Study of Elastic Scattering Effects of a Slowing-Down Model on D-T Neutron Porosity Logging

Authors: Shao-Heng Zhou, Ying-Long Tao, Bo Chang, Li-Juan Hao, Jie-Qiong Jiang, Ya-Zhou Li, Fang Wang, Quan Gan, Li-Juan Hao

Date: 2024-12-04T00:00:00+00:00

Abstract

measurement method used to determine the formation porosity. The americium-beryllium (Am-Be) source is commonly used for conventional porosity measurements. However, they pose potential risks to safety, health, and the environment [14]. Researchers have studied substituting chemical sources with deuterium-deuterium (D-D) neutrons [5-7]. However, the neutron yield of D-D neutron sources is relatively low, which restricts their application [8,9]. In 2005, the oilfield services company Schlumberger (now SLB) began using a deuterium-tritium (D-T) neutron source for compensated thermal neutron porosity logging [10]. Compared to the chemical source, the D-T neutron source has a higher neutron yield, higher neutron energy of 14 MeV, and a longer neutron slowing-down length; thus, D-T neutron logging can detect more thermal neutrons and has higher detection accuracy. Nevertheless, neutron slowing-down is facilitated by inelastic scattering owing to the higher neutron energy. Consequently, the D-T neutron recording exhibits reduced sensitivity to porosity compared to the chemical source [11,12], as measured by the conventional neutron porosity interpretation model founded on the Am-Be source [13-15]. Methods have been proposed to improve the porosity sensitivity of D-T neutron logging. The near and far thermal neutron counts were corrected using the density correction function, and the modified thermal neutron counting ratio was considered to be only associated with the hydrogen index [16]. Further, the density correction methods were summarized to modify the thermal neutron counting ratio and studied by MC simulation [17]. The relationship between the neutron slowing-down length, formation density, and hydrogen index was deduced and numerically simulated [18]. Based on data from the standard calibration experiment and actual logging [19,20], it was verified that the density correction results were consistent with the isotopic neutron source. In addition, the slowing-down length of the D-T neutron source can be converted into that of the Am-Be source. One method involves using the relationship between the neutron slowing-down length, cap-

ture cross-section, and density to predict the counting ratio [21]. The second involves the combination of the two-group diffusion theory with the formation density, reaction cross-section, and neutron slowing-down length [22].

Full Text

Preamble

Study of the Elastic Scattering Effects of a Slowing-Down Model on D-T Neutron Porosity Logging

Shao-Heng Zhou,[1,2,3] Ying-Long Tao,[3] Bo Chang,[3] Li-Juan Hao,[3,4,†] Jie-Qiong Jiang,[1,5] Ya-Zhou Li,[3,5,6] Fang Wang,[3,5,6] Quan Gan,[3,6] and FDS Consortium

[1]Hefei Institutes of Physical Science, Chinese Academy of Sciences, Hefei 230031, China
[2]University of Science and Technology of China, Hefei 230026, China
[3]International Academy of Neutron Science, Qingdao 266199, China
[4]Thinking Neutron Technology Co., Ltd., Hefei, 230031, China
[5]Institute of Nuclear Science and Technology, Shandong University, Jinan 250061, China
[6]Shandong Key Laboratory of Neutron Science and Technology, Qingdao 266199, China

The application of controllable neutron sources for measuring formation porosity in nuclear logging has garnered increased attention. Existing porosity algorithms based on thermal neutron counting ratios exhibit lower sensitivity in high-porosity regions. To enhance sensitivity, we theoretically analyzed the effects of elastic and inelastic scattering on fast neutron slowing-down and developed a corresponding slowing-down model. Based on this model, we proposed a density-correction porosity algorithm that incorporates the relationship between density, thermal neutron counting ratio, and porosity. Finally, we used the super multi-functional calculation program for nuclear design and safety evaluation (TopMC/SuperMC) to create a simulation model for porosity logging and examine its applicability. The results demonstrated that the relative error between calculated and actual porosities was less than 1%, and the influence of density measurement deviation was less than 2%. Therefore, the proposed density-correction algorithm based on the fast neutron slowing-down model can effectively improve sensitivity in high-porosity regions. This study is expected to serve as a reference for neutron porosity measurements using D-T neutron sources.

Keywords: D-T neutron source, Slowing-down model, Elastic scattering, Neutron porosity

Introduction

Neutron porosity logging is a conventional geophysical measurement method used to determine formation porosity. The americium-beryllium (Am-Be) source is commonly employed for conventional porosity measurements; however, it poses potential risks to safety, health, and the environment [?, ?, ?, ?]. Researchers have studied substituting chemical sources with deuterium-deuterium (D-D) neutron sources [?, ?, ?], but the relatively low neutron yield of D-D sources restricts their application [?, ?].

