# A Novel Ephemeris Model for Martian Moons Incorporating Their Free Rotation Postprint

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## Abstract

High-precision ephemerides not only support space missions, but can also be used to study the origin and future of celestial bodies. In this paper, a coupled orbit-rotation dynamics model that fully takes into account the rotation of the Martian moons is developed. Phobos and Deimos' rotations are first described by Eulerian rotational equations, and integrated simultaneously with the orbital motion equations. Orbital and orientational parameters of Mars satellites were simultaneously obtained by numerical integration for the first time. In order to compare the differences between our newly developed model and the one now used in the ephemerides, we first reproduced and simulated the current model using our own parameters, and then fit it to the Institut de Mécanique Céleste et de Calcul des Éphémérides ephemerides using least-square procedures. The adjustment test simulations show Phobos and Deimos' orbital differences between the refined model and the current model are no more than 300 m and 125 m, respectively. The orientation parameters are confirmed and the results are in good agreement with the International Astronomical Union results. Moreover, we simulated two perturbations (main asteroids and mutual torques) which were not included in our refined model, and find that their effects on the orbits are completely negligible. As for the effect on rotation, we propose to take care of the role of mutual attraction in future models.

## Full Text

## Preamble

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A Novel Ephemeris Model for Martian Moons Incorporating Their Free Rotation

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## Abstract

High-precision ephemerides not only support space missions, but can also be used to study the origin and future evolution of celestial bodies. In this paper, we develop a coupled orbit-rotation dynamics model that fully accounts for the rotation of the Martian moons. Phobos and Deimos' rotations are first described by Eulerian rotational equations and integrated simultaneously with the orbital motion equations. For the first time, orbital and orientational parameters of Mars' satellites were obtained simultaneously through numerical integration. To compare differences between our newly developed model and the one currently used in ephemerides generation, we first reproduced and simulated the current model using our own parameters, then fit it to the Institut de Mécanique Céleste et de Calcul des Éphémérides ephemerides using least-squares procedures. Adjustment test simulations show that the orbital differences between the refined model and the current model are no more than 300 m for Phobos and 125 m for Deimos. The orientation parameters are confirmed and the results are in good agreement with International Astronomical Union standards. Moreover, we simulated two perturbations (main asteroids and mutual torques) that were not included in our refined model, and find that their effects on the orbits are completely negligible. Regarding rotational effects, we propose that the role of mutual attraction be considered in future models.

Key words: planets and satellites: dynamical evolution and stability – methods: numerical – celestial mechanics – astrometry

## 1. Introduction

Mars is the planet most similar to Earth in the solar system and is the only terrestrial planet besides Earth to have natural satellites. Phobos and Deimos are the two moons of Mars. Since their discovery in 1877, the orbital motion of the Martian moons has been studied extensively. A variety of dynamical models have been developed to fit observational data, progressing from Earth-based observations to spacecraft observations. During the Mariner and Viking era, Sinclair (1971, 1978) and Shor (1975) employed analytical expressions to fit various sets of Earth-based observations to generate ephemerides and confirmed the secular tidal acceleration first studied by Sharpless (1945). Jacobson et al. (1989) and Sinclair (1989) used all available positional observations of Mars' satellites, including Earth-based data and observations from the Mariner 9 and Viking spacecraft, to re-determine the orbits of the Martian moons. These ephemerides remain available in the SPICE library to this day (Arora & Russell 2010).

The first completely numerical dynamical model of the Martian moons was developed by Lainey et al. (2007) during the Mars Express (MEX) mission. This model incorporated: (1) the aspherical Martian gravity field, (2) perturbations from the Sun, Jupiter, Saturn, Earth, and the Moon using planetary ephemerides, (3) the IAU2000 Martian precession/rotation, (4) the mass of each Martian moon, and (5) tidal effects modeled by the tidal bulge raised by each moon on Mars using physical formulations instead of fitting secular accelerations in the satellite longitudes. After fitting to MEX, Mars Global Surveyor (MGS), Phobos 2, Viking 1–2, Mariner 9, and ground-based observations, new ephemerides of the Martian moons were developed based on this first numerical dynamical model. It is worth mentioning that although the authors recognized that perturbations due to the librations of the Martian moons have a significant influence that cannot be ignored, this effect was not modeled due to the lack of accurate C20 coefficients and estimated libration angles (Lainey et al. 2007).

