Four-Dimensional Velocity Vector Based on Euclidean Metric and Its Applications

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Abstract

The four-velocity vector as defined in Minkowski spacetime is not constituted by the moving object's respective velocities in each of the four dimensions, lacks intuitive physical significance, and increases the complexity of computation and analysis. This paper proposes a four-velocity vector definition based on the Euclidean metric, wherein its four components correspond one-to-one with the object's motion velocities in four-dimensional spacetime, and establishes a corresponding spacetime coordinate system along with its line element calculation method. Employing this four-velocity vector and coordinate system, the paper conducts derivations for several problems in special and general relativity, thereby obtaining the reasons underlying the inapplicability of the Euclidean metric to Minkowski spacetime. In particular, from the perspective of momentum and energy changes arising from variations in the Euclidean four-velocity, it reveals the physical principle that gravity is a property of spacetime structure rather than a force, and demonstrates that the Euclidean metric exhibits more pronounced intuitiveness than the Minkowski metric in relativistic calculations.

Full Text

Preamble

Four-Dimensional Velocity Vector Based on the Euclidean Metric and Its Applications

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The four-dimensional velocity vector defined in Minkowski spacetime is not composed of the individual velocities of a moving object in each of the four dimensions, which obscures its physical meaning and increases the complexity of calculations and analysis. This paper proposes a definition of the four-dimensional velocity vector based on the Euclidean metric, where the four components correspond one-to-one to the object's velocity in four-dimensional spacetime. We establish a spacetime coordinate system compatible with this four-dimensional velocity vector and develop a method for calculating its line element. Using this four-dimensional velocity vector and coordinate system, we perform calculations on several problems in both special and general relativity, revealing why the Euclidean metric cannot be applied in Minkowski spacetime. In particular, by examining momentum and energy changes arising from variations in the Euclidean four-velocity, we reveal the physical principle that gravity is a property of spacetime structure rather than a force. This demonstrates that the Euclidean metric offers significantly greater intuitive clarity than the Minkowski metric in relativistic calculations.

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In classical special relativity, the four-velocity vector is defined based on d/dτ:

(cid:3091) = ((cid:3031)(cid:3047) (cid:3031)(cid:3099) (cid:3031)(cid:3051) (cid:3031)(cid:3099) (cid:3031)(cid:3052) (cid:3031)(cid:3099) (cid:3031)(cid:3053) (cid:3031)(cid:3099)

where t is coordinate time and τ is the proper time of the observed moving object. The four-velocity vector can also be written as:

(cid:3091) = (γ , γ (cid:3051), γ (cid:3052), γ (cid:3053))

where γ = (1 − (cid:2870)/ (cid:2870))(cid:2879)(cid:2869)/(cid:2870) is the Lorentz factor. The magnitude of (cid:3091) is:

| | =

The classical definition of four-velocity and its reliance on the Minkowski metric represent important mathematical tools in relativity that have greatly advanced related research. However, the first dimension of this four-velocity vector is not the "velocity" of the object in the time dimension, making its physical meaning insufficiently clear.

2. Four-Dimensional Velocity Vector Based on the Euclidean Metric

We attempt to define the four-velocity vector (cid:3091) under the Euclidean metric using d/dt rather than d/dτ:

(cid:3091) = ((cid:3031)(cid:3099) (cid:3031)(cid:3051) (cid:3031)(cid:3052) (cid:3031)(cid:3053) (cid:3031)(cid:3047)

According to the Lorentz transformation, = γ , so this definition of the four-velocity vector can also be written as:

(cid:3031)(cid:3047) (cid:3031)(cid:3047) (cid:3031)(cid:3047) (cid:3091) = ((cid:2913) (cid:2963) , (cid:3051), (cid:3052), (cid:3053))

In this four-vector (cid:3091), the first component is the object's velocity in the time dimension, while the remaining three are its velocities in the spatial dimensions. These four velocities combine to form a four-velocity vector with clear physical meaning that can accommodate the more familiar Euclidean metric. In subsequent sections, we refer to this as the Euclidean four-velocity vector.

