

Reducing Solar Magnetic Field Measurement Errors Using Stereoscopic Observations (Postprint)

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Abstract

The Zeeman effect is currently the primary method for measuring solar magnetic fields; however, observations utilizing the Zeeman effect suffer from two issues: in measured vector magnetic fields, the direction of the magnetic field component perpendicular to the line of sight (transverse field) exhibits a 180° ambiguity, while the measurement precision of the transverse field is also an order of magnitude lower than that of the field component along the line of sight (longitudinal field). Stereoscopic observations of the magnetic field from different lines of sight can eliminate the 180° ambiguity and enhance the measurement precision of the solar magnetic transverse field. This work focuses on the problem of improving transverse field measurement precision through combined observations from the Sun-Earth L5 point and the Sun-Earth line direction, and quantitatively analyzes models and data to determine the reduction in transverse field errors. Using spherical trigonometric formulas, the coordinate relationship between magnetic fields at Earth and the L5 point is derived; the standard deviation propagation formula is employed to obtain corrected measurement errors at Earth; the data distribution of solar magnetic field errors observed by the Helioseismic and Magnetic Imager (HMI) is statistically analyzed, and two datasets consistent with the solar magnetic field distribution are simulated to serve as error maps for Earth and the L5 point, respectively; by combining Earth and L5 point data, corrected transverse field error data for Earth are obtained and compared with original transverse field data, revealing that the transverse field error on the ecliptic plane observed at Earth can be reduced to approximately 17% of its original value.

Full Text

Preamble

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Reducing Solar Magnetic Field Measurement Errors Through Stereo Observations

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Abstract

The Zeeman effect is currently the primary method for measuring solar magnetic fields, but observations using this effect suffer from two fundamental problems: the direction of the magnetic field component perpendicular to the line of sight (the transverse field) has a 180° ambiguity, and the measurement precision of the transverse field is an order of magnitude lower than that of the line-of-sight component (the longitudinal field). Stereo observations from different viewing directions can eliminate the 180° ambiguity and improve the measurement accuracy of the solar magnetic field's transverse component. This study focuses on the problem of improving transverse field measurement precision through combined observations from the Sun-Earth L5 point and the Sun-Earth line direction. Through model construction and quantitative data analysis, we determine the reduction in transverse field error. Using spherical trigonometric formulas, we derive the coordinate relationship between magnetic fields observed from Earth and the L5 point, and obtain the corrected measurement errors at Earth through standard error propagation formulas. By statistically analyzing the error distribution of solar magnetic field data from the Helioseismic and Magnetic Imager (HMI), we simulate two datasets consistent with solar magnetic field distributions to serve as error maps for Earth and the L5 point. Combining data from both Earth and L5 point, we obtain corrected transverse field error data at Earth and compare it with the original transverse field data, finding that the transverse field error in the ecliptic plane can be reduced to approximately 17% of its original value.

Keywords: space vehicles: instruments, methods: statistical, Sun: magnetic fields

1. Introduction

Violent solar activities such as coronal mass ejections and solar flares frequently occur in the solar atmosphere. These activities can alter Earth's magnetic field, posing risks to aerospace operations and artificial satellites, reducing navigation

accuracy and communication reliability, and even causing large-scale power outages by affecting power transmission systems. Since these intense solar activities are intimately related to solar magnetic fields, a clear understanding of the structure and evolution of solar magnetic fields is crucial for predicting solar activity and preparing for its impacts.

Polarization measurement is the most important method for observing solar magnetic fields, and achieving more precise magnetic field observations has always been a primary goal. Research related to solar magnetic fields remains at the forefront of solar physics and space physics. Most violent solar activities are closely associated with coronal magnetic fields. However, to date, no effective means exist to accurately measure coronal magnetic fields. Current methods for obtaining coronal magnetic fields primarily involve extrapolation using appropriate magnetic field models with photospheric magnetic fields as boundary conditions. Therefore, improving the measurement accuracy of photospheric magnetic fields also helps us better understand coronal magnetic fields.