Jacobson (2010) upgraded the dynamical model of the Martian satellites, introducing the effect of Phobos' libration in the form of an analytical formula for the first time. Assuming Phobos rotates synchronously, its pole is perpendicular to the orbital plane, and its axis of minimum principal moment of inertia points toward Mars. The angle between Phobos' axis of minimum principal moment of inertia and the direction from Phobos to Mars, also called the libration angle, is small and can be described by

$$\langle MATH_0 \rangle$$

where $\mathcal{A}$ is the libration amplitude that can be calculated from the moments of inertia (Chao & Rubincam 1989), and $e$ and $M$ are Phobos' orbital eccentricity and mean anomaly, respectively. Revised orbits of the Martian moons were obtained by fitting this numerical dynamical model to all available Earth-based observations, imaging observations, and radio tracking data from spacecraft.

The above dynamical model has been used until now, and although observational data have accumulated over time, the dynamical model has seen only minor additions. Examples include (1) mutual attraction between satellites, (2) general relativity, and (3) the second gravitational field of satellites (Jacobson & Lainey 2014; Lainey et al. 2020). This is mainly due to the difficulty of determining the gravitational coefficients of Phobos and Deimos with current observations.

The Martian Moons eXploration (MMX) mission is under development by the Japan Aerospace Exploration Agency (JAXA) and is scheduled for launch in 2026. This mission is dedicated to surveying the two Martian moons and returning samples from Phobos (Kawakatsu et al. 2023). To achieve the objective of collecting samples from Phobos, a probe will land on Phobos' surface. At that time, tracking data between the lander and Earth will become available, offering new opportunities to study the orbital and rotational motions of the Martian moons.

Inspired by the MMX mission, this paper presents a refined numerical dynamical model for the libration of the Martian moons. The libration is described using the Euler-Liouville equations, i.e., with a state of complete free rotation that does not rely on any assumptions. This dynamical model will allow us to study the motions of the Martian moons in the future with more realistic scenarios based on new observational data, such as those from MMX. In Section 2 we briefly introduce the currently used dynamical model. In Section 3 we detail our optimized libration model of the Martian moons. Section 4 provides a detailed comparison of these two models, followed by Section 5 where we summarize and conclude.

## 2. Review of Numerical Dynamical Model

In this work, we apply the currently used numerical model as a reference to study its differences with our revised model. Hence, we first introduce the modeling process of an ephemeris model.

The equations of translational motion are described in a planetocentric (Mars) reference system with fixed axes that align with the International Celestial Reference System (ICRS). The position vectors of the eight planets and the Sun relative to Mars in this reference system can be easily retrieved from numerical ephemerides. Here we use the latest version of INPOP21a provided by the Institut de Mécanique Céleste et de Calcul des Éphémérides (IMCCE) to obtain the position and velocity vectors of the Sun and planets relative to Mars in the Barycentric Celestial Reference System (BCRS; Fienga et al. 2021). This ephemeris updates the Mars orbit relative to INPOP19a (Fienga et al. 2020), adding an additional 2 yr of data from MEX.

The orbital motion of the satellites around Mars can be described in terms of position $\mathbf{r} \equiv (r_x, r_y, r_z)$ and velocity $\mathbf{v} \equiv (v_x, v_y, v_z)$ in rectangular coordinates. The classical differential equations of relative motion in the planetocentric system can be written as

$$\langle MATH_1 \rangle$$

where $\mathbf{F}_s$ and $\mathbf{F}_0$ indicate all external forces exerted on the satellites and Mars, $m_s$ is the mass of the satellite, $m_0$ is the mass of Mars, and $t$ is time expressed in the Barycentric Dynamical Time (TDB) timescale.

The forces that induce the relative motion can be split into a two-body part, a third-body perturbation part, a mutual attraction part, a tidal perturbation part, a relativistic perturbation part, and a spin libration part. Hence, Equation (2) can be rewritten as

$$\langle MATH_2 \rangle$$

where $\mathbf{a}_{\text{two-body}}$, $\mathbf{a}_{\text{third-body}}$, and $\mathbf{a}_{\text{rel}}$ have the usual forms used in numerical ephemerides (Folkner et al. 2014; Pitjeva & Pavlov 2017; Viswanathan et al. 2017).

If we consider Deimos as a third body, we can calculate its effect on Phobos' orbit. According to the third-body perturbation equation, the acceleration of Phobos due to mutual attraction can be described as

$$\langle MATH_3 \rangle$$

Similarly, the acceleration of Deimos due to mutual attraction is

$$\langle MATH_4 \rangle$$

Here $\mu_d$, $\mu_p$, $\mathbf{r}_d$, and $\mathbf{r}_p$ are the gravitational parameters of Deimos and Phobos, and the position vectors of Deimos and Phobos relative to Mars in the inertial system, respectively. $r_p$ denotes the norm of $\mathbf{r}_p$, $r_d$ signifies the norm of $\mathbf{r}_d$, and $r_{dp} = r_{pd} = |\mathbf{r}_p - \mathbf{r}_d|$.