In 2005, the oilfield services company Schlumberger (now SLB) began using a deuterium-tritium (D-T) neutron source for compensated thermal neutron porosity logging [?]. Compared to chemical sources, the D-T neutron source offers higher neutron yield, higher neutron energy (14 MeV), and a longer neutron slowing-down length. Consequently, D-T neutron logging can detect more thermal neutrons and achieve higher detection accuracy.

Nevertheless, neutron slowing-down is facilitated by inelastic scattering due to the higher neutron energy. Consequently, D-T neutron measurements exhibit reduced sensitivity to porosity compared to chemical sources [?, ?] when using the conventional neutron porosity interpretation model based on the Am-Be source [?, ?, ?]. To overcome this problem, various density correction methods have been proposed to improve the porosity sensitivity of D–T neutron logging. The near and far thermal neutron counts were corrected using a density correction function, and the modified thermal neutron counting ratio was considered to be associated only with the hydrogen-containing index [?]. Further studies summarized density correction methods to modify the thermal neutron counting ratio and investigated them through Monte Carlo simulation [?]. The relationship between neutron slowing-down length, formation density, and hydrogen index was deduced and numerically simulated [?]. Based on data from standard calibration experiments and actual logging [?, ?], researchers verified that the density correction results were consistent with isotopic neutron source measurements. The slowing-down length of the D-T neutron source can be converted to that of the Am-Be source through two primary approaches: one involves the relationship between neutron slowing-down length, capture cross-section, and density to predict the counting ratio [?]; the second combines two-group diffusion theory with formation density, reaction cross-section, and neutron slowing-down length [?].

In these studies, the slowing-down length is expressed as $s = L^2_{E_0-E_1} + L^2_{E_1-E_t}$, where $L_{E_0-E_1}$ is the slowing-down length for neutrons from energy $E_0 = 14$ MeV to E_1, and $L_{E_1-E_t}$ is the slowing-down length for neutrons from energy E_1 to E_t (thermal neutron energy). The corresponding correction methods assume that the slowing-down length is determined by inelastic scattering from E_0 to E_1 and ignore the contribution of elastic scattering.

However, neutron slowing-down results from both elastic and inelastic scattering. At high porosity, the higher hydrogen content shortens the mean free path of elastic scattering [?, ?], making $L_{E_0-E_1}$ dominated by inelastic scattering. At low porosity, elastic scattering cannot be ignored because the long mean free path of D-T neutron elastic scattering for $L_{E_0-E_1}$ is primarily contributed by C, O, and Ca. Ignoring this leads to measurement deviations in porosity logging at low porosities. To improve the accuracy of D-T neutron porosity logging, Section II presents a neutron slowing-down model for the D-T source that considers elastic scattering throughout the entire process. In this model, both elastic and inelastic scattering contribute to neutron slowdown. Based on this model, Section III develops a porosity density-correction algorithm based on the relationship between density and porosity. Finally, Section IV verifies the effectiveness of the proposed method using a multifunctional calculation program for nuclear design and safety evaluation (TopMC/SuperMC) [?].

II. Theoretical Analysis

A. Distribution of Thermal Neutrons

Based on two-group diffusion theory, the distribution of thermal neutron flux density for a fast neutron point source in an infinite uniform medium is expressed as [?]:

$$\phi_t(r) = \frac{Q}{4\pi D_t r}\left(\frac{e^{-r/L_f} - e^{-r/L_t}}{L_f^2 - L_t^2}\right)$$

where Q is the strength of the fast neutron point source, r is the detector spacing, and D_t, L_f, L_t are the thermal neutron diffusion coefficient, fast neutron slowing-down length, and thermal neutron diffusion length, respectively. In typical formations, L_f is significantly larger than L_t, allowing Eq. (1) to be approximated as:

$$\phi_t(r) \approx \frac{Q}{4\pi D_t r L_f^2} e^{-r/L_f}$$

Assuming near and far detector spacings of r_s and r_l (as shown in Fig. 1 [Figure 1: see original paper]), with detector efficiencies η_l and η_s, the counting ratios for the near and far detectors are expressed as:

$$R_t = \frac{\eta_s \phi_t(r_s)}{\eta_l \phi_t(r_l)} = \frac{\eta_s r_l}{\eta_l r_s} e^{(r_l - r_s)/L_f}$$

The detector counting ratio is related to the fast-neutron slowing-down length. In conventional compensated porosity logging, neutrons emitted from the Am-Be source have energies below 5 MeV and are primarily slowed down by elastic scattering with hydrogen in the formation, making the thermal neutron counting ratio a direct reflection of formation porosity. However, neutrons produced by the D-T reaction have an energy of 14 MeV and are slowed down by both elastic and inelastic reactions with formation elements, meaning formation porosity cannot be directly determined from the thermal neutron counting ratio.

B. The Slowing-Down Model of Fast Neutrons

This subsection establishes a slowing-down model for D-T neutrons to investigate the relationship between the slowing-down length L_f and elastic scattering, particularly at low porosity.