We now calculate the magnitude of (cid:3091) using the Euclidean metric:

(cid:3030) | |(cid:2870) = (cid:4672) (cid:3082) (cid:3030) (cid:4673) (cid:4672) (cid:3082) (cid:4673) + ((cid:3051))((cid:3051)) + (cid:3435) (cid:3052)(cid:3439)(cid:3435) (cid:3052)(cid:3439
+ ((cid:3053))((cid:3053))

(cid:3030) | |(cid:2870) = (cid:4672) (cid:3082) (cid:2870) (cid:4673) + (cid:2870)

Since the Lorentz factor $\gamma = (1 -$ (cid:2870)/ (cid:2870))(cid:2879)(cid:2869)/(cid:2870), we have (cid:4672) (cid:3082) (cid:3030) (cid:2870) (cid:4673) = (cid:2870) − (cid:2870). Therefore, the magnitude of the Euclidean four-velocity is:

| | =

This verifies that the Euclidean four-velocity vector has a constant magnitude equal to the speed of light.

By comparing equations (5) and (2), we find that each component of the Euclidean four-velocity can be obtained through a linear transformation of the components of the Minkowski four-velocity. Therefore, the Lorentz covariance of the Minkowski four-velocity vector ensures that the Euclidean four-velocity vector maintains its form in all inertial frames. Equation (8) proves that the magnitude of the Euclidean four-velocity remains invariant in all inertial frames. Consequently, we can conclude that the Euclidean four-velocity vector possesses Lorentz covariance.

3.1 Momentum Based on the Euclidean Four-Velocity Vector

Since the Euclidean four-velocity vector is defined using d/dt, we calculate the Euclidean four-momentum vector P(cid:2972) in this reference frame using relativistic mass γ :

(cid:2913) P(cid:2972) = (cid:4672) (cid:2963) γ , γ V(cid:2934), γ V(cid:2935), γ V(cid:2936)(cid:4673)

This can be simplified to:

P(cid:2972) = (cid:3435) c, γ V(cid:2934), γ V(cid:2935), γ V(cid:2936)(cid:3439)

The time component of the Euclidean four-momentum is , while the spatial components are γ (cid:3036) corrected by the Lorentz factor.

3.2 Modulus of the Euclidean Four-Momentum

We now consider the modulus of the Euclidean four-momentum, (cid:3091) (cid:3091), and according to the Euclidean metric's definition of the sum of squares, we obtain:

P(cid:2972) (cid:3048) = (c)(cid:2870) + (γ V)(cid:2870)

Substituting γ = (1 − (cid:2870)/ (cid:2870))(cid:2879)(cid:2869)/(cid:2870) and simplifying yields:

(cid:3091) (cid:3091) = γ(cid:2870) (cid:2870) (cid:2870)

This result shows that the modulus of momentum under the Euclidean metric differs from that under the Minkowski metric. This is because in Minkowski metric, the modulus of four-momentum is obtained by subtracting the square of spatial momentum from the square of total momentum, yielding the square of momentum in the time dimension. In contrast, under the Euclidean metric, we obtain the square of the object's total momentum. Let us now examine the relationship between momentum and energy.

3.3 Relationship Between Euclidean Four-Momentum and Energy

In the special relativity framework, the total energy of a moving object is:

= γ (cid:2870)

The relationship with the calculated modulus of the Euclidean four-momentum is:

(cid:3091) (cid:3091) = γ(cid:2870) (cid:2870) (cid:2870) = (γ (cid:2870) (cid:2870)/)(cid:2870) = (/)(cid:2870)

Thus, the momentum modulus calculated under the Euclidean metric represents the object's total four-momentum, consistent with the relativistic energy-momentum relationship.

4. Coordinate System for the Euclidean Four-Velocity Vector

Minkowski constructed the Minkowski line element expression by adding a time dimension t to the Cartesian three-dimensional coordinate system and proposing the Minkowski metric:

(cid:2870) = − (cid:2870) (cid:2870) + (cid:2870) + (cid:2870) + (cid:2870)

The Minkowski metric cannot accommodate our defined Euclidean four-velocity, so we require a coordinate system that can adapt to the Euclidean metric.

We add =ct as the radial coordinate and =arcsin(1/γ) as the polar angle to the Cartesian three-dimensional coordinate system, forming a composite coordinate system. To facilitate understanding and calculation, we add a cτ axis perpendicular to the X axis as the projection axis of the object's worldline in the horizontal direction (τ depends on t and γ, not serving as an independent dimension). We call this the Cartesian-τ coordinate system, which describes Euclidean spacetime.