For current solar magnetic field measurements, even without considering any systematic errors, the vector field at a given location cannot be completely determined. Solar magnetic fields are primarily obtained by observing solar polarization and utilizing the Zeeman effect. For example, the Helioseismic and Magnetic Imager (HMI) obtains magnetic fields through the Very Fast Inversion of the Stokes Vector (VFISV), which results in a 180° ambiguity in the transverse component of the magnetic field. To determine the vector magnetic field, we must eliminate this ambiguity. Currently, we can only use theoretical models to resolve the 180° ambiguity, with the most widely used model being the minimum energy principle.

One model-independent method is to use multi-viewpoint observations to resolve the 180° ambiguity. If two satellites simultaneously observe the solar vector magnetic field from two different perspectives, this uncertainty can be eliminated. Zhou et al. also found that at a separation angle of about 30° , approximately 50 Gs of transverse field 180° ambiguity can be corrected. With the successful operation of the Solar Orbiter, simultaneous observations of the solar magnetic field from two different positions have been achieved, marking the beginning of the multi-viewpoint observation era for solar magnetic field measurements. Additionally, the successful launch of the Advanced Space-based Solar Observatory (ASO-S) represents China's first space-based solar magnetic field observation.

Furthermore, measuring the transverse component of the photospheric magnetic field has errors an order of magnitude higher than the longitudinal (or line-of-sight) component. This is an intrinsic limitation of magnetic field measurement using the Zeeman effect, resulting in inaccurate measurements of the solar magnetic field's transverse component. However, just as multi-viewpoint observations can remove the 180° ambiguity in the transverse field, they can also significantly reduce transverse field errors. For example, launching three satellites to simultaneously observe the line-of-sight magnetic field from appropriate viewing angles could achieve this. However, deploying satellites off the Sun-

Earth line presents many challenges, including higher requirements for launch, orbit insertion, and mission lifetime. Therefore, astronomers suggest first placing detectors at the Sun-Earth Lagrange point L5, then advancing solar polar orbit satellites, close-proximity detection, and panoramic solar observations.

This paper considers combined observations from L5 point and Earth perspectives to address the problem of inaccurate measurement of the solar magnetic field's transverse component. The specific research uses existing HMI solar magnetic field observation data as an example to quantitatively calculate the improvement in transverse field measurement accuracy through dual-viewpoint observations and verify the feasibility of this method through numerical simulation. In Section 2, we use spherical trigonometry to derive the relationship between Earth and L5 point coordinates and calculate the improved Earth transverse field errors through standard error propagation formulas. In Section 3, we statistically analyze the distribution of magnetic field errors with heliocentric angle, simulate two magnetograms as observation data for Earth and L5 point, and substitute them into the formulas from Section 2. Section 4 summarizes the research results and provides in-depth discussion.

2. Theoretical Model

2.1 Ideal Case Analysis

We first consider the simplest scenario where the magnetic field has only a line-of-sight component B_ζ and a transverse component B_ξ . The only influencing factor is the angle α shown in [Figure 1: see original paper].

[Figure 1: see original paper] shows the relationship between magnetic fields observed from Earth and a satellite under ideal conditions. To derive the relationship between the transverse magnetic field component observed at Earth, $B_{E\xi}$, and the two line-of-sight components $B_{E\zeta}$ and $B_{S\zeta}$, we decompose $B_{S\zeta}$ along the $B_{E\zeta}$ and $B_{E\xi}$ directions:

$$B_{S\zeta} = B_{E\zeta} \cos \alpha - B_{E\xi} \sin \alpha$$

Rearranging gives the relationship between $B_{E\xi}$ and $B_{E\zeta}$, $B_{S\zeta}$:

$$B_{E\xi} = \cot \alpha B_{E\zeta} - \csc \alpha B_{S\zeta}$$

When x and y are independent, the standard error propagation formula is:

$$\Delta F(x, y)^2 = \Delta x^2 + \Delta y^2$$

Since observations involve two satellites, $B_{E\zeta}$ and $B_{S\zeta}$ are independent. Using equations (2) and (3), we can derive the improved error $\Delta B'_{E\xi}$:

$$\Delta B'_{E\xi} = \cot^2 \alpha \Delta B_{E\zeta}^2 + \csc^2 \alpha \Delta B_{S\zeta}^2$$

If both satellites use identical instruments, their line-of-sight magnetic field errors should be of the same order. Assuming $\Delta B_{E\zeta}$ and $\Delta B_{S\zeta}$ are of the same magnitude, we can divide both sides of equation (4) by $\Delta B_{E\zeta}$:

$$\frac{\Delta B'_{E\xi}}{\Delta B_{E\zeta}} = \sqrt{\cot^2 \alpha + \csc^2 \alpha}$$

This ratio represents the relationship between the corrected error and the original line-of-sight magnetic field error. The smaller the ratio, the better the precision improvement effect.

