Postprint: A Deep-Water Wave Model for Near-Earth Object Impact-Induced Tsunamis Based on Hemispherical Instantaneous Impact Craters

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Date: 2024-10-15T09:46:43+00:00

Abstract

Using a hemispherical transient impact crater as the initial perturbation, a deep-water wave model for near-Earth object impact-induced tsunamis was established and applied to a specific scenario (140 m asteroid diameter, Chelyabinsk impact parameters for velocity and density, and a Pacific-average ocean depth of 4 km) to investigate the evolution of maximum tsunami wave height with distance and time. Comparison of different transient impact crater models and propagation models revealed that: hemispherical craters induce greater tsunami hazards than parabolic craters; the shallow-water wave model yields larger-scale tsunamis compared to the deep-water wave model; and for medium-sized asteroids (diameters of several hundred meters), a deep-water wave model with hemispherical transient impact crater geometry is more suitable for studying ocean impact-induced tsunamis.

Full Text

Preamble

Vol. 65 No. 5

September 2024

ACTA ASTRONOMICA SINICA Vol. 65 No. 5 Sept., 2024 doi: 10.15940/j.cnki.0001-5245.2024.05.009

Deep Water Wave Model of Near-Earth Object Impact-Induced Tsunami Based on Hemispherical Instantaneous Crater*

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Abstract: Using a hemispherical instantaneous impact crater as the initial disturbance condition, we establish a deep water wave model for tsunamis induced by near-Earth object impacts. Applied to a specific scenario (an asteroid with a diameter of 140 m, impact velocity and density adopting Chelyabinsk impact parameters, and average ocean depth in the impact region using the Pacific average depth of 4 km), we investigate the evolution of the maximum tsunami wave height with distance and time following a near-Earth object impact on the ocean. By comparing the effects of different instantaneous impact crater models and propagation models on tsunami wave height, we find that tsunamis induced by hemispherical instantaneous impact craters pose greater hazards than those from parabolic craters; relative to the deep water wave model, the shallow water wave model yields larger scales of impact-induced tsunamis; for medium-sized asteroids (with diameters in the range of several hundred meters), the deep water wave model with a hemispherical instantaneous impact crater shape is more suitable for studying ocean impact-induced tsunamis.

Keywords: minor planets, asteroids: general, Earth, hydrodynamics, methods: analytical **Classification**: P183; Document code: A

Near-Earth Objects (NEOs) refer to small solar system bodies with perihelion distances less than 1.3 au, most of which are asteroids with some comets. Near-Earth asteroids can be classified into four categories based on orbital characteristics: Apohele (also known as Atiras), Aten, Apollo, and Amor. Among these, NEOs with Earth orbit intersection distances less than 0.05 au and sizes larger than 140 m are designated as Potentially Hazardous Objects (PHOs). Such objects pose significant threats to Earth and are the primary monitoring targets for asteroid defense systems.

Following the 2013 Chelyabinsk impact event in Russia, research on the hazards of asteroid impacts has gained increasing attention. NASA has established the Planetary Defense Coordination Office (PDCO) to study potential threats from asteroid impacts and held a workshop in 2016 on tsunami-related issues caused by asteroid impacts [1]. China has also confirmed the establishment of an asteroid defense system. Since approximately 70% of Earth's surface is covered by water, asteroid impact locations are also likely to be in oceanic regions. Morais et al. [2] quantified the potential consequences following an asteroid impact and concluded that when the impact occurs in ocean areas far from populated regions, tsunami effects represent the most threatening consequence.

Generally, there are two types of impact tsunamis [3]: one is generated by asteroid airburst. The shock wave from the airburst phenomenon strikes the water surface and further propagates through the water body, producing two distinct waves—response waves and gravity waves. Response waves refer to sea surface disturbances directly coupled with the moving shock wave and propagating at the same speed. Gravity waves are generated by pressure increases at the initial impact point and move more slowly, with an upper speed limit of $\sqrt{(gh)}$ (where g is gravitational acceleration and h is the undisturbed water depth). The other type of tsunami is produced by the collapse oscillation of the instantaneous crater formed by the asteroid striking the sea surface. This paper focuses on the latter type of tsunami wave.

