Measurement and compensation of combined resonance driving terms in SESRI

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Abstract

In high-intensity synchrotrons, usually with strong effect of nonlinearity, like the High Intensity heavy-ion Accelerator Facility (HIAF), the inevitable large incoherent space-charge-induced tune spread will cross resonance lines, leading to beam instability. As a result, the space charge resonance compensation scheme becomes a vital technology for achieving high intensity. In this paper, the feasibility of resonance compensation and the stabilization of particles extremely close to the resonance line, $Q_x = \frac{5}{3}$, are demonstrated in the experiments conducted at the Space Environment Simulation and Research Infrastructure (SESRI) synchrotron. A study on Resonance Driving Terms (RDTs) applied to SESRI is conducted with a particular focus on the Combined RDTs (CRDTs). A linear relevance between the strength of sextupole correctors and CRDTs was experimentally established, through which the effective resonance compensation was finally achieved. Furthermore, the issue of frequency leakage was mitigated by introducing a new simple method. Although limited by corrector precision, this work shows a great compensation effect in SESRI. Meanwhile, this study confirms the stabilization of particles extremely close to the resonance line. It is the foundation for space charge resonance compensation scheme in the HIAF, as well as in other high-intensity synchrotrons. In these high-intensity synchrotrons, the tune spread is limited by the resonance lines, because particles that cross resonance lines will lose under the resonance caused by the combined effect of space charge resonance crossing and magnet errors. In order to break through this limitation, the space charge resonance compensation scheme is our subsequent target.

Full Text

Preamble

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In high-intensity synchrotrons with strong nonlinear effects, such as the High Intensity heavy-ion Accelerator Facility (HIAF), the inevitable large incoherent space-charge-induced tune spread will cross resonance lines, leading to beam instability. Consequently, space charge resonance compensation becomes a vital technology for achieving high intensity. This paper demonstrates the feasibility of resonance compensation and the stabilization of particles extremely close to the resonance line Qx = 5/3 through experiments conducted at the Space Environment Simulation and Research Infrastructure (SESRI) synchrotron. A study on Resonance Driving Terms (RDTs) applied to SESRI is presented with particular focus on Combined RDTs (CRDTs). A linear relationship between the strength of sextupole correctors and CRDTs was experimentally established, enabling effective resonance compensation. Furthermore, the issue of frequency leakage was mitigated by introducing a new simple method. Although limited by corrector precision, this work shows excellent compensation effects in SESRI. Meanwhile, this study confirms the stabilization of particles extremely close to the resonance line, providing a foundation for space charge resonance compensation schemes in HIAF and other high-intensity synchrotrons. In these high-intensity synchrotrons, the tune spread is limited by resonance lines because particles that cross resonance lines will be lost due to resonance caused by the combined effect of space charge resonance crossing and magnet errors. To break through this limitation, the space charge resonance compensation scheme is our subsequent target.

Keywords: SESRI, Nonlinearity, Resonance compensation, Frequency leakage

Introduction

Achieving accurate magnetic fields is essential in synchrotrons. However, field imperfections inevitably arise from numerous factors, including manufacturing tolerances, alignment errors, power supply fluctuations, eddy currents [**?**], and hysteresis effects. These imperfections introduce nonlinearities that lead to nonstructural resonances, diminishing beam lifetime, reducing dynamic aperture, and increasing beam loss. Compensation for these magnetic field errors thus plays a significant role in accelerator physics research.

This paper describes two experiments conducted at the Space Environment Simulation and Research Infrastructure (SESRI) synchrotron [**?**], constructed by the Institute of Modern Physics and located at the Harbin Institute of Technology. These experiments were performed near the resonance line Qx = 5/3, where over

90% beam loss occurs when sextupole correctors are deactivated. In this work, a notable resonance compensation effect was achieved, experimentally demonstrating that particles can remain stable in the vicinity of third-order resonance lines under appropriate compensation. The primary aim of these experiments was to verify the resonance compensation scheme by measuring and minimizing RDTs and to conduct preliminary research for the High Intensity heavy-ion Accelerator Facility (HIAF) [?].