By varying α from 0° to 90° , we obtain the function of improved $\Delta B'_{E\xi}$ versus separation angle α . Analyzing HMI error files and comparing the full-disk average of $\Delta B_{E\xi}$ with that of $\Delta B_{E\zeta}$ reveals that $\Delta B_{E\xi}$ is approximately 8 times larger than $\Delta B_{E\zeta}$, allowing us to derive the function of improved $\Delta B'_{E\xi}$ versus separation angle α .

[Figure 2: see original paper] shows the variation of improved $\Delta B'_{E\xi}$. From equation (5) and Figure 2: see original paper, when the separation angle α is 60° , the improved $\Delta B'_{E\xi}$ is about 1.28 times $\Delta B_{E\zeta}$. When α is 30° , the improved $\Delta B'_{E\xi}$ is about 2.65 times $\Delta B_{E\zeta}$.

As shown in Figure 2: see original paper, when α is 60° , the improved $\Delta B'_{E\xi}$ is about 16.0% of the original $\Delta B_{E\xi}$. It should be noted that although theoretically a 90° separation between two satellites (directly measuring the transverse field) would maximize error reduction, too large a separation angle reduces the common observation area, and limb darkening effects also increase measurement errors near the solar limb.

2.2 Actual Solar Conditions

After converting HMI observation data to map structure and reading the b_0 parameter, we can determine the coordinate relationship between Earth and the L5 point using spherical trigonometric formulas. In spherical triangles $\triangle ABL_1$ and $\triangle ADL_5$, the law of sines gives:

$$\frac{\sin b_0}{\sin i} = \frac{\sin b_1}{\sin i} = \frac{\sin a}{\sin 90^\circ}, \quad \frac{\sin(a - 60^\circ)}{\sin 90^\circ}$$

[Figure 3: see original paper] shows the coordinate relationship between Earth and the L5 point under actual solar conditions. The specific value of b_0 can be obtained from the Solar Software (SSW) package in IDL (Interactive Data Language) using the `pb0r()` function with the specific universal time of observation. It is worth noting that when determining b_1 from equations (6) and (7),

we must use the monotonicity of b_0 to determine the value of a . Also, since b_1 is the angle between the line from L5 to the solar center and the equatorial plane, its absolute value should be less than or equal to the solar ecliptic obliquity i .

Finally, we only need to find the β angle. In spherical triangle $\Delta PL_{1L}5$, using the cosine law for sides:

$$\cos 60^\circ = \cos(90^\circ + b_1) \cos(90^\circ + b_0) + \sin(90^\circ + b_1) \sin(90^\circ + b_0) \cos \beta$$

From [Figure 3: see original paper], to transform coordinates from L5 point to L1 point, we first rotate by $-b_1$ around the ξ axis, then by β around the η axis, and finally by b_0 around the ξ axis. Using rotation matrices and the angles b_1 , b_0 , and β , we obtain:

$$\mathbf{B}_E = R_\xi(b_0)R_\eta(\beta)R_\xi(-b_1)\mathbf{B}_S$$

Expanding the rotation matrix:

$$\begin{pmatrix} B_{E\zeta} \\ B_{E\xi} \\ B_{E\eta} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} B_{S\zeta} \\ B_{S\xi} \\ B_{S\eta} \end{pmatrix}$$

Focusing on the first and second row expansions:

$$\begin{aligned} B_{E\zeta} &= a_{11}B_{S\zeta} + a_{12}B_{S\xi} + a_{13}B_{S\eta} \\ B_{E\xi} &= a_{21}B_{S\zeta} + a_{22}B_{S\xi} + a_{23}B_{S\eta} \end{aligned}$$