For tidal acceleration $\mathbf{a}_{\text{tide}}$, Lainey et al. (2007) and Jacobson (2010) employed different but similar formulations in their numerical integrations. In this paper, we simulate differences between our new model and the French Numerical Orbit and Ephemerides (NOE); hence, we refer to Lainey et al. (2007) for a complete description of the tidal force $\mathbf{F}_T$ acting on the satellite of the form

$$\langle MATH_5 \rangle$$

where: $k_2$ is the Love number of Mars, $\omega_0$ is the Martian angular velocity vector, $m_s$ is the satellite mass, $\Delta t$ is the time delay due to viscoelastic response of Mars, and $R_0$ is the equatorial radius of Mars.

Finally, we present the libration model used in current ephemerides of Martian moons. Under the assumption that the spin pole is normal to the orbital plane, according to Equation (1), the quadrupole force on Mars exerted by a satellite can be computed as

$$\langle MATH_6 \rangle$$

where $C_{20}$ and $C_{22}$ are the second zonal and sectorial harmonics of the satellite, respectively, $\hat{\mathbf{r}}$ denotes the unit vector directed from Mars toward the satellite, and $\hat{\mathbf{t}}$ signifies the unit vector in the satellite's orbit plane normal to $\hat{\mathbf{r}}$ and in the direction of its orbital motion. Therefore, the reaction force acting on the satellite is

$$\langle MATH_7 \rangle$$

with $\mu_m$ denoting the gravitational parameter of Mars.

Utilizing Equations (4)–(8) to calculate $\mathbf{a}_{\text{mutual}}$, $\mathbf{a}_{\text{tide}}$, and $\mathbf{a}_{\text{libr}}$ in turn and summing them together with $\mathbf{a}_{\text{two-body}}$, $\mathbf{a}_{\text{third-body}}$, and $\mathbf{a}_{\text{rel}}$, we obtain the orbital equations of motion in planetocentric coordinates.

To describe the rotational motion, the body-fixed frame of Phobos was aligned with its principal axes (PA). The orientation of Phobos' frame with respect to (w.r.t.) the inertial frame is determined by three Euler angles: $\phi$, $\theta$, and $\psi$, which vary with time $t$. The transformation from the inertial system to the body-fixed system (PA) is given by the matrix

$$\langle MATH_8 \rangle$$

where the rotation matrices $R_z$ and $R_x$ are right-handed rotations around the z-axis and x-axis, respectively. For simplicity, the argument $t$ will be omitted where appropriate.

In a rotating system, the rate of change of angular velocity $\omega$ is related to the torque $\mathbf{T}$ and determined by Euler-Liouville's equations of rotation

$$\langle MATH_9 \rangle$$

where $\mathbf{I}$ represents the moment of inertia tensor. This leads to the equations for $\omega$

$$\langle MATH_{10} \rangle$$

Consequently,

$$\langle MATH_{11} \rangle$$

The effect of elastic deformation is not considered in the current modeling because its effect on the Phobos ephemeris is less than 100 m even if Phobos' $k_2$ is as large as $1 \times 10^{-4}$ (Yang et al. 2024), corresponding to an extremely porous Phobos (Le Maistre et al. 2013). We consider this unlikely, since the porosity of Phobos is limited by the density of the material that constitutes the matrix of the bulk material. Hence, $\mathbf{I}$ is diagonal and constant. Solving Equation (11) for $\dot{\omega}$, we find that the resultant angular acceleration takes a simple form

$$\langle MATH_{12} \rangle$$

The components of the angular velocity vector in the body-fixed system are easily expressed in terms of reference Euler angles $(\phi, \theta, \psi)$ (Goldstein et al. 2002):

$$\langle MATH_{13} \rangle$$

## 3. Rotation Model of the Martian Moons

In this paper, the Martian moons are modeled as rigid bodies, as in our previous study of Phobos' libration (Yang et al. 2020). The orientations of Phobos and Deimos are integrated from differential equations for their angular velocities. The angular momentum vector of a satellite is the product of angular velocity and moment of inertia. The angular momentum vector varies with time due to external torques.

We present the rotation modeling process using Phobos as an example. To describe variations of Phobos' rotation in the inertial frame, for convenience we define

$$\langle MATH_{14} \rangle$$

where $(\phi, \theta, \psi)$ are the precession angle, nutation angle, and rotation angle, respectively.