Nonlinear beam dynamics remains a major challenge for high-quality beams. Studies on systematic coupling resonances [?], slow extraction [?], and proton therapy system design [?] demonstrate the importance of nonlinearity in accelerator physics. RDTs are valuable tools for studying the nonlinear effects of magnetic fields on particle beams in synchrotrons [?], focusing on identifying and quantifying specific perturbations that induce resonant behavior. The system Hamiltonian can be expressed as a series of Fourier coefficients depending on the amplitude and phase of beam oscillations [?]. RDTs have been applied in various experiments, including measurements of linear coupling, sextupole RDTs, and octupole RDTs at different energies and intensities in the CERN Super Proton Synchrotron (SPS) [?]. RDTs were measured to validate nonlinear optics commissioning [?] and achieve chromatic coupling correction in the Large Hadron Collider (LHC) [?]. At the BNL Relativistic Heavy Ion Collider (RHIC), the Three Beam Position Monitors (BPMs) method [?], derived from RDTs, was utilized to measure global and local resonance terms [?]. At the European Synchrotron Radiation Facility (ESRF), the Combined RDTs (CRDTs) method, which requires only one BPM for measuring and correcting synchrotron nonlinearity, was also confirmed through RDT applications [?]. Additionally, RDTs play an important role in analyzing resonances excited by space charge potential in synchrotrons [?]. Further theoretical research on RDTs can be found in [?].

RDTs require harmonic analysis of BPM data to identify spectral components associated with different multipole fields. Recording particle oscillation data necessitates a Turn-by-Turn (TbT) acquisition system to capture beam position information during oscillations. After normalizing this position information, Fourier analysis can extract the individual spectral components required for RDTs. The development of TbT acquisition systems dates back to the 1990s [?].

Stabilizing particles crossing resonance lines is a vital issue for high-intensity synchrotrons like HIAF because the large tune spread (Δ x -0.23, Δ y -0.37) due to space charge effects [?] causes inevitable resonance crossing and beam instability, representing the main limitation. In this paper, stabilization of particles extremely close to Qx = 5/3 was realized through the resonance compensation scheme described in Sec. III D.

The paper is structured as follows. Section II presents theoretical derivations and simulation work for the compensation scheme employed in the experiments. Section III shows the progress, results, and analysis of the two experiments, including raw data acquisition, the frequency leakage issue, compensation scheme, and experimental results. Section IV contains a summary of the entire paper.

II. Formalism and Simulation

A. CRDTs Formalism

The RDT formalism provides a theoretical framework for nonlinear beam dynamics based on the normal form description of beam oscillations shown in Eq. (1). According to RDTs, the horizontal TbT data of the complex Courant-Snyder variables at the BPM location, denoted as b, can be decomposed into a series of Fourier components [?]:

$$[(\tilde{x}-i\tilde{p}_x](N,b) = \sqrt{2I_x}e^{i(2\pi\nu_x N+\phi_{xb})}\sum_{jklm} f^{(b)}_{jklm}(2I_x)^{j+k-1}(2I_y)^{l+m}\times e^{i[(1-j+k)(2\pi\nu_x N+\phi_{xb})+(m-l)(2\pi\nu_y N+\phi_{yb})]}$$

where $\tilde{x}$ and $\tilde{p}_x$ are normalized coordinates, $I_{x,y}$ and $\phi_{x,y}$ are nonlinear action-angle variables, and $\nu_{x,y}$ are tunes influenced by detuning from linear coupling or amplitude-dependent effects. The horizontal and vertical phases of the lattice at the BPM location b are denoted as ϕ_{xb} and ϕ_{yb}. According to Eq. (1), the fundamental tune line ν_x and numerous secondary lines, such as $(1-j+k)\nu_x+(m-l)\nu_y$, generated by corresponding nonlinear magnets will appear within the spectrum of $\tilde{x}-i\tilde{p}_x$.