[Figure 1: see original paper] Cartesian-τ coordinate system

In the Cartesian-τ coordinate system, the x-axis represents null worldlines, while the τ-axis represents worldlines of objects at rest in the reference frame. Worldlines of various inertial frames lie between the X-axis and τ-axis.

As shown in the figure, two objects A and B moving with constant velocities V1 and V2 have corresponding Lorentz factors $ $1 and $ 2.Takinganarcwithpolarradiusρ = ct0thatintersectstheworldlinesofobjectsAandBatpointsAandBrespectively,thepolarcoordinatesofAare(ct(c 2868), arcsin(1/ (cid : 2869)))andthoseofBare(ct(cid : 2868), arcsin(1/ $(cid:2870))). The projections of their worldlines OA and OB onto the X-axis are the spatial distances traveled, (cid:2869) and (cid:2870), while their projections onto the τ-axis are the products of their proper times and the speed of light, (cid:2869) and (cid:2870). Based on this coordinate system, the line element length is:

(cid:2870) (cid:2870) = (cid:2870) (cid:2870) + (cid:2870) + (cid:2870) + (cid:2870)

In one-dimensional space, the worldline line elements ct for OA and OB satisfy:

(cid:2870) (cid:2868) (cid:2870) = (cid:2870) (cid:2869) (cid:2870) (cid:2870) + (cid:2869)

(cid:2870) (cid:2868) (cid:2870) = (cid:2870) (cid:2870) (cid:2870) (cid:2870) + (cid:2870)

Equations (17) and (18) demonstrate that the "time" and "space" components obtained through orthogonal decomposition along the same object's worldline have a mutually "perpendicular" relationship. The reason the Euclidean metric cannot be applied in Minkowski spacetime is that the vertical axis of the coordinates describes the time experienced by an observer at rest at the coordinate origin, while the horizontal axis describes the distance traveled by an observer on the moving object. These represent the time and space experienced by observers in two different inertial frames, which are not physically "perpendicular" and thus cannot accommodate the Euclidean metric.

5. Application of Euclidean Four-Velocity in General Relativity

Based on equation (16), the Euclidean line element and equation (15), the Minkowski line element represent different expressions of the same physical principle. Therefore, the Euclidean metric is equally applicable to general relativity.

We now analyze and compare using the well-known Schwarzschild metric tensor as an example. In Minkowski spacetime, the line element expression for the Schwarzschild metric is:

(cid:2870) = − (cid:4672)1 − (cid:4673) (cid:2870) (cid:2870) + (cid:3045)(cid:3294) (cid:3045) (cid:2869) (cid:3293)(cid:3294) (cid:3293) (cid:2869)(cid:2879) (cid:2870) + (cid:2870)((cid:2870) + (cid:2870) (cid:2870))

where (cid:3046) = (cid:2870)(cid:3008)(cid:3014) (cid:3030)(cid:3118) is the Schwarzschild radius.

Comparing equations (16) and (15), we see that the Minkowski spacetime line element (cid:2870) is equivalent to − (cid:2870) (cid:2870) in the Cartesian-τ coordinate system. Substituting into equation (19) yields:

− (cid:2870) (cid:2870) = − (cid:4672)1 − (cid:4673) (cid:2870) (cid:2870) + (cid:3045)(cid:3294) (cid:3045) (cid:2869) (cid:3293)(cid:3294) (cid:3293) (cid:2869)(cid:2879) (cid:2870) + (cid:2870)((cid:2870) + (cid:2870) (cid:2870))

Rearranging gives the line element expression in the Cartesian-τ coordinate system:

(cid:2870) (cid:2870) = (cid:3030)(cid:3118)(cid:3031)(cid:3099)(cid:3118) (cid:3293)(cid:3294) (cid:2869)(cid:2879) (cid:3293) (cid:3031)(cid:3045)(cid:3118) (cid:3293)(cid:3294) (cid:3293) (cid:4672)(cid:2869)(cid:2879) (cid:4673) (cid:3118) + (cid:3045)(cid:3118) (cid:3293)(cid:3294) (cid:2869)(cid:2879) (cid:3293) ((cid:2870) + (cid:2870) (cid:2870))

Considering only radial changes (= 0 and =0), we can simplify to:

