretor Spin-up

The progenitors of binary systems with Be-stars and long-period Algols comprise stars with radiative envelopes and convective cores. Mass exchange in these progenitor systems begins after hydrogen exhaustion in the donor's core. The donor fills its Roche lobe and loses matter on a thermal timescale (Paczynski 1971). The material falls into the accretor's gravitational field, forming an accretion disk around the accretor (Lubow & Shu 1975; Richards & Ratliff 1998; Bisikalo et al. 2000; Raymer 2012). The material then joins the accretor, bringing with it Keplerian angular momentum.

The arrival of angular momentum into the accretor's upper layer strongly affects the velocity field of meridional circulation (Staritsin 2019). In the upper layer, a circulation cell forms in the meridional plane, with circulation rates several orders of magnitude higher than in single stars. This circulation transfers the angular momentum of the attached matter within the cell (Staritsin 2021, 2022). The mass of matter in the cell increases over time for two reasons: layers located beneath the cell bottom become incorporated into the cell, and new accreted layers are added from above.

After the accretor mass increases by several percent, the surface rotation speed at the equator becomes equal to the Keplerian value (Staritsin 2022, 2024). This rotational state is called critical rotation. Let J and M be the angular momentum and mass of a star in critical rotation, and $j_{\rm Kep}$ be the specific Keplerian angular momentum at its equator. Accretion onto a critically rotating star can continue due to removal of excess angular momentum from the star by an accretion disk (Paczynski 1991; Bisnovatyi-Kogan 1993). Matter rotating at Keplerian velocity joins the star, forming another circulation cell in the attached matter. In this cell, circulation carries angular momentum to the star's surface, from which it may be removed by the accretion disk (Paczynski 1991; Bisnovatyi-Kogan 1993). As a result of this angular momentum decrease, the accreted layers contract, as normally occurs during accretion. The removal of excess angular momentum from accreted matter by meridional circulation, and its subsequent removal from the star by an accretion disk, ensures continued mass and angular momentum increase for a star in critical rotation.

2.2. The Arrival of Angular Momentum in the Accretor

At the beginning of mass exchange, the angular velocity of the accretor surface is less than the Keplerian value, resulting in a boundary layer between the star's surface and the inner edge of the Keplerian accretion disk. The details of angular momentum transfer between the boundary layer and the star, as well as between the boundary layer and the disk, do not affect the final angular momentum of the accretor (Staritsin 2024). Therefore, in current calculations, the angular velocity of added matter is assumed equal to the angular velocity of the accretor surface.

At the start of mass exchange, the accretor's mass increase is accompanied by an increase in its size (Benson 1970; Kippenhahn & Meyer-Hofmeister 1977). A situation may arise where a stream of matter from the Lagrangian point L1 is directed directly or tangentially onto the accretor surface. In this case, the disk around the accretor has sub-Keplerian rotation (Kaitchuck & Honeycutt 1982; Kaitchuck 1988, 1989; Richards & Ratliff 1998; Raymer 2012; Richards et al. 2014). Lowering the disk rotation rate below Keplerian does not affect the accretor's final angular momentum (Staritsin 2024). Therefore, after the accretor surface angular velocity increases to the Keplerian value, the angular velocity of added matter is assumed equal to the Keplerian velocity.

Angular momentum transfer in the radiative envelope is carried out by meridional circulation and shear turbulence, both considered within the shellular rotation model (Zahn 1992). Angular momentum transfer is described by the principle of angular momentum conservation (Tassoul 1978). The meridional circulation velocity u is determined from the law of energy conservation in stationary form (Maeder & Zahn 1998). In these equations, Ω is angular velocity, ϖ is distance to the rotation axis, ρ is density, ν_v is turbulent viscosity in the vertical direction, P is pressure, T is temperature, s is specific entropy, ε_n is nuclear energy release rate, χ is thermal conductivity, and F_h is turbulent enthalpy flow in the horizontal direction: $F_h = -\nu_h \rho T \partial s/\partial\theta$, where ν_h is turbulent viscosity in the horizontal direction.