The resonance driving terms f_{jklm} in Eq. (1) are:

$$f^{(b)}_{jklm} = \frac{\sum_\omega h_{\omega,jklm}e^{i[(j-k)\Delta\phi^b_{\omega,x}+(l-m)\Delta\phi^b_{\omega,y}]}}{1-e^{2\pi i[(j-k)Q_x+(l-m)Q_y]}}$$

where n is the order of resonance, $n = j+k+l+m$, K is the normal multipole coefficient, J is the skew multipole coefficient, and L is the length of the ωth multipole. For example, when $jklm = 3000$, $\Omega_{\omega,n-1} = K_{2\omega}L$, and when $jklm = 0030$, $\Omega_{\omega,n-1} = iK_{2S\omega}L$.

According to Eq. (2), f_{jklm} will diverge when:

$$(j-k)Q_x + (l-m)Q_y = \ell$$

where   is any integer. When Eq. (4) is satisfied, resonances occur, and the most important values of $jklm$ can be identified through Eq. (4). For example, when $Q_x = 1.67$, meaning that $(3-0)Q_x + (0-0)Q_y \approx 5$, attention should be paid to the case $jklm = 3000$. Substituting $jklm = 3000$ into Eq. (1) and ignoring other f_{jklm}, it is evident that both the tune line ν_x and the secondary line driven by sextupole fields, $-2\nu_x$, in the horizontal spectrum should be considered.

Returning to Eq. (1), $\tilde{x}$ can be determined using the TbT acquisition system, while reconstruction of $\tilde{p}_x$ requires data from two BPMs [?]. Fourier analysis of $\tilde{x} - i\tilde{p}_x$ can extract information about the coefficients f_{jklm}, which is crucial for resonance measurement and compensation. Unfortunately, successful $\tilde{p}_x$ reconstruction requires a region without nonlinearities between the two synchronized BPMs. In the SESRI synchrotron, such a region does not exist because dipoles with nonlinearities are present between any two horizontal or vertical BPMs. However, measurement of CRDTs remains possible even without $\tilde{p}_x$ information.

CRDTs are linear combinations of RDTs designed to analyze the real part of $\tilde{x} - i\tilde{p}_x$. As previously discussed, with $Q_x = 1.67$, the $-2\nu_x$ spectral line in $\tilde{x} - i\tilde{p}_x$ is influenced by the coefficient f_{3000}. However, the $-2\nu_x$ spectral line in $\tilde{x}$ comprises both the $+2\nu_x$ and $-2\nu_x$ spectral lines from $\tilde{x} - i\tilde{p}_x$. This implies that this CRDT includes not only f_{3000} at the $-2\nu_x$ line but also f_{1200} at the $+2\nu_x$ line. A simple derivation can be found in equations D15-D17 of the arXiv version of [?]:

$$\tilde{x}(N,b) = \sqrt{2I_x}[e^{i(2\pi\nu_x N+\phi_{xb})}+e^{-i(2\pi\nu_x N+\phi_{xb})}]-2iI_x(3f^{(b)}_{3000}-f^{(b)*}_{1200})e^{-i2(2\pi\nu_x N+\phi_{xb})}+2iI_x(3f^{(b)*}_{3000}-f^{(b)}_{1200})e^{i2(2\pi\nu_x N}$$

from which the CRDT corresponding to f_{3000} is:

$$F^{(b)N}_{S3} = 3f^{(b)}_{3000} - f^{(b)*}_{1200}$$

According to Eqs. (2) and (3), Eq. (6) can be rewritten as:

$$F^{(b)N}_{S3} = \sum_{\omega} \frac{3h_{\omega,3000}[e^{i3\Delta\phi^b_{\omega,x}} - e^{i\Delta\phi^b_{\omega,x}}]}{1 - e^{i6\pi Q_x}}$$

with $h_{\omega,1200} = 3h_{\omega,3000}$. Two key conclusions can be drawn from Eq. (7). Firstly, $F^{(b)N}_{S3}$ has a linear relationship with each sextupole magnetic field within the synchrotron. Secondly, under conditions where the tune satisfies $3Q_x \approx N$ (with $Q_x \neq N$), in most cases when errors are randomly distributed in the synchrotron, the amplitude of $f^{(b)}_{3000}$ greatly exceeds that of $f^{(b)}_{1200}$, yielding the approximation $F^{(b)N}_{S3} \approx 3f^{(b)}_{3000}$. However, this approximation might break down when $|f^{(b)}_{1200}|$ is too large. For example, in the realm of linear coupling, it was observed that $|f_{1001}|$ exceeded $|f_{1010}|$ even though the tune was closer to the difference resonance (1,-1) excited by f_{1001} than to the sum resonance (1,1) excited by f_{1010} at the ESRF [?].