According to the diffusion equation [?], the slowing-down length L_f can be expressed as:

$$L_f = \sqrt{\frac{\overline{r^2}}{6}}$$

where $\overline{r^2}$ is the mean square straight-line distance from a D-T neutron slowed down to thermal energy. As shown in Fig. 2 [Figure 2: see original paper], the neutron slowing-down process can be calculated using the multigroup method [?].

According to neutron transport theory, $\overline{r^2} = \sum_{k=1}^{n} r_k^2$, where r_k is the distance traveled by the neutron between the $(k-1)$th and kth scattering events, E_0 is the D-T neutron energy (14 MeV), and E_t is the thermal neutron energy.

When a neutron slows down, two types of scattering occur: elastic and inelastic. In elastic scattering, the neutron strikes the nucleus, which remains in its ground state. Inelastic scattering is identical except that the nucleus remains in an excited state. Neutrons lose energy through numerous elastic and inelastic scatterings. Therefore, $\overline{r^2}$ in Equation (5) is expressed as:

$$\overline{r^2} = \bar{n}_{in}\overline{r_{in}^2} + \bar{n}_{el}\overline{r_{el}^2}$$

where $\bar{n}_{in}$ and $\bar{n}_{el}$ are the average numbers of inelastic and elastic scattering events, respectively. Further, $\overline{r_{in}^2}$ and $\overline{r_{el}^2}$ can be expressed as:

$$\overline{r_{in}^2} = 2\lambda_{in}^2 = \frac{2}{\Sigma_{in}^2}$$

$$\overline{r_{el}^2} = 2\lambda_{el}^2 = \frac{2}{\Sigma_{el}^2}$$

where λ_{in} and λ_{el} are the inelastic and elastic mean free paths, respectively, and Σ_{in} and Σ_{el} are the macroscopic inelastic and elastic cross-sections, respectively. Substituting Equations (6) and (7) into Equation (4), L_f is rewritten as:

$$L_f = \sqrt{\frac{\bar{n}_{in}}{3\Sigma_{in}^2} + \frac{\bar{n}_{el}}{3\Sigma_{el}^2}}$$

A neutron slowing-down model was established to obtain $\bar{n}_{in}$ and $\bar{n}_{el}$. In this model, neutrons are slowed by elastic scattering or both elastic and inelastic scattering. To simplify calculations, we assumed that inelastic scattering is effective such that neutron energy falls below E_1 (the inelastic scattering threshold) through a single inelastic scattering event.

We define $a = \frac{\Sigma_{in}}{\Sigma_{in}+\Sigma_{el}}$ and $b = \frac{\Sigma_{el}}{\Sigma_{in}+\Sigma_{el}}$ as the probabilities of inelastic and elastic scattering, respectively. When a neutron' s energy drops to E_1 after i elastic collisions, the relationship is given by [**?**]:

$$\left(\frac{E_1}{E_0}\right) = \left(1 - \frac{2}{A+1}\right)^i = e^{-i\zeta}$$

where ζ is the lethargy of the formation. Based on this assumption, $\bar{n}_{in}$ and $\bar{n}_{el}$ can be obtained as:

$$\bar{n}_{in} = a\frac{1-b^i}{1-b} = \frac{\Sigma_{in}}{\Sigma_{in}+\Sigma_{el}}\frac{1-b^i}{1-b}$$

$$\bar{n}_{el} = i \cdot b\frac{1-b^i}{1-b} + b\frac{1-b^i}{1-b} - \frac{ib^i}{1-b}$$

These equations indicate that $\bar{n}_{in}$ and $\bar{n}_{el}$ are affected by Σ_{el}, Σ_{in}, and ζ, which are determined by the nuclides in the formation. Furthermore, $b\frac{1-b^i}{1-b}$ represents the average number of elastic scattering events during the neutron slowing-down process from E_0 to E_1. As the elastic scattering cross-section Σ_{el} in the formation is considerably larger than the inelastic scattering cross-section Σ_{in}, we have $1 - b^i \approx i(1-b)$. By substituting Equation (9) into Equation (8), L_f can be rewritten as:

$$L_f = \sqrt{L^2_{in,E_0-E_1} + L^2_{el,E_0-E_1} + L^2_{el,E_1-E_t}}$$

where $L_{in,E_0-E_1} = \sqrt{\frac{2}{3\zeta\Sigma_{in}(\Sigma_{in}+\Sigma_{el})}}$ is the slowing-down length of inelastic scattering, $L_{el,E_0-E_1} = \sqrt{\frac{2}{3\zeta\Sigma_{el}(\Sigma_{in}+\Sigma_{el})}}$ is the slowing-down length of elastic scattering in the neutron slowing-down process from energy E_0 to E_1, and $L_{el,E_1-E_t} = \sqrt{\frac{2}{3\zeta\Sigma_{el}}\ln\left(\frac{E_1}{E_t}\right)}$ is the slowing-down length of elastic scattering through which the neutron is slowed down from energy E_1 to E_t. Equation (10) shows that elastic scattering of high-energy neutrons corrects the slowing-down length.