The impact tsunami generated by crater collapse can be divided into several processes: First, the asteroid impact creates an instantaneous impact crater, whose parameters can be determined by combining the semi-analytical crater empirical law of Chesley and Ward [4] with the asteroid's physical parameters. Previous studies (such as those conducted by Stokes et al. [5]) have mostly used rotating paraboloid-shaped craters as initial conditions for subsequent derivations and simulations, while experiments by Gault and Sonett [6] in 1982 and Olevson [7] in 1969 demonstrated that water impact craters should be hemispherical.

Subsequently, the collapse of the impact crater will cause water movement, leading to the formation of the initial tsunami wave. The shock wave generated as the asteroid penetrates the water body compresses both the water and the asteroid. Most of the asteroid's initial kinetic energy is transferred to the water through this compression. As the shock wave pressure is released, the remaining internal energy causes water vaporization, and part of the energy is converted into kinetic energy, initiating radial water flow. The final evolution of this flow field results in crater formation. Since shock wave excavation is often more effective than water displacement caused by direct asteroid penetration, high-velocity impacts produce larger craters than low-velocity impacts. In this process, the energy conversion efficiency during impact is commonly used to relate crater parameters to asteroid physical parameters. In 2000, Ward et al. [8] provided a theoretical derivation from fundamental energy laws and crater scaling rules, linking the depth and diameter of parabolic impact craters to the asteroid's density, radius, and impact velocity.

Finally, seawater propagates outward from the impact point. Different models are used to study impact-induced tsunamis compared to earthquake-generated tsunamis. Typically, earthquake-generated tsunamis have long wavelengths and are simulated using the shallow-water equation (SWE). Previous impact tsunami studies have also analogized them to earthquake tsunamis and adopted the shallow-water equation. However, Wünnemann et al. [9] in 2014 noted that impact-generated tsunami waves often exhibit characteristics of short period and large amplitude, which differ significantly from earthquake and landslide tsunamis. Robertson et al. [10] also demonstrated through numerical simulation that deep-water impacts produce tsunami waves more closely resembling deep water waves. In cases such as continental shelf regions or impacts by larger asteroids several kilometers in diameter in deep water regions, tsunami wave propagation is more appropriately modeled by the shallow-water equation.

In Section 2, we establish a deep water wave model for near-Earth object impact-induced tsunamis with an initial disturbance in the form of a hemispherical instantaneous impact crater. In Section 3, assuming an asteroid with specific physical parameters impacts Earth, we use the model developed in Section 2 to study the evolution of the maximum tsunami wave height and its location over time following an ocean impact. Section 4 further investigates the variation of impact tsunami wave height with time and distance under this model, compares it with previous impact tsunami models, and analyzes the effects of different models on impact-induced tsunamis. Section 5 presents the conclusions of this study.

2 Impact Tsunami Model

Gisler et al. [11] suggest that under vertical impact conditions, tsunami wave propagation exhibits axisymmetric distribution. Robertson et al. [10] conducted three-dimensional simulations of oblique impacts using the Los Alamos Rage hydrocode and found that tsunami waves remain approximately symmetric in such cases, allowing the three-dimensional study to be reduced to two dimensions. Using a cylindrical coordinate system with the impact center as the origin, the horizontal plane as the polar coordinate plane (r,), and the axis perpendicular to the horizontal plane through the origin as the z-axis, we can conduct the study in the (r, z) plane due to the axisymmetric distribution of tsunami waves during propagation. Based on the deep water wave model for multi-ring basins [12], the propagation of impact-generated tsunami waves is derived from the hydrodynamic control equations as follows.

For two-dimensional incompressible irrotational fluid, the governing equation is the Laplace equation, which simplifies in cylindrical coordinates to:

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r} + \frac{\partial^2\Phi}{\partial z^2} = 0,$$

where Φ(r, z, t) is the velocity potential.

The boundary conditions are:

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} = 0, \quad (z = 0, t > 0).$$

The initial conditions are:

$$\Phi = 0, \quad \frac{\partial\Phi}{\partial t} = -g\eta(r, t), \quad (z = 0, t = 0),$$

where (r, t) is the tsunami wave height. Assuming that under initial conditions at r = 0 and t = 0, the seawater is stationary and the initial impact crater is hemispherical, we have:

$$\eta(r,0) = \begin{cases} \sqrt{R^2 - r^2}, & (r \leq R), \\ 0, & (r > R). \end{cases}$$

The initial impact crater is shown in [Figure 1: see original paper], where R is the initial impact crater radius.