In summary, although only $\tilde{x}$ is measurable, it is possible to set the tune in the vicinity of the $3Q_x \approx N$ condition to achieve $F^{(b)N}_{S3} \approx 3f^{(b)}_{3000}$. Then, the linear relationship between $F^{(b)N}_{S3}$ and the intensity of each sextupole magnet can be measured. Consequently, the measured linear relationship between $F^{(b)N}_{S3}$ and the strength of sextupole correctors can be effectively used to compensate for sextupole field errors using only one BPM, producing similar effects as compensation through $f^{(b)}_{3000}$. It is important to add that the general compensation scheme aims to match and minimize (C)RDTs throughout the ring [?], meaning that (C)RDTs at all BPMs should typically be measured and minimized. However, in later experiments, sextupole fields in the ring arose only from alignment errors, high-order magnetic field errors of dipoles and quadrupoles, and sextupole hysteresis, meaning no strong sextupole fields were present. This caused the amplitude of $f^{(b)}_{3000}$ to remain almost constant around the ring, making the single-BPM compensation scheme introduced in this paper effective in this situation, though it might not achieve optimal compensation when strong sextupole fields such as chromaticity sextupoles are activated.

In conclusion, the key idea of this section is to realize the approximation $F^{(b)N}_{S3} \approx 3f^{(b)}_{3000}$ when $3Q_x \approx N$ through Eq. (7), enabling measurement and compensation with even a single BPM. Furthermore, as the entire scheme operates under the $3Q_x \approx N$ condition, the amplitude of secondary lines driven by sextupoles in the horizontal spectrum should be large enough for BPM measurement.

B. Simulation

The concept was tested using MADX-PTC single-particle tracking [?, ?] within the ideal lattice model of the SESRI synchrotron. To induce resonance, a small sextupole magnet was introduced into the lattice while maintaining the tune at $Q_x = 1.67$, $Q_y = 1.74$. The beam oscillation spectrum, depicted in Fig. (1), provides information for calculating $f^{(b)}_{3000}$, $f^{(b)}_{1200}$, and $F^{(b)N}_{S3}$ values: $3f^{(b)}_{3000} = -1.840 - 3.265i$, $f^{(b)}_{1200} = -0.015 + 0.082i$, and $F^{(b)N}_{S3} = -1.825 - 3.183i$.

By adjusting the sextupole magnet and analyzing the spectra of $\tilde{x} - i\tilde{p}_x$ and $\tilde{x}$, two clear linear relationships are shown in Fig. (2). Through measurement of the relationship between $K_{\omega,2}L$ and both $F^{(b)N}_{S3}$ and $f^{(b)}_{3000}$, the two earlier conclusions are confirmed: (1) $F^{(b)N}_{S3}$ displays linear dependence on sextupole strength, and (2) when the tune satisfies $3Q_x \approx N$, $F^{(b)N}_{S3} \approx 3f^{(b)}_{3000}$. Consequently, when $3Q_x \approx N$, it is feasible to reduce the amplitude of $F^{(b)N}_{S3}$ by exploiting its linear relationship with the sextupole field and making corresponding adjustments to sextupole correctors. This approach ultimately reduces the amplitude of $f^{(b)}_{3000}$, stabilizing beam oscillations and mitigating resonance.

In conclusion, as the two key conclusions described in Sec. II A have been confirmed by simulation, the compensation scheme for subsequent experiments can be designed as follows. Step 1: Acquire reliable beam oscillation data through the TbT acquisition system while setting the tune as close as possible to $Q_x = 5/3$. To obtain such TbT data, excitation such as kickers or RF-knockout (RF-KO) [?, ?] should be introduced during data acquisition while avoiding excessive beam loss to ensure TbT data precision. Step 2: Process the

TbT data through Fourier analysis, check corresponding lines in the spectrum, and calculate $F_{S3}^{(b)N}$ using information from these lines. Step 3: Repeat measurement and calculation several times to obtain average and standard deviation. Step 4: Adjust sextupole corrector strength and return to Step 3 until linear relationships are obtained. Step 5: Calculate compensation through the linear relationships and detect beam intensity to verify compensation effectiveness.