C. Analysis of the Slowing-Down Model

To analyze the effects of elastic and inelastic scattering on the neutron slowing-down length, Table 1 presents theoretical calculation results for the slowing-down length of D–T neutrons in limestone with porosities ranging from 0% to 100%. Specifically, the inelastic scattering threshold was set to 3.6 MeV for limestone [**?**], and the cross-sections for inelastic and elastic scattering corresponded to neutron energies of 14 MeV and 10 MeV.

When the limestone porosity was 0%, no water was present; the macroscopic inelastic scattering cross-section was 0.0279 cm^{-1}, and the macroscopic elastic scattering cross-section was 0.3248 cm^{-1} [**?**]. Therefore, the mean free path of inelastic scattering was approximately 35 cm, whereas that of elastic scattering was approximately 3 cm. However, the nuclides in limestone are primarily C, O, and Ca, which have a weaker ability to slow down neutrons, resulting in more elastic collisions (about 165 times). This yielded $(L_{el,E_0-E_t})^2 = 525.56$ cm^2 and $(L_{in,E_0-E_t})^2 = 262.36$ cm^2. Thus, the effect of elastic scattering on the slowing-down length exceeded the contribution of inelastic scattering, resulting in a theoretically calculated slowing-down length of D–T neutrons up to 28 cm.

As the water content in limestone increased with porosity, the macroscopic cross-section of inelastic scattering decreased slowly, while the macroscopic cross-section of elastic scattering increased rapidly. In particular, hydrogen—the nuclide with the strongest ability to slow down neutrons—caused the number of elastic scatterings to decrease rapidly. For example, when porosity increased to 10%, the number of elastic scatterings decreased to 43, the elastic scattering mean free path was 2.0 cm, the number of inelastic scatterings decreased to 0.14, and the inelastic scattering mean free path was 38 cm. At this point, the contribution of elastic scattering to the slowing-down length was less than that of inelastic scattering. When porosity exceeded 50%, the contribution of elastic scattering was significantly lower than that of inelastic scattering and could be ignored.

Table 1 also indicates the contribution of D-T neutron elastic scattering in the high-energy region (14 MeV to E_1). The variation of the slowing-down length of elastic scattering in the high-energy region (L_{el,E_0-E_1}) was consistent with the variation of the whole elastic scattering slowing-down length; that is, the slowing-down length of elastic scattering decreased rapidly as porosity increased. However, the slowing-down length of elastic scattering from E_1 to E_t decreased faster than that in the high-energy region.

When porosity was 0%, the proportion of $(L_{el,E_0-E_1})^2$ in the elastic scattering reached approximately 4%, whereas at 30% porosity, the proportion was approximately 7%. Therefore, the contribution of elastic scattering cannot be ignored for accurate porosity measurements. When addressing the slowing-down length above the inelastic threshold energy, although the contribution primarily originates from inelastic scattering, the contribution of elastic scattering cannot be neglected, as it affects the accuracy of porosity measurement.

In existing D-T neutron porosity logging methods, elastic scattering in the high-energy region is directly ignored, and porosity measurement is associated only with elastic scattering below the threshold energy. As shown above, although high-energy neutron elastic scattering has minimal contribution to the slowing-down length during the D-T neutron slowing-down process, ignoring it can still cause an error of approximately 4%. Therefore, using D-T neutrons for porosity logging requires establishing a suitable method that accounts for these effects.

III. Density Correction Porosity Algorithm

According to Equation (3), the thermal neutron counting ratio R_t remains almost unchanged in the high-porosity region because of inelastic scattering [?], making porosity less sensitive to the thermal neutron counting ratio for the D-T neutron source at high porosity. To enhance porosity sensitivity, this section introduces a density correction method based on Equation (11). By combining Equations (9) and (10), the slowing-down length can be written as:

$$L_f = \sqrt{L^2_{in,E_0-E_1} + L^2_{el,E_0-E_1} + L^2_{el,E_1-E_t}}$$

Equation (11) shows that L_f correlates with the macroscopic cross-sections of elastic and inelastic scattering, with the contribution of elastic scattering to the slowing-down length divided into terms above and below E_1. This demonstrates that elastic scattering cannot be ignored compared with inelastic scattering, particularly for low-porosity cases.

If the pore medium is pure water, the relationship between formation density and porosity can be described as:

$$\rho = \phi\rho_w + (1-\phi)\rho_b$$

where ρ is the formation density in g/cm^3, ρ_b is the matrix density, and ϕ is the porosity.