(cid:2870) (cid:2870) = (cid:3030)(cid:3118)(cid:3031)(cid:3099)(cid:3118) (cid:3293)(cid:3294) (cid:2869)(cid:2879) (cid:3293) (cid:3031)(cid:3045)(cid:3118) (cid:3293)(cid:3294) (cid:3293) (cid:4672)(cid:2869)(cid:2879) (cid:3118) (cid:4673)

Comparing equations (17) and (22), we see that in the flat spacetime of special relativity, the spacetime experienced by the same object is mutually "perpendicular," satisfying the Pythagorean theorem. In Schwarzschild spacetime, due to the influence of massive objects, the "perpendicular" relationship between time and space dimensions is broken. These two dimensions require correction by their respective coefficients (terms related to 1 − (cid:3045)(cid:3294) (cid:3045)) to satisfy the Pythagorean theorem. These correction coefficients embody the geometric structure of spacetime.

5.1 Time Dilation Effects from Spacetime Curvature

To study a stationary object in Schwarzschild spacetime, we can imagine a spacecraft hovering at a distance r from the singularity in the outer region of a black hole, using its own engines to resist gravity. In equation (22), setting

=0 yields the relationship between the proper time dτ experienced by the spacecraft's observer and the coordinate time dt of an observer at infinity:

τ = dt(cid:3495)(1 − (cid:3045)(cid:3294) (cid:3045)

5.2 Euclidean Four-Velocity Vector and Momentum-Energy Relations in Schwarzschild Spacetime

Based on equations (5) and (23), the Euclidean four-velocity vector of a stationary object in Schwarzschild spacetime is:

(cid:3091) = (cid:4672) (cid:3046) (cid:2914)(cid:2980) (cid:3031)(cid:3047) , 0,0,0(cid:4673) = ((cid:3495)(1 − (cid:3045)(cid:3294) (cid:3045)), 0,0,0)

Its corresponding momentum is:

(cid:3091) = (cid:3436)mc(cid:3495)(1 − (cid:3046) (cid:3045)(cid:3294) (cid:3045)), 0,0, 0(cid:3440)

According to equation (14), the corresponding total energy of the object is:

(cid:3046)(cid:2868) = (cid:3493) (cid:3091) (cid:3091) = m (cid:2870)(cid:3495)(1 − (cid:3045)(cid:3294) (cid:3045)

[Figure 2: see original paper] Variation of object's rest energy with radial coordinate r

As shown in Figure 2, in Schwarzschild spacetime, an object's rest energy decreases sharply with r near r=rs, rapidly converting into kinetic energy of free-fall motion. (cid:3046)(cid:2868) represents the total energy of a stationary object in Schwarzschild spacetime. If we consider the object's kinetic energy, we obtain the total energy of a moving object in Schwarzschild spacetime:

(cid:3046) = γm (cid:2870)(cid:3495)(1 − (cid:3045)(cid:3294) (cid:3045)

Comparing equation (26) with the total energy of a stationary object in flat spacetime, = (cid:2868) (cid:2870), we see that the total energy of an object with the same mass is reduced in Schwarzschild spacetime. Of course, energy does not disappear 凭空消失; rather, it is converted into kinetic energy of the object in free fall within Schwarzschild spacetime. Therefore, objects accelerating continuously in a gravitational field due to spacetime curvature differ fundamentally in energy transformation from objects accelerating continuously in flat spacetime due to applied thrust. The former does not gain energy but rather has its "rest energy" converted into "kinetic energy" under a special spacetime structure, while the latter has work done on it by another object, increasing its kinetic energy without losing rest energy, resulting in increased total energy.