If we differentiate Equations (13) with respect to time $t$ and rearrange them, we obtain a linear system of equations containing $\ddot{\phi}$, $\ddot{\theta}$, and $\ddot{\psi}$. The above equations model the Euler angle equations of motion, and the key is to calculate the angular acceleration $\dot{\omega}$, which can be evaluated by Equations (10)–(12). Having established the mathematical relationship between Euler angles and external torques through angular acceleration, we now derive the calculation of external torques.

The torque $\mathbf{T}$ applied to a satellite is usually divided into two parts: a torque from point mass (body) $A$ to the satellite's figure and a torque from the oblateness ($J_2$) of Mars to the satellite's figure. A detailed description of $\mathbf{T}$ and $\mathbf{I}$ can be found in Williams et al. (2001), Rambaux et al. (2012), Folkner et al. (2014), Pavlov et al. (2016), and Yang et al. (2020).

The instantaneous rotational state of a rigid body can be completely defined by six quantities: the Euler angles defined above and their rates of change. In this paper, we use this approach to model the libration of Phobos and Deimos. Unlike models used in current ephemerides (which assume the pole is normal to the orbital plane), the Martian moons in our model are completely free to rotate without any assumptions.

## 4. Comparison

The studies of the rotation and orbital motion of the Martian moons described in the previous sections are motivated by the high-precision observational data that may become available from future missions, such as a probe's orbital data and satellite image data when the probe is at very close range. In addition, lander tracking measurements on Phobos have been proposed (Kawakatsu et al. 2017; Usui et al. 2018). In this section, we simulate our new dynamical model incorporating the free rotation of the Martian moons and compare it with current ephemeris models.

### 4.1. Methodology

In our numerical model, numerous parameters significantly affect the orbit, such as the Martian gravitational field and the satellites' initial positions and velocities. To clarify differences between the fully coupled approach and the simple libration model (Equations (7)–(8)) used previously, we followed previous approaches (Yang et al. 2024) and first simulated the current simple model (Lainey et al. 2007, 2020; Jacobson 2010; Jacobson & Lainey 2014) but with our own selected physical parameters listed in Table 1. We then fitted the twelve initial parameters (positions and velocities of Phobos and Deimos) as solve-for parameters to match the current ephemeris. This fit resolves issues arising from differing parameters and provides the best reference for investigating differences between the new full model and the previously used ephemeris model. To fit model parameters, we introduce the common relational formula

$$\langle MATH_{15} \rangle$$

where $\Phi$ and $\dot{\Phi}$ are the Euler angles and their rates, respectively. In particular, $\Phi$ and $\dot{\Phi}$ are not modeled in the ephemeris model and should be omitted. $P_j$ denotes unspecified model parameters to be fit (such as initial positions, velocities, Euler angles, etc.), and the variational equations are integrated simultaneously with the dynamical model.

### 4.2. Fit to NOE Ephemerides

We selected the latest Martian ephemerides NOE-4-2020 (Lainey et al. 2020) as "observational data" and fit our simple model to them. The adjustment was performed by least-squares in Cartesian planetocentric coordinates J2000, using a sample set of 3650 points with a step size of one day (ten years), with no weights assigned. We started with the initial epoch at JD 2451545.0 (J2000.0, TDB timescale) and integrated the model over a decade. The residuals after fitting are shown in Figure 1. The resulting position differences are likely explained by different physical parameters (such as the physical libration amplitude $\mathcal{A}$, the $C_{20}$ and $C_{22}$ of the satellites, the Martian dissipation factor $Q$, etc.) in the two models.

[Figure 1: see original paper]

### 4.3. Fit to the Ephemeris Model

To explore differences between our refined new dynamical model and previous ephemeris models (Jacobson 2010; Jacobson & Lainey 2014; Lainey et al. 2020), we adjusted our new refined model to the simulated ephemeris integrated in Section 4.2, with the physical parameters of the two models being identical, so the differences are mainly due to model details.

[Figure 2: see original paper]

We selected the twenty-four initial conditions (each satellite's position, velocity, Euler angles, and their rates) as solve-for parameters to fit the simulated results from the previous section. The initial Euler angles and rates were taken from the International Astronomical Union (IAU) rotational elements (Archinal et al. 2018). Least-squares procedures were then applied to the model. Figure 2 shows the distance differences after adjustment. The positional deviation may be due to the newly introduced Phobos latitudinal libration and different longitudinal librations in the refined model.

One advantage of our refined model is that the Euler angles of the Martian moons and their rates can be obtained by numerical integration. Here we plot the evolutions of the Euler angles defined with respect to the inertial reference frame for 10 yr from 2000 to 2010 January 1 (TDB timescale) in Figures 3–6. Phobos and Deimos' precession angle $\phi$ and nutation angle $\theta$ are plotted alongside IAU modeling results in Figures 3 and 5, respectively. The rotation angles $\psi$ obtained by integration and their differences from the IAU results are shown in Figures 4 and 6, respectively. Although the reference frame used by the IAU is slightly different from ours (Archinal et al. 2018), the results are still in pretty good agreement.