C. Discussion On The Systematic Resonance

Nonlinear resonances are classified as systematic and random. Systematic nonlinear resonances occur at $\ell = P \times$ integer, where P is the superperiod of an accelerator [**?**]. A systematic resonance may be encountered even without magnet errors. If a systematic resonance occurred at $Q_x = 5/3$, beam compensation would be impossible using only sextupole correctors. Analysis of systematic resonances in the SESRI lattice is shown in Fig. (3). As the analysis demonstrates, the two closest resonance lines, $3Q_x = 5$ and $4Q_y = 7$, do not satisfy the condition $\ell = 6 \times$ integer, meaning systematic resonances will not occur at the target configuration.

III. Experiment

A. Parameter Setting

^{209}Bi^{32+} ions at 7.03 MeV/nucleon were used in the experiments described in this paper. The radio frequency (RF) cavity [**?**, **?**] remained switched on throughout the entire ramping process to ensure single-bunch beam conditions during measurement. The RF-KO was activated with constant amplitude and single frequency for 0.004 s (approximately 3400 turns) at the extraction platform to excite beam oscillations.

B. Data Acquisition

In the experiments, when $Q_x > 5/3$, it was observed that if the tune configuration fell below $Q_x = 1.676$, the decay of free beam oscillation became excessively rapid, making reliable TbT data acquisition challenging. Therefore, the final tune during measurement was set to $Q_x = 1.676$. An example of raw TbT data measured at the SESRI synchrotron is shown in Fig. (4), with a maximum TbT data length of 65536 turns.

The subsequent step involves spectrum analysis using Fast Fourier Transform (FFT). FFT is a common algorithm for Fourier analysis capable of transforming signals from the time or space domain into the frequency domain. Key factors affecting FFT analysis include signal length, noise levels, sampling rate, and window function. For data acquired via the TbT system, the sampling rate corresponds to the cyclotron frequency of the synchrotron, and no window function was used in this paper. Consequently, the primary factors affecting FFT results are the available TbT data length and data noise.

Fig. (5) illustrates an example of FFT results using data from turns 20,000 to 36,383. Several conclusions can be drawn from the figure: (1) TbT data noise is approximately within $±$1 μm, which is sufficient precision since the $-2\nu_x = 0.3456$ line would not be observable if noise exceeded $±$10 μm; (2) The actual tune is $Q_{x,\mathrm{real}} = 1.6729$, while the value set on the user interface is $Q_{x,\mathrm{set}} = 1.676$, indicating that experiments operate under the condition $Q_{x,\mathrm{set}} - Q_{x,\mathrm{real}} \approx 0.0031$, as confirmed by simulation in Fig. (6).

Quadrupole errors were introduced into the lattice using the EFCOMP order of MAD-X, causing the difference between $Q_{x,\mathrm{set}}$ and $Q_{x,\mathrm{error}}$. A difference of $Q_{x,\mathrm{set}} - Q_{x,\mathrm{error}} \approx 0.0031$ was observed by changing $Q_{x,\mathrm{set}}$ while keeping the random seed unchanged. By changing the seed, the gradient varied from 0.98 to 1.02. In conclusion, $Q_{x,\mathrm{set}} - Q_{x,\mathrm{real}} \approx 0.0031$ is not a strict relationship but an approximation to infer $Q_{x,\mathrm{real}}$ when it is unmeasurable due to strong nonlinearity near $Q_x = 5/3$.