The coefficients are determined by expressions involving the critical Richardson number Ri_c, buoyancy frequency N, thermal diffusivity K, amplitude of the vertical component of meridional circulation velocity $U(r)$, and Legendre function of order 2, $P_2(\cos\theta)$. The condition for Zahn model applicability (Zahn 1992) is $\nu_V < \nu_h < K$. The convective core rotates as a solid body. These equations are solved together with the equations of stellar structure and evolution (Staritsin 1999, 2005, 2007). Once $U(r)$ and $\Omega(r)$ are determined, the angular momentum flux carried by circulation (advective flux) can be calculated (Zahn 1994) along with the turbulent angular momentum flux and total flux.

3.1. Initial Parameters of the Systems Under Study

We calculated accretor spin-up during conservative mass exchange in systems with initial parameters from Table 1. The first system is similar to the progenitor of the binary system ϕ Per, which comprises a Be-star and a subdwarf O-star and is the best-studied system of this type (Gies et al. 1998; Schootemeijer et al. 2018). We adopted the progenitor parameters proposed by Vanbeveren et al. (1998) for this system to enable multiplication and variation of the initial parameters (Table 1). The observed characteristics of ϕ Per are reproduced in calculations of conservative mass exchange (Schootemeijer et al. 2018). The second system has parameters close to the progenitor of BD-011603, another Be-star/subdwarf O-star binary (Chojnowski et al. 2018) that is not as well studied. The third system (Table 1) is similar to the progenitor of the long-period Algol-type system RX Gem (Van Rensbergen & De Greve 2020). The observed parameters of long-period Algols can also be reproduced (except for rotation) through simulations of conservative mass exchange.

The accretion rate is assumed equal to the average value, with the accretor mass increase equal to the difference between the donor's initial mass and remnant mass. The remnant mass can be determined using the formula from Massevitch and Tutukov (1988). The coefficient ζ depends on input physics, initial chemical composition, and overshooting parameter; here it is determined using calculations by Vanbeveren et al. (1998) for ϕ Per. The mass exchange duration is three times the thermal timescale t_{KH} (Paczynski 1971), where R_{d0} and L_{d0} are the donor's size and luminosity before mass exchange, respectively. The average accretion rates are shown in Table 1.

The initial distance between components is smallest in the third system (Table 1). The synchronization time of axial rotation with orbital motion (Zahn 1975; Hurley et al. 2002) for the less massive star in this system is shorter than the evolution time of the more massive star on the main sequence. Therefore, the less massive star achieves synchronous rotation before mass exchange begins, with angular velocity $ $10% of the Keplerian value. In the first and second systems (Table 1), the lower-mass component may have non-synchronous rotation. For uniformity of initial conditions, the surface angular velocity of components in these systems is assumed to be 10% of the Keplerian velocity.

In all cases, the accretor's angular momentum before mass exchange, J_0, is small (Table 1). The meridional circulation rate is $\sim 10^{-6}$ cm s^{-1} in a 5 $M_\odot$ star and $\sim 10^{-7}$ cm s^{-1} in a 2.5 $M_\odot$ star, making angular momentum transfer in stellar interiors ineffective. The hydrogen content in the convective cores of 5 $M_\odot$ and 2.5 $M_\odot$ components before mass exchange is 0.4. During accretion, the core mass and hydrogen content increase. To simplify calculations, we assumed the hydrogen distribution in the convective core and overlying layer matches that of a single star with the same mass and helium content as the accretor.

3.2. Accretor Spin-up in Systems with $A_0 = 30R_\odot$

In both systems (Table 1), a new meridional circulation cell forms at the beginning of mass exchange in the accreted layers and underlying regions. The maximum meridional circulation velocity is 8×10^{-2} cm s^{-1} for the accretor from the first system and 8×10^{-3} cm s^{-1} for the second system. The circulation transfers angular momentum from the accreted matter into the cell, while a cascade of cells with opposite angular momentum transfer direction forms below. The maximum circulation velocity in these cascade cells is significantly lower, decreasing with depth. In the cascade cells, there is slight redistribution of the initial angular momentum among the trapped layers.