Applying separation of variables to the governing equation and substituting the boundary conditions yields the particular solution for the potential function and the dispersion relation:

$$\Phi_k(r,z,t) = e^{kz} e^{-i\omega t} J_0(kr), \quad \omega^2 = gk,$$

where J_0(kr) is the zeroth-order Bessel function and i is the imaginary unit, ω is frequency, and k is wavenumber.

Assuming the general solution for the potential function is:

$$\Phi(r,z,t) = \int F(k)\Phi_k \, dk.$$

Substituting into the boundary and initial conditions gives:

$$\int F(k) J_0(kr)\omega \, dk = g(R^2 - r^2).$$

After Hankel transformation, we obtain:

$$F(k) = -\frac{8}{3\pi} R_1^{2j}(kR),$$

where j_1(kR) is the first-order spherical Bessel function.

Substituting the expressions for F(k) and Φ_k into the general solution yields the velocity potential:

$$\Phi(r,z,t) = \int_0^\infty \frac{8}{3\pi} R_1^{2j}(kR) e^{kz} \sin(\omega t) J_0(kr) \, dk.$$

The tsunami wave height expression is therefore:

$$\eta(r,t) = -\frac{1}{g} \left. \frac{\partial \Phi}{\partial t} \right|_{z=0} = \int_0^\infty \frac{8}{3\pi} R_1^{2j}(kR) \cos(\omega t) J_0(kr) \, dk.$$

For kr 1, J_0(kr) √(2/πkr)cos(kr - π/4). Substituting this into the wave height expression and using the stationary phase method yields:

$$\eta(r,t) = A \cos\left(\frac{gt^2}{4r}\right),$$

where the tsunami wave amplitude is:

$$A = \frac{8R^2}{3\pi}\sqrt{\frac{2}{\pi}}\frac{g^{2t^2}}{4r^2\sqrt{r}}.$$

For a specific asteroid impact scenario, the hemispherical crater size can be calculated from the asteroid's parameters and used as the initial condition in the wave height equations to study the evolution of wave height with time and distance.

For impact tsunami problems, medium-sized asteroids with diameters of 100–1000 m are commonly selected as research targets. Asteroids in this range are large enough to avoid complete ablation in the atmosphere and reach Earth's surface, yet small enough not to produce large-scale effects such as climate change [13]. We select an asteroid diameter of 140 m, use the Pacific average depth of 4 km for the impact region's ocean depth, and adopt meteorite parameters from Olga et al. [14] regarding the 2013 Chelyabinsk impact event, where the impactor density is 3.3 $\mathrm{g \cdot cm^{-3}}$, impact velocity is 19.3 $\mathrm{km \cdot s^{-1}}$, and we assume a vertical impact scenario.

For calculating hemispherical crater dimensions, we employ the empirical formula from Robertson et al. [15], who conducted experimental simulations of impact processes and modified the semi-analytical model of Schmidt and Holsapple [16]. The modified empirical law incorporates shallow water conditions and background atmospheric pressure:

$$\frac{V_i}{V_c} = 0.29\left(\frac{\rho_i}{\rho_c}\right)^{0.65}\left(\frac{v^2}{gD}\right)^{0.7}\left(\frac{P_0}{\rho_c v^2}\right)^{-0.1} + 0.01\left(\frac{H}{D}\right)^{0.9},$$

where V_i is the impactor volume, V_c is the hemispherical crater volume, _c is water density, _i is impactor density, D is impactor diameter, v is impact velocity, P_0 is surface pressure, and H is water depth.

Using this formula, we calculate an initial crater radius of 1526.6 m for the assumed scenario, and determine the energy conversion efficiency = E_T/E_I, where E_T represents the initial tsunami wave energy calculated as _c g $_0$ z^2 2πr dr and E_I represents the impactor energy as ½mv^2. The calculated energy conversion efficiency under these conditions is approximately 1.1%. Substituting the hemispherical crater radius into the wave height equations yields the evolution relationship of tsunami wave height with propagation distance and time for the assumed scenario. presents the maximum tsunami wave height values and their locations at specific times for the hemispherical crater initial condition.