It is important to note that the FFT result in Fig. (5) is based on raw x rather than normalized $\tilde{x}$ due to incomplete β_x measurement. Furthermore, using the BPM calibration factor alone would be meaningless, so this factor was not considered. Obviously, results calculated from raw x without considering β_x and BPM calibration factor are not $F_{S3}^{(b)N}$. To maintain rigor in our analysis, we introduce a new quantity denoted as $k \times F_{S3}^{(b)N}$, where k is a dimensionless non-zero real number serving as a scaling factor between values obtained from our spectrum calculations and the actual value of $F_{S3}^{(b)N}$. However, even without the precise value of k, the linear relationship can still be measured, resulting in the same compensation configuration. Therefore, compensation for sextupole field errors can be accomplished by experimentally measuring the linear relationship between $k \times F_{S3}^{(b)N}$ and sextupole corrector strength, then adjusting the correctors accordingly. Insisting on the actual value of $F_{S3}^{(b)N}$ would only complicate the compensation scheme.

C. Frequency Leakage Issue

Initially, $2^{14} = 16384$ was chosen as the FFT length. When the data length is a power of 2, the FFT algorithm achieves its lowest computational complexity, making powers of 2 the conventional choice for FFT data length. However, in practice, FFT implementations such as scipy.fftpack.fft used in this paper can handle any data length.

To fully utilize the TbT data, the FFT starting turn number was incrementally adjusted from 20,000 to 40,000 while maintaining a constant FFT data length of 16384. The amplitudes and phases of all $k \times F_{S3}^{(b)N}$ values were visualized in Fig. (7). From this figure, two significant conclusions were drawn: (1) amplitude variation reached $±$0.7, corresponding to a large error of $±$18%; (2) the phase exhibited oscillatory behavior, alternating between two values differing by approximately π.

It was eventually determined that these strange phenomena derived from frequency leakage. Frequency leakage refers to the undesired phenomenon where signal or frequency components unintentionally leak into adjacent frequency grids, commonly resulting from imperfections in signal processing systems that cause energy from target frequency lines to bleed into neighboring grids.

Previous studies have introduced several mathematical methods to mitigate frequency leakage [**?**]. However, these methods are effective only when frequency leakage is not severe. In situations where amplitudes of key peaks and their neighboring points are nearly equal, as illustrated in Fig. (8), even these mathematical approaches cannot produce accurate results.

This paper introduces a new method for mitigating frequency leakage effects: adjusting the FFT data length. This approach is based on the principle that the accuracy of frequency, amplitude, and phase in spectral analysis depends on alignment between normalized frequency grids and specific spectral lines. When a normalized frequency grid aligns with the target frequency, calculated values are precise. However, if the target frequency falls between two normalized frequency grids, frequency and amplitude values become less accurate, leading to errors.

For example, as shown in Fig. (9), a complex array $Z(N) = e^{2\pi i \times 0.2N}$ was analyzed via FFT with data lengths of 1024 (left) and 1025 (right). In the left figure, FFT with length 1024 could not provide a normalized frequency grid aligning with 0.2, whereas in the right figure, FFT with length 1025 could provide a normalized frequency grid of 205/1025 aligning with 0.2. Consequently, in the left figure, the frequency, amplitude, and phase are all incorrect due to frequency leakage.

In conclusion, to solve the frequency leakage issue, an appropriate FFT data length can be chosen to ensure normalized frequency grids align with both the fundamental line and the sextupole-driven line. For example, the degree of frequency leakage, denoted as FL, can be defined as:

$$FL_n = \left| \frac{A_{n-1} - A_{n+1}}{A_{n-1} + A_{n+1}} \right|$$

where n represents the position of interest in the frequency domain, A_{n-1} is the amplitude of the previous point, and A_{n+1} is the amplitude of the next point. This definition in Eq. (8) is adopted for convenience in subsequent data processing and may not be optimal; the appropriate definition should be carefully considered based on specific analysis requirements.

To visually demonstrate the impact of frequency leakage in our experiment, spectra with FFT starting points at turns 21,900 and 22,000 were compared. The spectra were analyzed with last points of key lines marked in green and next points in blue, as shown in Fig. (8). The data segments used for the two figures differed by only 100 turns, implying the segments were nearly identical.

However, the calculated $k \times F_{S3}^{(b)N}$ values were $-3.357-1.76i$ and $2.874+2.690i$ respectively, indicating a phase difference of 3.384. Serious frequency leakage clearly occurred in the sextupole-driven lines in both figures, rendering the calculated results unreliable. Therefore, solving the frequency leakage issue is vital for obtaining accurate results.