The lethargy of the formation from elastic collisions ζ can be expressed as:

$$\zeta = \frac{\sum_{i=1}^{n}\zeta_i N_i \sigma_i^{el}}{\sum_{i=1}^{n} N_i \sigma_i^{el}} = \frac{\zeta_b N_b \sigma_b^{el} + \zeta_O N_O \sigma_O^{el} + \zeta_H N_H \sigma_H^{el}}{N_b \sigma_b^{el} + N_O \sigma_O^{el} + N_H \sigma_H^{el}}$$

where ζ_b, ζ_O, ζ_H are the lethargies of the matrix, oxygen, and hydrogen, respectively, and ζ_i, N_i, σ_i^{el} are the lethargy, nucleon density, and microscopic elastic scattering cross-section of the ith nuclide in the formation. According to Equations (12) and (13), the macroscopic cross-section can be rewritten as a function of formation porosity and density.

A. Monte Carlo Simulation

$$\Sigma_{in} = N_A \left[(1-\phi)\frac{\rho_b}{M_b}\sigma_b^{in} + \frac{\rho-\rho_b}{1-\rho_b} \cdot 2\sigma_O^{in} \right]$$

$$\Sigma_{el} = N_A \left[(1-\phi)\frac{\rho_b}{M_b}\sigma_b^{el} + \frac{\rho-\rho_b}{1-\rho_b} \cdot 2\sigma_H^{el} \right]$$

$$\zeta\Sigma_{el} = N_A \left[(1-\phi)\frac{\rho_b}{M_b}\rho_b\zeta_b\sigma_b^{el} + \frac{\rho-\rho_b}{1-\rho_b} \cdot 2\zeta_H\sigma_H^{el} \right]$$

where N_A is Avogadro' s constant, ρ_b is the matrix density, M_b is the molecular weight of the matrix, and $\sigma_b^{in/el}$, $\sigma_O^{in/el}$, and σ_H^{el} are the corresponding inelastic/elastic cross-sections of the matrix, oxygen, and hydrogen, respectively. As evident, Σ_{in} exhibits a linear relationship with ρ, while Σ_{el} and $\zeta\Sigma_{el}$ have linear relationships with ϕ.

Substituting Equations (11) and (14) into Equation (3), the relationship between porosity ϕ, density ρ, and ratio R_t is obtained as:

$$\ln(R_t) = \frac{r_l - r_s}{\sqrt{\frac{2}{3\zeta\Sigma_{in}(\Sigma_{in}+\Sigma_{el})} + \frac{2}{3\zeta\Sigma_{el}(\Sigma_{in}+\Sigma_{el})} + \frac{2}{3\zeta\Sigma_{el}}\ln\left(\frac{E_1}{E_t}\right)}} + \ln\left(\frac{\eta_s r_l}{\eta_l r_s}\right)$$

To verify the performance of the proposed algorithm, TopMC/SuperMC was used to build the tool model and simulate D-T neutron porosity logging. Referencing the NeoScope logging tool from Schlumberger and considering practical instrument constraints—including instrument size, detector size, and shield layer thickness—the instrument design is shown in Figure 3 [Figure 3: see original paper]. The neutron porosity logging instrument was developed using a D-T neutron source, two ^{3}He neutron detectors, and two shields. The D-T neutron source yield was set as isotopic, with near and far source-to-detector spacings of 35 cm and 70 cm. To reduce direct measurement interference from the neutron source, tungsten alloy neutron shields were placed between the neutron source and the near detector, as well as between the near and far detectors. The borehole had a diameter of 20 cm, filled with fresh water and surrounded by limestone rocks of varying porosities with a diameter and height of 200 cm. The instrument was compressed against the borehole wall, and the limestone density was 2.703 g/cm^3.

The equation above is a second-order function of ϕ, and its root can be obtained as:

$$\phi = \frac{-Y \pm \sqrt{Y^2 - 4XZ}}{2X}$$

In Equation (16), $Z = C + D \cdot f(R_t) \cdot (g(\rho) + E)$ and $Y = A + B \cdot f(R_t) \cdot g(\rho)$, where $f(R_t)$ is a second-order polynomial function of $\ln(R_t)$, and $g(\rho)$ has a linear relationship with $1/\rho$. The constants A, B, C, D, E are calibration factors related to formation lithology and detector source distances, which can be calibrated during actual logging.

Equation (16) shows a complex relationship between porosity, density, and thermal counting ratio. In general, porosity cannot be simply written as the product of $f(R_t)$ and $g(\rho)$. Therefore, using the product of $f(R_t)$ and $g(\rho)$ results in deviations in D-T neutron porosity logging.

IV. Simulation and Analysis

Monte Carlo numerical simulation technology is widely used in nuclear logging [**?**, **?**, **?**, **?**, **?**, **?**]. It can effectively, accurately, and reliably simulate particle propagation and interaction in formations, thereby predicting various nuclear logging parameters. This capability is significant for optimizing logging design and improving logging interpretation accuracy. To verify the validity of Equation (16), we constructed a porosity logging instrument model and a standard well model using TopMC/SuperMC. Consequently, this section investigates the application of neutron porosity logging.