Although equation (12) gives an expression for $B_{E\xi}$, it does not incorporate line-of-sight component data from L1 point and is not the optimal algorithm for error reduction. Therefore, we can combine equation (11) to obtain the minimum error. We introduce an undetermined coefficient c_1 and use equation (12) minus c_1 times equation (11):

$$B_{E\xi} = (a_{21} - c_{1a_{11}})B_{S\zeta} + (a_{22} - c_{1a_{12}})B_{S\xi} + (a_{23} - c_{1a_{13}})B_{S\eta} + c_1B_{E\zeta}$$

Applying the standard error propagation formula (3):

$$\Delta B_{E\xi} = \sqrt{(a_{21} - c_{1a_{11}})^2 \Delta B_{S\zeta}^2 + (a_{22} - c_{1a_{12}})^2 \Delta B_{S\xi}^2 + (a_{23} - c_{1a_{13}})^2 \Delta B_{S\eta}^2 + c_1^2 \Delta B_{E\zeta}^2}$$

Since all variables except c_1 are either constants related to the rotation matrix or observational errors, the expression under the square root is a quadratic function of c_1 . Finding the axis of symmetry yields the value of c_1 that minimizes $\Delta B_{E\xi}$:

$$c_1 = \frac{a_{21}a_{11}\Delta B_{S\zeta}^2 + a_{22}a_{12}\Delta B_{S\xi}^2 + a_{23}a_{13}\Delta B_{S\eta}^2}{a_{11}^2\Delta B_{S\zeta}^2 + a_{12}^2\Delta B_{S\xi}^2 + a_{13}^2\Delta B_{S\eta}^2 + \Delta B_{E\zeta}^2}$$

Substituting this c_1 into $\Delta B_{E\xi}$ yields its minimum value. By substituting equation (15) into equations (13) and (14), we can obtain the ξ -direction magnetic field from dual-viewpoint observations and calculate its error.

2.3 Using Only Satellite Line-of-Sight Magnetic Field Data

Since we are most concerned with satellite line-of-sight magnetic field errors—as these measurements have the highest precision and are more valuable than transverse field measurements—observing only the line-of-sight component would save significant space and resources. Therefore, we calculate how much the error can be reduced using only satellite line-of-sight data combined with Earth's three-component data. First, we find the relationship between satellite line-of-sight data and Earth's three-component data by inverting the rotation matrix in equation (10):

$$\begin{pmatrix} B_{S\zeta} \\ B_{S\xi} \\ B_{S\eta} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} B_{E\zeta} \\ B_{E\xi} \\ B_{E\eta} \end{pmatrix}$$

Expanding the first row:

$$B_{S\zeta} = b_{11}B_{E\zeta} + b_{12}B_{E\xi} + b_{13}B_{E\eta}$$

Rearranging to isolate $B_{E\xi}$:

$$B_{E\xi} = \frac{B_{S\zeta} - b_{11}B_{E\zeta} - b_{13}B_{E\eta}}{b_{12}}$$

Applying the standard error propagation formula (3):

$$\Delta B_{E\xi} = \sqrt{\left(\frac{1}{b_{12}}\right)^2 \Delta B_{S\zeta}^2 + \left(\frac{b_{11}}{b_{12}}\right)^2 \Delta B_{E\zeta}^2 + \left(\frac{b_{13}}{b_{12}}\right)^2 \Delta B_{E\eta}^2}$$

This is the theoretical model for correcting $\Delta B_{E\xi}$ using only the satellite's line-of-sight magnetic field component. Only the vector magnetic field observed from the current Earth perspective and the line-of-sight magnetic field component from a future space satellite are needed to calculate the improved transverse field and its error.