As the accretor mass increases, its surface angular velocity grows. After a $ $10% mass increase, the surface angular velocity approaches the Keplerian value (Figure 1). Once the accretor surface reaches Keplerian rotation, another circulation cell forms in the newly accreted layers. In this cell, circulation transfers part of the angular momentum from the accreted matter to the accretor surface, from which it is removed by a disk (Paczynski 1991; Bisnovatyi-Kogan 1993). This reduces the angular momentum of the accreted layers, causing them to contract while the accretor surface angular velocity remains at the Keplerian value.

The primary role of meridional circulation is transferring part of the angular momentum from accreted layers to the accretor surface when the surface angular velocity equals the Keplerian value. The outward angular momentum flux in the outer circulation cell results exclusively from the large-scale matter flow (Figure 2). The maximum meridional circulation velocity in this cell is 0.9 cm s^{-1} (first system) and 1.2 cm s^{-1} (second system). Notably, the angular velocity increases outward in the cell's lower part and decreases in the upper part (Figure 1).

Angular momentum transfer inside the accretor continues in the circulation cell formed at the beginning of mass exchange (Figure 2). The maximum circulation velocity in this cell is 2×10^{-2} cm s^{-1} (first system) and 2×10^{-3} cm s^{-1} (second system). Turbulence's contribution to angular momentum transfer varies from insignificant in the cell's upper part to dominant at its lower boundary. The coefficient of turbulent viscosity in the horizontal direction, ν_h, is determined with $C = 1/20$. This coefficient has two minima at the boundaries of the considered circulation cell (Figure 3), while the coefficient of turbulent viscosity in the vertical direction, ν_V, has one minimum in the outer circulation cell (where angular velocity is maximum) and another below the considered cell.

At the lower boundary of the cell under consideration, the advective angular momentum flux $F_{\rm ad}$ becomes zero, but the turbulent flux F_t remains non-zero and directed inward (inset in Figure 2(b)). This causes spin-up of layers below the cell's lower boundary, which are subsequently incorporated into the circulation cell. Turbulence contributes to the descent of the cell's lower boundary into the accretor.

The second minimum in the ν_V distribution (Figure 3) is located at the intersection of swirled and non-swirled accretor layers. Below, in one of the cascade cells, there is slight redistribution of initial angular momentum among the trapped layers, where turbulent viscosity decreases to values comparable to radiation viscosity ν_r and matter viscosity ν_m (Figure 3).

The mass fraction of matter swirled due to angular momentum transfer from accreted material increases over time similarly in accretors with different initial masses. However, the mass fraction of the convective core is greater for more massive accretors. The lower boundary of the cell where circulation and turbulence transfer angular momentum inward descends to the convective core in a 5 $M_\odot$ accretor before mass exchange ends, allowing angular momentum to flow into the convective core during mass exchange. In a 2.5 $M_\odot$ accretor, the cell's lower boundary reaches the convective core only at the very end of mass exchange (Figure 1), with angular momentum beginning to flow into the core after mass exchange concludes.

After mass exchange ends, 18% of the angular momentum brought by accreted matter remains in the accretor for the first system and 16% for the second.

3.3. Accretor Spin-up in Systems with Different Separations

The thermal timescale of a star depends on its size. Mass exchange duration is longer and the mass exchange rate is smaller in systems with smaller component separations (Table 1). The accretor size increases less during accretion when the accretion rate is lower. The ratio of accretor size for $A_0 = 15R_\odot$ to that for $A_0 = 30R_\odot$ is shown in Figure 4. Matter is added to the accretor with Keplerian rotational velocity, so the angular momentum brought into the accretor in the $A_0 = 15R_\odot$ case is less than in the $A_0 = 30R_\odot$ case. Nevertheless, the accretor's angular momentum after mass exchange is the same for both $A_0 = 15R_\odot$ and $A_0 = 30R_\odot$ (Figure 5). The angular momentum transferred by circulation to the accretor surface and subsequently lost depends on the initial separation between components.