Fig. 3 [Figure 3: see original paper]. (Color online) Model of the instrument and the formation

Based on the calculation model shown in Fig. 3, thermal neutron counts were simulated for different porosities. By combining the simulated results with the least-squares method, the coefficients in Equation (16) were determined as follows:

In Eq. (17),
$Z = 0.1431 \cdot f(R_t) \cdot (g(\rho) + 0.092) - 0.0046$
and
$Y = 4.6069 - 0.9431 \cdot f(R_t) \cdot g(\rho)$.

B. Verification of the Algorithm

According to the algorithm in Equation (17), D-T neutron porosity logging was simulated for limestone porosities of 0.1%, 1%, 2%, 3%, 5%, 7%, 10%, 13%, 17%, 21%, 26%, 31%, 37%, 43%, 50%, 57%, 65%, 73%, 82%, 91%, and 99.9%. Figure 4 shows the performance of Equation (17) for different porosity values, verifying its accuracy. The calculated values almost always fell on the 45° line, demonstrating that porosity values calculated using Eq. (17) were highly consistent with true porosity values. Additionally, Figure 4 shows the relative errors of the calculated porosity values, indicating that the relative error between calculated and real porosities was less than 2% when porosity exceeded 10%, while the absolute error was less than 0.1% when porosity was below 10%.

If L_{el,E_0-E_1} in Eq. (11) is neglected, the slowing-down length can be described as $L_f = \sqrt{L^2_{in,E_0-E_1} + L^2_{el,E_1-E_t}}$. Thus, Equation (17) is simplified, and the algorithm of the model can be obtained as:

$$\phi = 1.768 \times \ln(0.811 \times R_t)^2 - 0.000176$$

Figure 5 [Figure 5: see original paper] presents a comparison of results calculated using Equations (17) and (18). As evident, porosity values calculated using both equations were consistent with actual porosity values (Figure 5(a)). Figure 5(b) compares the relative errors obtained using Equations (17) and (18) for porosity values exceeding 10%. Compared with Equation (17), the relative error of Equation (18) was larger, particularly at low porosities. Table 2 indicates that the absolute error reached 3.6% for Equation (18) when actual porosity was less than 7%, demonstrating that the influence of elastic scattering from E_0 to E_1 on porosity measurement cannot be ignored.

Influence of Density Accuracy on Porosity Measurement

The relationship between density and porosity can be observed from Equations (12) and (16). As density decreases, porosity tends to increase. The introduction of density correction raises concerns about the accuracy of porosity measurements caused by density measurement errors. In conventional density measurements, the measurement error $\Delta\rho$ should generally be less than 0.015 g/cm^3. To evaluate the effect of density precision on neutron porosity measurement, calculated results were obtained for density errors of +0.015 g/cm^3 and -0.015 g/cm^3, as shown in Fig. 6 [Figure 6: see original paper] and Table 3 .

As shown in Fig. 6 and Table 3, the absolute error between calculated and actual porosities was less than 0.3% when actual porosity was below 7%, while the relative error was less than 4% in areas where actual porosity exceeded 7%. In particular, when porosity exceeded 2%, the relative error was less than 30%. As porosity increased, the relative error gradually decreased. Therefore, the measurement error of density exerts minimal effect on porosity in conventional density measurements.

This method considers the nonlinear relationship between detector count and porosity and density during D-T neutron transport in the formation, indicating that the proposed algorithm can effectively improve porosity measurement accuracy compared with existing density correction algorithms. Furthermore, it greatly eliminates the influence of density measurement error in actual measurement processes.

V. Conclusion

The fast neutron slowing-down length of the D-T source is affected by both elastic and inelastic scattering. A slowing-down model for fast neutrons was established to explain the neutron slowing-down process. In this model, the contribution of elastic scattering to neutron slowdown was considered above the inelastic scattering threshold. The model shows that the slowing-down length of D-T neutrons comprises L_{in,E_0-E_1}, L_{el,E_0-E_1}, and L_{el,E_1-E_t}. Further analysis indicated that for formations with relatively low porosity, the probability of neutron slowing-down through inelastic scattering was higher. As porosity increased, the probability of inelastic scattering decreased. However, due to the rapid increase in the elastic scattering macroscopic cross-section, the contribution of inelastic scattering to the slowing-down length became significant. The results also indicated that ignoring elastic scattering above the inelastic scattering threshold energy would impact the slowing-down length by approximately 4%, which exceeds the required accuracy for porosity measurement. Therefore, the effect of elastic scattering cannot be ignored at high energies in porosity measurement.