3. Data Analysis

3.1 Calculating Vector Magnetic Field Errors

First, we download seven files from the HMI website containing magnetic field strength B and its error ΔB , azimuth angle ϕ and its error $\Delta\phi$, inclination angle θ and its error $\Delta\theta$, and files with the 180° ambiguity removed. After reading these with SSW, we use formulas from Hagyard et al. and related studies to obtain line-of-sight and transverse magnetic field components from the above data. The geometric relationships give:

$$\begin{aligned} B_\zeta &= B \cos \theta \\ B_\xi &= B \sin \theta \cos(\phi + 270^\circ) \\ B_\eta &= B \sin \theta \sin(\phi + 270^\circ) \end{aligned}$$

We also read the correlation coefficients between B , ϕ , and θ from the HMI website. These are on the order of 10^{-3} or lower, allowing us to approximate B , ϕ , and θ as independent. Using the standard error propagation formula (3):

$$\begin{aligned} \Delta B_\zeta &= \sqrt{B^2 \sin^2 \theta \Delta\theta^2 + \cos^2 \theta \Delta B^2} \\ \Delta B_\xi &= \sqrt{B^2 \sin^2 \theta \cos^2 \phi \Delta\phi^2 + B^2 \cos^2 \theta \sin^2 \phi \Delta\theta^2 + \sin^2 \theta \sin^2 \phi \Delta B^2} \\ \Delta B_\eta &= \sqrt{B^2 \sin^2 \theta \sin^2 \phi \Delta\phi^2 + B^2 \cos^2 \theta \cos^2 \phi \Delta\theta^2 + \sin^2 \theta \cos^2 \phi \Delta B^2} \end{aligned}$$

Equation (21) allows us to calculate the line-of-sight magnetic field error ΔB_ζ and transverse magnetic field errors ΔB_ξ and ΔB_η .

3.2 Statistical Distribution of Vector Magnetic Field Errors

In solar magnetic field observations, limb darkening causes measurement precision to decrease with increasing heliocentric angle. Therefore, we need to statistically analyze the distribution of solar magnetic field errors with respect to heliocentric angle.

Using magnetic field data from HMI observed on September 6, 2017 at 11:36 UT, we calculate line-of-sight and transverse field errors using the algorithm in Section 3.1. For heliocentric angles from 0° to 90° , we randomly select 50 points at intervals of 0.5° to obtain ΔB_ζ , ΔB_ξ , and ΔB_η data, creating scatter plots of error distribution versus heliocentric angle as shown in [Figure 4: see original paper].

[Figure 4: see original paper] shows that ΔB_ξ and ΔB_η errors gradually increase with heliocentric angle, with average values around 50 Gs. More errors exceed

100 Gs at larger heliocentric angles, while ΔB_ζ shows no obvious relationship with heliocentric angle. Since scatter plots alone cannot precisely simulate magnetic field errors, we also need to statistically analyze the mean values of ΔB_ξ , ΔB_η , and ΔB_ζ versus heliocentric angle, as shown in [Figure 5: see original paper].

To simulate error data using Gaussian distributions, we must also analyze the standard deviation distribution of errors with heliocentric angle, which helps us choose appropriate standard deviation values when adding Gaussian noise. The standard deviation σ of magnetic field errors versus heliocentric angle is shown in [Figure 6: see original paper].

[Figure 5: see original paper] shows that transverse errors slowly increase with heliocentric angle, approximately linearly, while line-of-sight field errors appear more scattered. [Figure 6: see original paper] shows that the transverse error curves are relatively flat, with peaks around 40° caused by sunspots at that heliocentric angle resulting in extremely large standard deviations.

3.3 Data Simulation

Based on the mean values of ΔB_ξ and ΔB_η versus heliocentric angle from Figure 5: see original paper and (b), we add Gaussian noise according to the error standard deviations in Figure 6: see original paper and (b). We use each heliocentric angle's error mean as the Gaussian distribution peak and the statistical standard deviation as the distribution's σ . This assignment is applied for heliocentric angles from 0° to 80° . For the 80° to 90° range, where limb darkening makes errors less reliable, we extrapolate using regression on data from 0° to 80° . The ΔB_ζ case is more complex: since σ in Figure 6: see original paper exceeds the mean ΔB_ζ , and scatter plots show a few very large ΔB_ζ values causing large standard deviations, directly using this σ would produce many negative ΔB_ζ values, which is unphysical. Therefore, we use the scatter plot data from Figure 4: see original paper where σ is about 1.5 Gs.

This method simulates two magnetic field datasets, which we treat as magnetic field data for L5 point and Earth. Substituting both datasets into the theoretical models from Section 2 yields the corrected Earth ξ -component error.