5.3 Momentum-Energy Relations Using the Minkowski Four-Velocity Vector in Schwarzschild Spacetime

To verify whether calculations using the two four-velocity definitions yield consistent results in a gravitational field, we perform the same calculations as in Section 5.2 using the Minkowski four-velocity vector in Minkowski spacetime. Noting that the definition in equation (1) requires the speed of light constant c, we must also determine the "effective speed of light" in Schwarzschild spacetime at radius r and modify equation (1) accordingly. In equation (22), setting = 0 gives the radial motion of light in Schwarzschild spacetime:

= (1 − (cid:3045)(cid:3294) (cid:3045)

To obtain the "effective speed of light" experienced by objects in Schwarzschild spacetime, we need to find dr/dτ to maintain the same reference frame as dt/dτ. Combining equations (23) and (28) yields:

C(cid:2915) = (cid:3031)(cid:3045) (cid:2914)(cid:2980) (cid:2913)(cid:2914)(cid:2930)((cid:2869)(cid:2879) (cid:2914)(cid:2930)(cid:3495)((cid:2869)(cid:2879) (cid:3293)(cid:3294) (cid:3293) (cid:3293)(cid:3294) (cid:3293) = c(cid:3495)1 − (cid:3045)(cid:3294) (cid:3045)

Using this coefficient for correction, we obtain the definition of the Minkowski four-velocity vector for a stationary object in a gravitational field:

(cid:3091) = (cid:4672)C(cid:2915) ∗ (cid:3046) (cid:3031)(cid:3047) (cid:3031)(cid:3099) , 0,0,0(cid:4673) = ((cid:3495)1 − (cid:3045)(cid:3294) (cid:3045) (cid:3031)(cid:3047) (cid:3031)(cid:3099) , 0,0,0)

Substituting equation (23) into equation (30) yields the Minkowski four-velocity vector for a stationary object in Schwarzschild spacetime:

(cid:3091) = ((cid:3046) (cid:3030)(cid:3495)(cid:2869)(cid:2879) (cid:3293)(cid:3294) (cid:3293) (cid:3495)((cid:2869)(cid:2879) (cid:3293)(cid:3294) (cid:3293) , 0,0,0)=(, 0,0,0)

Through derivation using the four-velocity vector in equation (31), we should obtain the same energy result as equation (26). First, we obtain the Minkowski four-momentum from equation (31):

(cid:2972) = (c, 0,0, 0)

Then, using the momentum-energy relation E=P(cid:2868)*C, we obtain the energy. Note that here C must use the effective speed of light C(cid:2915):

(cid:3046) = ∗ C(cid:2915) = (cid:2870)(cid:3495)(1 − (cid:3045)(cid:3294) (cid:3045)

This matches equation (26). This demonstrates that as long as the physical meaning of every factor in the four-velocity vector and energy-momentum operations is thoroughly understood, the four-velocity vector, momentum, and energy from special relativity can be successfully extended to general relativity.

5.4 "Gravitational Force" on Objects in Schwarzschild Spacetime

Through the analysis in Section 5.2, we have learned that objects in Schwarzschild spacetime do not experience "gravitational force" but rather undergo energy conversion due to spacetime geometry. To determine the "gravity" that objects perceive due to this energy conversion, we attempt to derive the "force" through the kinetic energy formula. We know that an object's kinetic energy equals its total energy minus its rest energy. Based on equations (13) and (26), we can derive the kinetic energy (cid:3038) of a free-falling object in Schwarzschild spacetime as the "rest energy" in flat spacetime minus the "rest energy" in Schwarzschild spacetime:

(cid:3038) = (cid:2870) − (cid:2870)(cid:3495)(cid:4672)1 − (cid:3045)(cid:3294)
(cid:3045) (cid:4673) = (cid:2870)(1 − (cid:3495)(cid:4672)1 − (cid:3045)(cid:3294)
(cid:3045) (cid:4673))

According to the definition of work:

(cid:3031)(cid:3006)(cid:3286) (cid:3031)(cid:3045) (cid:3031)(cid:3040)(cid:3030)(cid:3118)((cid:2869)(cid:2879)
(cid:3293)(cid:3294) (cid:3293) (cid:4673))) (cid:3031)(cid:3045) (cid:2869)
(cid:2870) (cid:2870) (cid:3045)(cid:3294) (cid:3045)(cid:3118) (1 −)(cid:2879)
(cid:3117) (cid:3118) (cid:3045)(cid:3294) (cid:3045)

This indicates that an object in Schwarzschild spacetime experiences a force opposite to the direction of increasing r, i.e., directed toward the black hole's center. Substituting (cid:3046) = (cid:2870)(cid:3008)(cid:3014)
(cid:3030)(cid:3118) and simplifying yields:

|F| = (cid:3008)(cid:3014)(cid:3040) (cid:3045)(cid:3118) (1 − (cid:3045)(cid:3294)
(cid:3045))(cid:2879) (cid:3117) (cid:3118)

[Figure 3: see original paper] Relationship between gravitational force and radial coordinate r in Schwarzschild spacetime

This is the calculation formula for the virtual "gravitational force" experienced by an object in Schwarzschild spacetime. When r=∞, this formula matches Newton's law of universal gravitation. As shown in Figure 3, the gravitational force experienced by an object in Schwarzschild spacetime increases dramatically near = (cid:3046).