Based on the slowing-down model, a porosity density-correction algorithm was developed to improve the porosity sensitivity of D-T neutrons. The algorithm represents a complex function of density and porosity and cannot be simply written as $f(\rho)g(\phi)$. Finally, a porosity logging model was created using TopMC/SuperMC, and the effectiveness of the established algorithm was verified through simulations. The results showed that the relative error between calculated and actual porosities was less than 2% when porosity exceeded 10%, while the absolute error was less than 0.1% when porosity was below 10%. If L_{el,E_0-E_1} was ignored, the relative error in porosity reached 19%. The comparison demonstrated that high-energy neutron elastic scattering exerts significant influence on porosity measurement accuracy.

Additionally, the effect of density precision on porosity measurement was evaluated. The results showed that the absolute error was less than 0.3% when actual porosity was below 7%, while the relative error was less than 4% in areas where actual porosity exceeded 7%.

The algorithm, proposed through theoretical deduction based on fixed lithology and detector-to-source distance, can improve the overall measurement accuracy of the density correction method. This study provides valuable reference for porosity measurements using D–T neutron sources.

This study was primarily conducted using simulation instruments in a standard well. Future work will involve scaling and testing based on actual instruments, with further analysis of influencing factors in complex environments, particularly the effects of lithology and borehole diameter.

VI. Author Contributions

All authors contributed to the conception and design of this study. Material preparation, data collection, and analyses were performed by Shao-heng Zhou, Ying-long Tao, Bo Chang, Li-juan Hao, Jie-qiong Jiang, Ya-zhou Li, Fang Wang, and Quan Gan. The first draft of the manuscript was written by Shao-heng

Zhou, and all authors commented on previous versions of the manuscript. All authors have read and approved the final version of the manuscript.

VII. Acknowledgement

This study was based on a TopMC/SuperMC simulation of the described model and received strong support from other members of the FDS Consortium.

VIII. Bibliography

[1] T. Gjerdingen, J. Hilton, N. Bounoua et al., Sourceless LWD porosity determination: A fit for purpose formation evaluation with significant HS&E benefits. SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers, (2012). DOI:10.2118/159522-MS

[2] J.D. Aitken, R. Adolph, M. Evans et al., Radiation sources in drilling tools: comprehensive risk analysis in the design, development and operation of LWD tools. SPE International Conference on Health, Safety and Environment in Oil and Gas Exploration and Production, March 20–22 (2002). DOI:10.2118/73896-MS

[3] A. Badruzzaman, S. Bames, F. Bair et al., Radioactive sources in petroleum industry: applications, concerns and alternatives. AsiaPacific Health, Safety, Security and Environment Conference, 4–6 (2009). DOI:10.2118/123593-MS

[4] Non-isotopic alternative technologies working group. Non-Radioisotopic alternative technologies white paper. U.S. Department of Homeland Security Cybersecurity and Infrastructure Security Agency (2019)

[5] F. Zhang, C. Yuan, Monte Carlo simulation on compensated neutron porosity logging with D-D neutron generator. Well Logg. Techno. 34(3), 227–232 (2010). DOI:10.16489/j.issn.1004-1338.2010.03.004 (in Chinese)

[6] Y. Yan, B.Y. Li, S.P. Zheng et al., Monte Carlo simulation of the neutron porosity logging using D-D neutron generator. J. Lanzhou Univ. (Nat. Sci.) 48(03), 123–127 (2012). DOI:10.13885/j.issn.0455-2059.2012.03.009 (in Chinese)

[7] A. Badruzzaman, A. Schmidt, A. Antolak et al., Neutron generator as alternatives to Am-Be source in well logging: an assessment of fundamentals. Petrophysics 60(1), 136–170 (2019). DOI:https://doi.org/10.30632/PJV60N1-2019a10

[8] L. Bond, K. Denslow, J. Griffin et al., Evaluation of non-nuclear techniques for well logging technology evaluation (PNNL, Washington, 2010). DOI:10.2172/1006309

[9] J. Griffin, T. Moran, L. Bond, Radiation source replacement workshop (PNNL, Washington, 2010). DOI:10.2172/1062523

[10] G. Weller, R. Griffiths, C. Stoller et al., A new integrated LWD platform brings next-generation formation evaluation services. In: SPWLA 46th Annual Logging Symposium. Society of Petrophysicists and Well-Log Analysts, Louisiana, USA, (2005).

[11] D.V. Ellis, J.M. Singer, Well logging for earth scientists. 2nd Ed. New York: Inc, (2007), 383–413. DOI: 10.1007/978-1-4020-4602-5_{15}

[12] H. Guo, K. Zhao, N. Lu et al., The design of "multiple energies from one source" in D-T neutron tube. Nucl. Sci. Tech. 27(05), 127–132 (2016). DOI:10.1007/s41365-016-0119-1

[13] A. Badruzzaman, A. Schmidt, A. Antolak. An Assessment of Fundamentals of Neutron Porosity Interpretation: Americium-Beryllium Source Versus Neutron Generator-Based Alternatives. SPWLA 58th Annual Logging Symposium. Society of Petrophysicists and Well-Log Analysts, Oklahoma, USA (2017).