3.4. Evolution of Angular Velocity Profile After Mass Exchange

After mass exchange ends, the accretor restores thermal equilibrium. A decrease in accretor size while conserving angular momentum would cause the surface angular velocity to exceed the Keplerian value. Thermal equilibrium restoration occurs with slight angular momentum loss, maintaining the surface angular velocity at the Keplerian value. The need to reduce angular momentum may trigger decretion disk formation immediately after mass exchange ends. Such a system could be observed as a binary with a disk but no inter-component mass flow.

The main process during thermal equilibrium restoration is angular momentum transfer into the accretor's inner layers (Figure 6). This increases the convective core's angular velocity while decreasing that of the outer radiative envelope (Figure 1). This change reduces meridional circulation velocity, which reaches a maximum of 9×10^{-3} cm s^{-1} in a 4.9 $M_\odot$ accretor by the end of thermal equilibrium restoration.

Over longer timescales, evolutionary structural changes affect angular velocity. Slight core compression and envelope expansion contribute to increasing the core angular velocity and decreasing the envelope angular velocity. With continued angular momentum transfer into the star, the core begins rotating faster than the envelope. Shortly thereafter, the direction of angular momentum transfer reverses, and circulation and turbulence begin transferring angular momentum outward, as occurs in main-sequence rotating stars.

4. Discussion and Conclusions

In all considered cases, the accretor's convective core mass increases during mass exchange, with hydrogen content rising to \$ \$0.6 by mass exchange end. The accretor acquires characteristics of a slightly evolved main-sequence star, with a moment of inertia 14% greater than that of a zero-age star of the same mass. In principle, the accretor's angular momentum could exceed the maximum angular momentum of a zero-age star model. However, due to removal of part of the angular momentum by circulation and the disk during mass exchange, the accretor's angular momentum is less than the zero-age star model's maximum. On the other hand, the accretor achieves faster rotation than observed in young early-spectral-subclass stars (B0–B3). The evolution of stars with such angular momentum (Table 1) is accompanied by an increasing V_e/V_c ratio over time (Meynet & Maeder 2005; Staritsin 2007; Ekstrom et al. 2008; Granada et al. 2013). An accretor may develop Be-star characteristics during subsequent binary evolution if the component separation is large enough for tidal interaction to be negligible. Thus, conservative mass exchange in binary systems may explain the rapid rotation of Be-stars across the entire mass range observed in pairs with O-subdwarfs.

Population synthesis methods enable tracing the evolution of numerous binary systems through simplified descriptions of the mass exchange stage, often assuming rigid rotation of the accreting star (Mink et al. 2013). This assumption is justified by the slight deviation from rigid rotation in single main-sequence stars. Detailed calculations of angular momentum transfer in accreting star interiors reveal differential rotation, but after mass exchange ends and thermal equilibrium is restored, the accretor's rotation is not significantly different from rigid. In conservative mass exchange, the accreted mass is about half the accretor's mass, and angular momentum transfer from accreted matter to the interior during thermal equilibrium restoration does little to reduce surface angular velocity. Therefore, the population synthesis assumption of rigid rotation is justified for conservative mass exchange.

The conclusion about the independence of the accretor's final angular momentum from the angular velocity of accreted matter was obtained previously in calculations where the accreted matter angular velocity was varied (Staritsin 2023b, 2024).

Accretor spin-up during conservative mass exchange in the Hertzsprung gap does not depend on the initial separation between binary components. The removal of part of the angular momentum from the accretor by circulation and the disk during mass exchange cannot explain the observed rotation of accreting components in long-period Algols. The shear flow that forms inside the accretor during mass exchange could be a source of magnetic field. Under suitable conditions, magnetic field-disk interaction could decelerate the accretor's rotation, but this mechanism's effect does not depend on the initial separation and is also unable to explain the observed rotation of accreting components in long-period Algols (Van Rensbergen & De Greve 2020).

The only deceleration mechanism whose effect depends on the distance between binary components is tidal interaction. Further study of this mechanism's details may help resolve the rotation issue for accreting components in long-period Algols.

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