4 Results and Discussion

Previous impact process studies have commonly employed the Schmidt and Holsapple crater model [16] and explosion shock wave experiments by Van Dorn et al. [17] and LeMéhauté et al. [18], using parabolic crater shapes. Regarding the energy conversion efficiency parameter, Ward and Asphaug [8] considered it either constant or a function related to impactor properties and target material. Using formulas from the Ward and Asphaug model [19] with an energy conversion efficiency of 15%, we obtain a parabolic crater depth of 1553.8 m, inner radius of 2312.2 m, and outer radius of 3269.9 m for the assumed scenario. Robertson and Gisler [10] determined through numerical simulation that the energy conversion efficiency should be 1%, yielding a parabolic crater depth of 728.4 m, inner radius of 1273.4 m, and outer radius of 1809.9 m; subsequent calculations in this study are based on this efficiency value. They also argued that ocean impacts differ from underwater explosions or impacts on rock and soil, and that crater shapes should be hemispherical rather than parabolic.

For impact tsunami propagation studies, previous research has analogized earthquake-generated tsunamis and employed the shallow water wave equation. Generally, waves with wavelengths less than ten times the water depth are considered deep water waves [20]. Numerical simulations show that impact tsunami wavelengths are typically comparable to the initial crater diameter [10]. Therefore, with an assumed average depth of 4 km, only impacts by larger asteroids (diameters above 10 km) produce shallow water waves. Additionally, since medium-sized asteroids generate tsunami wavelengths of several kilometers, propagation over continental shelf regions with depths of several hundred meters can also be treated as shallow water waves. To compare the effects of deep water and shallow water equations on maximum tsunami wave height, presents the maximum tsunami wave heights and their locations at specific times for parabolic craters under both shallow water [21] and deep water [12] propagation models. The data clearly show that the deep water equation yields significantly greater attenuation of maximum wave height. While the shallow water equation also shows attenuation of maximum wave height over time, the locations where maxima occur are farther from the impact point, indicating that the shallow water model simulates tsunamis propagating to greater distances. For example, at 50 s versus 10 s after impact, the shallow water model's maximum wave height decreases from 1724 m to 568 m, with the location shifting from 644 m to 4737 m from the impact point. In contrast, the deep water model's maximum wave height decreases from 2520 m to 504 m, with the location moving from 316 m to 1579 m.

To investigate the influence of different crater models on subsequent tsunami propagation, [Figure 2: see original paper]–[Figure 6: see original paper] show the attenuation of impact tsunami wave height with propagation distance at 10 s, 50 s, 100 s, 200 s, and 600 s after impact for both hemispherical and parabolic crater shapes, with hemispherical craters on the left and parabolic craters on the right. Under the deep water wave equation, different crater models significantly affect subsequent propagation. The hemispherical crater model shows a clear outward propagation process over time, with notable overall waveform changes. The parabolic crater model, however, exhibits intense fluctuations near the impact point even after considerable time, with no significant waveform evolution over time. Furthermore, we examine the effects of different crater models on tsunami propagation at fixed distances. [Figure 7: see original paper]–[Figure 10: see original paper] present the variation of deep water tsunami wave height with time at distances of 5 km, 10 km, 20 km, and 50 km, with hemispherical craters on the left and parabolic craters on the right. Both models show varying degrees of wave height fluctuations near the impact point, with fluctuations appearing later at greater distances. At larger distances, the propagation differences between the two crater models become clear. The hemispherical crater model continues to show wave fluctuations, while the parabolic model does not exhibit this phenomenon. Comparing the deep water data in and reveals that overall, hemispherical craters produce larger maximum wave heights that occur farther from the impact point than parabolic craters. For instance, at 200 s after impact, the hemispherical crater model yields a maximum deep water tsunami wave height of 177 m at 7799 m from the impact point, whereas the parabolic crater model gives a maximum of 126 m at 6316 m.