[14] F. Zhang, X. Jin, S. Hou, Monte Carlo simulation in LWD compensated porosity logging with D-T pulsed neutron generator. Isot. 23, 15–20 (2010). DOI:10.7538/tws.2010.23.01.0015 (in Chinese)

[15] H. Yu, Y. Zhou, X. Chen et al., Analysis of factors affecting the sensitivity of neutron porosity logging. Nucl. Tech. 41, 53–58 (2018). DOI:10.11889/j.0253-3219.2018.hjs.41.020502 (in Chinese)

[16] Y. Ge, Q. Luo, H. Chen et al., A new porosity evaluation parameter based on neutron porosity logging. Appl. Radiat. Isot. 190, 110481 (2022). DOI:10.1016/j.apradiso.2022.110481

[17] J. Liu, F. Zhang, R.P. Gardner et al., A method to improve the sensitivity of neutron porosity measurement based on D-T source. Nat. Gas Sci. Eng. 33, 879–884 (2016). DOI: 10.1016/j.jngse.2016.06.028

[18] H. Wang, W. Wu, H. Wang et al., Formation density affects and corrections in pulsed neutron porosity logging. Nucl. Tech. 41, 80406 (2018). DOI:10.11889/j.0253-3219.2018.hjs.41.080406 (in Chinese)

[19] N. Yang, Data correction and experimental study of neutron porosity logging using D-T source. Nucl. Tech. 43, 90–94 (2020). DOI:10.11889/j.0253-3219.2020.hjs.43.040502 (in Chinese)

[20] H. Wang, A. Yue, X. Zhang et al., Study on density correction of pulsed neutron porosity logging. Well Logging Technol. 45, 552–556 (2021). DOI:10.16489/j.issn.1004-1338.2021.06.002 (in Chinese)

[21] S. Fricke, D.P. Madio, B. Adolph et al., Thermal neutron porosity using pulsed neutron measurements. In: SPWLA 49th Annual Logging Symposium. Society of Petrophysicists and Well-Log Analysts, Edinburgh, Scotland (2008).

[22] H. Yu, Z. Wang, Z. Xue et al., Corrections of fast neutron inelastic scattering effects on D-T neutron porosity logging. Appl. Radiat. Isot. 190, 110486 (2022). DOI:10.1016/j.apradiso.2022.110486

[23] K. H. Beckurts, K. Wirtz, Neutron Physics, 1st edn. (Springer-Verlag New York, 1964), pp.121-123

[24] M.B. Chadwick, M. Herman, P. Obložinský et al., ENDF/B-VII.1 Nuclear Data for Science and Technology: Cross Sections, Covariances, Fission Product Yields and Decay Data. Nuclear Data Sheets 112(12), 2887–2996 (2011).

[25] Y. Wu, Multifunctional neutronics calculation methodology and program for nuclear design and radiation safety evaluation. Fusion Sci. Technol. 74, 321–329 (2018). DOI: 10.1080/15361055.2018.1475162

[26] D. McKeon, H. Scott, SNUPAR-a nuclear parameter code for nuclear geophysics applications. Nucl. Sci. IEEE Trans. 36, 1215–1219 (1989). DOI:10.1109/23.34634

[27] Y. Qiu, L. Xia, H. Peng et al., On Benchmark Verification of MCNP Modeling in Compensated Neutron Logging. Well Logging Technol. 28, 471–477 (2004). DOI:10.16489/j.issn.1004-1338.2004.06.001 (in Chinese)

[28] Y. Zhou, H. Yu, M. Wang et al., Numerical simulation of controllable source neutron porosity logging based on CLYC detector. Nucl. Tech. 44, 74–79 (2021). DOI:10.11889/j.0253-3219.2021.hjs.44.040501 (in Chinese)

[29] H. Yu, H. Xiao, Monte Carlo Study on Shielding of D-T Neutron Porosity Logging-While-Drilling Tool. Nuclear Electronics & Detection Technology 33, 1123–1126 (2013). DOI:10.3969/j.issn.0258-0934.2013.09.020 (in Chinese)

[30] P. Parvaresh, M. Sohrabpour. Design of a neutron porosity probe using MCNP code. J. Radioanal. Nucl. Chem. 260(2), 299–303 (2004). DOI:10.1023/b:jrnc.0000027106.91120.1a

[31] R.C. Little, M. Mickael, K. Verghese et al., Benchmark neutron porosity log calculations: a comparison of MCNP and the specific purpose code McDNL. Nucl. Sci. IEEE Trans. 36(1), 1223–1226 (1989). DOI:10.1109/23.34636

[32] L. Zhang, H. Yu, Y. Li et al., Improved formation density measurement using controllable D-D neutron source and its lithological correction for porosity prediction. Nucl. Sci. Tech. 33(01), 26–36 (2022). DOI:10.1007/s41365-022-00988-1

Note: Figure translations are in progress. See original paper for figures.