To mitigate frequency leakage in both the fundamental line and the sextupole-driven line, the relationship between $FL_n(\nu_x) + FL_n(-2\nu_x)$ and FFT data length is plotted in Fig. (10). The left figure shows the FFT starting point at 21,900, while the right shows it at 22,000. This analysis helps identify the optimal FFT data length that minimizes frequency leakage at these specific spectral lines. In the top two plots of Fig. (10), the most suitable FFT data lengths were found within the range [16284, 16484], as the smallest $FL_n(\nu_x) + FL_n(-2\nu_x)$ values were achieved with lengths of 16,438 and 16,328 respectively. The corresponding data segments were used in Fourier analysis, producing the bottom two plots. Comparison between Fig. (8) and Fig. (10) reveals significant reduction in frequency leakage effects at spectral lines after applying this FFT data length adjustment approach. Using results from Fig. (10), calculated $k \times F_{S3}^{(b)N}$ values were $-3.916-2.646i$ and $-3.919-2.615i$, eliminating the large phase difference observed in Fig. (8).

By applying this method, Fig. (7) was reanalyzed to obtain Fig. (11). The reanalyzed figure shows amplitude variation reduced to $\pm$0.15 (approximately $\pm$3%), while phase remains stable at -2.54 ± 0.04. The phase oscillations previously observed in Fig. (7) have been eliminated.

In conclusion, frequency analysis accuracy can be improved by aligning normalized frequency grids with specific spectral lines through FFT data length adjustment, effectively reducing frequency leakage impact. The key contribution of this section is introducing a new simple method of adjusting FFT data length to mitigate frequency leakage.

D. Experiment Results

In the first experiment, large amplitude errors and strange phase discontinuities were observed. However, the cause—frequency leakage—had not yet been identified at that time. These issues seriously affected the precision of measured $k \times F_{S3}^{(b)N}$, resulting in unreliable linear relationships with sextupole corrector strength. The first compensation configuration calculated from this unreliable linear relationship improved beam loss from 95% to 75%. However, another attempt through manual scanning of sextupole correctors near this configuration fortuitously achieved relatively good compensation. Results from the first experiment are shown in Fig. (12). The top figure compares intensity with and without compensation on DC current transformers (DCCT) [**?**] during the first experiment, with the extraction platform tune set at 1.673 while the actual tune was approximately 1.670. In the initial three ramping events, sextupole correctors remained inactive, resulting in approximately 95% beam loss. Conversely, during the subsequent two ramping events, activated sextupole correctors achieved approximately 25% beam loss, substantially better than before compensation. Notably, as shown in the bottom figure with the same configuration, compensation effects underwent dynamic changes, with almost no beam loss in optimal results. This phenomenon might be caused by power supply instability, β beating, and closed-orbit instability. Based on improved beam performance with compensation, it can be concluded that systematic resonances did not occur.

However, when the tune was set closer to 5/3, the compensation configuration from the first experiment rapidly lost effectiveness. For instance, when $Q_{x,\mathrm{real}} \approx 1.667$, beam loss still exceeded 90% even with compensation activated, indicating incomplete compensation. Consequently, after solving the frequency leakage issue, the second experiment commenced.

In the second experiment, the linear relationship between $k \times F_{S3}^{(b)N}$ and sextupole corrector strength was successfully established, as shown in Fig. (13). The experimental scheme, guided by Sec. II, proceeded as follows. Step 1: Acquire reliable beam oscillation data as in Fig. (4). Step 2: Process TbT data as in Fig. (11) to obtain $k \times F_{S3}^{(b)N}$ values. Step 3: Repeat measurement and calculation 10 times to obtain average and standard deviation. Step 4: Adjust sextupole corrector strengths $K_2(4S02)$ and $K_2(4S04)$, then return to Step 3 until satisfactory linear relationships are obtained. Step 5: Calculate compensation through the linear relationships and detect beam intensity to verify compensation effectiveness.