These simulation results demonstrate the differences in impact-induced tsunami scales when using different initial crater models and tsunami propagation models. Regarding crater model effects, comparisons of propagation time ([Figure 2: see original paper]–[Figure 6: see original paper]) and distance ([Figure 7: see original paper]–[Figure 10: see original paper]) show that the parabolic crater model produces tsunami waveforms that remain relatively similar across different times, with overall wave height attenuation over time but strong oscillations near the impact point. The hemispherical crater model shows less intense fluctuations near the impact point over time, which better matches reality. At locations near the impact point, both models exhibit strong vibrations shortly after impact that gradually attenuate over time. At greater distances, the hemispherical crater model shows earlier wave height changes and more frequent oscillations than the parabolic model. Additionally, the variation of wave height with distance at fixed times differs between crater models. Overall, the hemispherical crater model simulates tsunamis with earlier onset of fluctuations, more frequent oscillations, and potentially greater hazards than the parabolic model.

Regarding the differences between deep water and shallow water models, these two wave types have different attenuation rates. Deep water waves experience dispersion during propagation [22–23], resulting in an attenuation rate of 1/r, which is greater than the $1/\sqrt{r}$ rate for shallow water waves [20]. clearly shows this attenuation difference for parabolic craters under both models. Consequently, during long-distance propagation, the shallow water model produces larger wave heights and thus greater tsunami hazards than the deep water model. For example, at 50 s after impact, the parabolic deep water model gives a maximum wave height of 504 m at 1579 m, while the parabolic shallow water model yields 568 m at a much greater distance of 4737 m. Furthermore, wave height evolves over time: at 600 s, the shallow water model still shows a maximum wave height of 14 m at 604.8 km, whereas the deep water model's maximum of 42 m occurs only 19 km from the impact point.

Generally, impact-induced tsunamis produce two wave types [9]: rim waves and collapse waves. The former are generated by ejecta curtain material at the crater edge falling back under gravity and propagating away from the crater, while the latter are formed by the collapse of the central water column created as water near the crater fills inward, which then falls under gravity and propagates outward. Wünnemann and Weiss [9] further noted that in shallow water impacts, rim wave effects dominate because ejecta contain mostly seafloor debris with relatively little water. For deep water impacts, the crater does not reach the seafloor, the crater water mass is about three times the ejecta mass, and the collapse wave trough overlaps the rim wave crest, making collapse wave effects dominant. Therefore, to simplify the model, ejecta at the edge of hemispherical craters can be neglected in deep water impact scenarios. The study [9] also noted that impact-generated tsunamis have shorter periods and larger wave heights compared to earthquake and landslide tsunamis. Our comparison of different crater and propagation models shows that the hemispherical deep water model produces tsunami waves with these characteristics, making it the appropriate choice for studying medium-sized asteroid impacts in deep ocean environments.

5 Conclusions

This study first establishes a deep water wave model with a hemispherical impact crater as the initial disturbance, then discusses the effects of initial crater models and different tsunami propagation models on asteroid impact-induced tsunamis. The following conclusions are drawn:

(1) The hemispherical crater tsunami model simulates faster tsunami propagation and more frequent wave fluctuations than the parabolic model, though with greater overall wave height attenuation over time. Due to earlier onset and more frequent fluctuations, hemispherical craters may pose greater tsunami hazards than parabolic craters. For example, at 20 km from the impact point, the hemispherical deep water model's first wave peak appears around 220 s with numerous subsequent peaks, reaching a maximum wave height of approximately 70 m. In contrast, the parabolic deep water model's first wave peak appears around 600 s with no significant subsequent fluctuations and a maximum wave height of only 40 m.

(2) Different propagation models produce differences in subsequent tsunami scales. The shallow water model exhibits smaller attenuation during long-distance propagation, resulting in larger tsunami scales and potentially greater hazards than the deep water model. For instance, at 50 s after impact, the parabolic deep water model shows a maximum wave height of 504 m at 1579 m, while the parabolic shallow water model shows 568 m at a much greater distance of 4737 m. Additionally, wave height evolves over time: at 600 s, the shallow water model still shows a maximum of 14 m at 604.8 km, while the deep water model's maximum of 42 m occurs only 19 km from the impact point.

(3) By comparing simulation results from different impact tsunami models, we find that for medium-sized asteroids (diameters in the range of several hundred meters) impacting the ocean, the deep water model with a hemispherical impact crater is generally more suitable for study.

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Note: Figure translations are in progress. See original paper for figures.