5.5 Gravitational Redshift

Photon energy originates from the energy difference when electrons transition between different energy levels or orbits. In Schwarzschild spacetime, the reduction in an object's rest energy leads to a corresponding loss in the energy of emitted light. Therefore, the redshift ratio should correspond to the proportion of rest energy reduction. Let (cid:2869) represent the photon energy emitted by an object in Schwarzschild spacetime, while (cid:2868) represents the photon energy emitted by the same object in flat spacetime. Their relationship can be expressed as:

(cid:3035)(cid:3092)(cid:3117) (cid:3035)(cid:3092)(cid:3116) (cid:3040)(cid:3030)(cid:3118)(cid:3495)(cid:2869) (cid:3293)(cid:3294) (cid:3293) (cid:3040)(cid:3030)(cid:3118) = (cid:3495)1 − (cid:3045)(cid:3294) (cid:3045)

This equation shows that photon energy emitted in Schwarzschild spacetime is reduced by a factor of (cid:3495)1 − (cid:3045)(cid:3294) (cid:3045) compared to photon energy in flat spacetime. Consequently, the photon's frequency decreases, causing gravitational redshift. This derivation aligns well with the traditional explanation of gravitational redshift, confirming that the observed redshift is a direct result of gravitational time dilation described by the Schwarzschild metric. Furthermore, this conclusion provides fundamental support for reference [10].

5.6 Gravitational Lensing

In a gravitational field, the effective speed of light varies, and light passing through Schwarzschild spacetime can be treated as traveling through a medium with spatially varying refractive index. Therefore, geometric optics methods can also be applied when calculating starlight deflection caused by gravitational lensing. The refractive index ( ) of this medium is defined as the ratio of the speed of light in vacuum c to the effective speed of light in the medium (cid:3032)(cid:3033)(cid:3033):

( ) = (cid:3030) (cid:3030)(cid:3280)(cid:3281)(cid:3281)((cid:3045)) (cid:3030) (cid:3030)((cid:2869)(cid:2879) (cid:3293)(cid:3294) (cid:3293) (cid:2869) (cid:3293)(cid:3294) (cid:3293) (cid:2869)(cid:2879) 1 + (cid:3045)(cid:3294) (cid:3045)

Note that (cid:3032)(cid:3033)(cid:3033) is derived from formula (28) and represents the effective speed of light relative to dt. In this case, we are calculating the lensing effect for an observer at infinity, so the effective speed of light differs from the "local effective speed of light" C(cid:2915) experienced by objects in Schwarzschild spacetime. According to formula (37), this expression matches the first-order approximation of equation [10] given in reference [8]. Consequently, we obtain the total deflection angle of light caused by the Sun's gravitational field:

(cid:2870)(cid:3045)(cid:3294) (cid:3029) 1.75 arcseconds

where the impact parameter is the perpendicular distance between the incident light's asymptotic (unbent) path and the Sun's center, with the solar radius used in actual calculations.

6. Conclusion

This paper defines the Euclidean four-velocity vector based on d/dt, verifies its validity, and defines a Cartesian-τ coordinate system compatible with the Euclidean metric. Through calculations verifying the Euclidean metric in the

Cartesian-τ coordinate system, we derive the reasons why Minkowski geometry cannot accommodate the Euclidean metric. Based on the Euclidean metric line element calculation formula, we also analyze the Schwarzschild solution to Einstein's field equations. By comparing the Euclidean four-velocity vector in flat spacetime and Schwarzschild spacetime, this paper reveals from an energy transformation perspective that gravity is a property of spacetime structure rather than a force. The Euclidean four-velocity vector can describe the velocity, energy, and even spacetime metric of any matter or particle, offering more intuitive physical meaning than the Minkowski four-velocity vector and enabling applications in broader physics research domains.

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Note: Figure translations are in progress. See original paper for figures.