In Fig. (13), each data point represents the average of 10 measurements with the same K_2 configuration. As shown in Fig. (11), error from a single $k \times F_{S3}^{(b)N}$ measurement is quite small. However, error bars from 10 measurements remain relatively large, likely due to factors including power supply instability, β beating, and sextupole corrector instability. Nonetheless, linearity composed from averages is clear.

SUSSIX is a program developed by Frank Schmidt and Riccardo Bartolini at CERN for post-processing tracking or experimental turn-by-turn data via frequency analysis [?]. Fig. (14) was plotted by replacing constant FFT length in Fig. (11) with SUSSIX. Average amplitude and phase from SUSSIX and FFT length adjustment are almost identical. Furthermore, 4S02 is used as an example to show linear relationship comparison in Fig. (15), demonstrating the reliability of our FFT length adjustment method.

As mentioned previously, β function measurement in the synchrotron had not been completed, preventing attainment of the real lattice model. Nevertheless, the linear relationship obtained experimentally was chosen as the compensation basis, formulated into a response matrix R:

$$\begin{pmatrix} 1.962 & 0.520 \\ - & - \end{pmatrix} \to \text{Error} = 0$$

$$\begin{pmatrix} K_2(4S02) \\ K_2(4S04) \end{pmatrix} = \begin{pmatrix} 1.678 \\ 0.387 \end{pmatrix} \mathrm{m}^{-3}$$

$$\begin{pmatrix} 3.49 \\ - \end{pmatrix}$$

Finally, by applying the K_2 values for 4S02 and 4S04 and setting the tune at 1.670 (while the actual tune was approximately 1.667, the closest configuration to 5/3), DCCT screen captures are presented in Fig. (16). Observed beam loss varied from 10% to 50%, substantially better than both the uncompensated case and the first experiment's compensation. This indicates strong resonance effects extremely close to the resonance line, where any tiny disturbance can cause huge beam loss. The primary compensation limitations were sextupole corrector accuracy and overall synchrotron stability, as manual scanning of sextupole correctors did not yield further improvements.

In conclusion, by setting the tune as close as possible to 5/3, the compensation method has proved successful. The key point of this section is experimentally demonstrating that particles extremely close to the resonance line can remain stable with appropriate compensation.

IV. Conclusion

This article presents resonance measurement and compensation extremely close to the resonance line, beginning with establishment of important mathematical connections: (1) the linear relationship between $F_{S3}^{(b)N}$ and the sextupole field, and (2) the correlation of $F_{S3}^{(b)N}$ with $f_{3000}^{(b)}$ when the tune approaches $3Q_x \approx N$. MADX-PTC simulation was conducted to validate these ideas.

Subsequently, resonance measurement and compensation experiments were performed. The initial experiment faced frequency leakage issues, achieving successful results only at $Q_{x,\text{real}} \approx 1.670$, while beam loss exceeded 90% at $Q_{x,\text{real}} \approx 1.667$. After analyzing various factors, the frequency leakage issue was identified.

A new solution for the frequency leakage issue was established in this experiment: adjusting FFT data length to align normalized frequency grids with target spectral lines, which reduces frequency leakage. In the subsequent experiment, a linear relationship between $k \times F_{S3}^{(b)N}$ and sextupole corrector strength was successfully established. Despite not fitting the ideal lattice model, it was chosen as the compensation basis.

Finally, under the condition $Q_{x,\mathrm{real}} \approx 1.667$, the compensation effect is significant. Manual adjustments by scanning correctors did not yield improvement, leading to the conclusion that current compensation in the SESRI synchrotron might be limited by corrector precision, though the compensation method itself proved successful.

This article describes CRDT measurement with the tune extremely close to the resonance line, enabling effective resonance compensation. Through these experiments, it has been shown that stabilization of particles extremely close to the resonance line can be maintained with appropriate compensation. This achievement establishes a foundation for future resonance compensation work at the High Intensity heavy-ion Accelerator Facility (HIAF), particularly for particles undergoing resonance crossing due to tune spread. Research on space charge resonance compensation schemes in high-intensity synchrotrons—the vital technology for overcoming space charge effect limitations—represents our subsequent important work, including a simulation code for high-intensity beams [**?**].

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Note: Figure translations are in progress. See original paper for figures.