[Figure 3: see original paper]

[Figure 4: see original paper]

To characterize the high-frequency spectrum of the difference between our numerical integration results and the IAU polynomials, we decomposed the difference in the frequency domain using the method from Yang et al. (2017, 2019). The period and frequency of the largest amplitude term are shown in Table 2. These high-frequency oscillations are due to the rotational motion of the satellites and their orbital motion around Mars.

[Figure 5: see original paper]

[Figure 6: see original paper]

To demonstrate differences between the librations of the two models, Figure 7 presents the longitude of the direction from the Martian moons to Mars and the moons' axis of minimum principal moment of inertia for both models. By comparison, we can see that the differences between the two dynamical models are very small, indicating that the deviation in the longitude direction can be well described by the simple model (Equation (1)). The moons' obliquities are shown in Figure 8. Based on the assumption that the pole is normal to the orbital plane, these parameters were not considered in the simple model, and these values are an order of magnitude smaller than the longitudinal librations.

[Figure 7: see original paper]

[Figure 8: see original paper]

### 4.4. Consideration of Two Minor Perturbations

With the fitted initial conditions, we test here two perturbations that were not introduced in the newly established dynamical model. An easy way to quantify them is to compute the difference between a first simulation including the perturbation and a second simulation without it. The differences between simulations with and without each tested perturbation, integrated over 10 yr, are presented in Figures 9–11.

The first perturbation tested is the influence of the three largest asteroids: 1 Ceres, 2 Pallas, and 4 Vesta. Their orbital information is taken from the JPL SPICE kernel files (https://naif.jpl.nasa.gov/pub/naif/). The simulations indicate that this perturbation introduces only several centimeters of influence on the satellites' orbits, and the effect on rotation is also very small, less than 0.2 milliarcsecond.

The second perturbation tested is the presence of mutual mass torques between the two Martian moons. For example, the point torque from Deimos to Phobos' figure can be calculated by

$$\langle MATH_{16} \rangle$$

where $\mathbf{f}_{\text{figP-pmD}}$ is the force acting on Deimos as a point mass in Phobos' gravitational field, and in turn one can calculate the torque of the point mass Phobos on Deimos. The main related effect is that the torque on Deimos from Phobos' point mass can lead to differences in rotation angle reaching up to 1  over 10 yr. The results show that neither of the two perturbations considered in this section appears to have an effect on the orbit at an observable level. For rotation, however, the effect of the point mass torque by Phobos on Deimos' rotation can reach up to 1 arcsecond, so this factor is recommended to be taken into account in the modeling process.

[Figure 9: see original paper]

[Figure 10: see original paper]

[Figure 11: see original paper]

## 5. Conclusion

High-precision numerical ephemerides typically provide information on the position, velocity, and orientation parameters of celestial bodies over time, in addition to enabling detailed studies of their evolution and internal structure. This work developed a new numerical dynamical model of the motions of the Martian satellites, fully accounting for their rotation, and constructed a coupled orbit-rotation dynamical model using a method often applied in lunar motion studies (Folkner et al. 2014; Pavlov et al. 2016). To study differences between the newly developed model and the currently used dynamical model, we first reproduced the ephemerides model, fitting it to the NOE-4-2020 ephemerides published by the Paris Observatory (Lainey et al. 2020) using the least-squares method, and then used it as the reference to which the newly developed model was fitted. The differences between the post-fit orbits of Phobos and Deimos for the two models are no more than 300 m and 125 m, respectively.

For the first time, we have simultaneously computed the Euler angles and their rates for Phobos and Deimos by numerical integration (Rambaux et al. 2012), and confirmed that the results are in good agreement with IAU values. Moreover, we simulated two possible perturbations that were not adopted in our refined model and found that their effects on the orbits are completely negligible. Regarding rotational effects, we propose that the role of mutual attraction on rotation be considered.

This revised numerical model of Martian satellite motion provides opportunities for further study of the Martian moons using high-precision observations from future missions such as MMX. In the future, not only positions but also orientation parameters of the satellites can be derived from the refined dynamical model (Yang et al. 2024). Finally, our improved model of Martian satellite dynamics employs a generalized approach that can be extended to systems beyond the Martian system, such as Saturn and Jupiter, by appropriately treating the rotational model.

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*Note: Figure translations are in progress. See original paper for figures.*