3.4 Magnetogram Conversion

Processing error data from both magnetic field datasets requires direct comparison of Earth and L5 point magnetograms, meaning one magnetogram must be interpolated to match the other's coordinates. Since we focus on improving Earth's transverse field precision, we interpolate the L5 point magnetogram to Earth's perspective, creating a new vector magnetogram. We assume identical solar magnetic field instruments at both points, ignoring errors from instrument differences.

First, we create a blank magnetogram. From HMI data, we obtain the solar

radius, the ξ and η coordinates of the solar center, and the arcseconds per pixel, allowing us to calculate ξ and η coordinates for all points. Points beyond the original solar radius are assigned boundary values. For points within the solar radius, we calculate their ξ , η , and ζ coordinates. Magnetic field conversion uses the inverse of the rotation matrix in equation (9) to transform L5 point coordinates to Earth coordinates. If transformed coordinates have fractional parts, we use bilinear interpolation in ξ and η to assign magnetic field errors. Points with $\zeta < 0$ after transformation are on the Sun's far side and cannot be observed from Earth, so these are also assigned boundary values.

For combined Earth and L5 point observations, approximately 120° of the solar surface can be commonly observed. Using HMI magnetic field strength data B observed on September 6, 2017 at 11:34 UT as an example, we process it to obtain two magnetograms shown in [Figure 7: see original paper].

Figure 7: see original paper shows the image after interpolating Figure 7: see original paper, with coordinates now corresponding to the common observation region with L1 point. Substituting both datasets into equations (14) and (19), we calculate the improved $\Delta B'_{E\xi}$ and divide it by the original $\Delta B_{E\xi}$ and $\Delta B_{E\zeta}$ to obtain correction factors. lists the averaged simulation results for the common observation region.

The method for improving transverse field errors through combined observations from other positions in the ecliptic plane and the Sun-Earth line is consistent with the L5 point method, requiring only modification of the 60° angle in equation (7) to the corresponding angle. We also present results for a 30° separation angle in the ecliptic plane, shown in .

4. Conclusions and Discussion

Our research demonstrates that through appropriate theoretical calculations combined with simultaneous observations from L5 point and Earth perspectives, we can significantly improve the precision of traditionally poorly measured transverse magnetic fields. Using only the satellite's line-of-sight magnetic field data, while slightly less precise than the optimal calculation method, requires only one-third of the data volume and represents the most cost-effective approach—a negligible precision difference. Moreover, even using only the satellite's line-of-sight component substantially improves transverse field precision.

Multi-viewpoint solar magnetic field observations would enable more in-depth solar activity research. If satellites are deployed not only in the ecliptic plane but also outside it, the η -component error can also be significantly reduced. For combined observations with Earth, off-ecliptic satellites would ideally be positioned in a plane perpendicular to the ecliptic and intersecting Earth. This would allow just two satellites' line-of-sight observations plus L1 point's three-component observations to reduce errors in both transverse components of the commonly observed region by approximately an order of magnitude.

For future simultaneous observations from satellites at other angles, one need only calculate the coordinate relationship between that satellite' s perspective and Earth' s perspective, modify the rotation matrix in equation (10) accordingly, and use the magnetogram conversion method from Section 3.4 to transform the satellite' s perspective to Earth' s perspective before substituting into the theoretical formulas from Section 2.2 or 2.3. The specific formula choice depends on whether the satellite can provide three magnetic field components.

For three-satellite simultaneous observations, the problem can be decomposed into pairwise combinations, improving magnetic field errors from two directions independently. Care should be taken that the separation angle between any satellite and the plane containing the other two should not be too large or too small. Excessively large angles reduce the common observation area and exacerbate limb darkening effects. Angles exceeding 90° yield poorer correction factors than 90° while reducing common observation area, so separation angles should not exceed 90° . Meanwhile, equation (5) and [Figure 1: see original paper] show that angles smaller than 20° produce insignificant error improvement. Therefore, ideal separation angles between different satellites range from 30° to 60° .

This work aims to provide theoretical methods and preliminary attempts for calculating errors in multi-viewpoint observations for future solar space missions, offering an approach to improve magnetic field precision for observations from L5, L4, or other angles in the ecliptic plane, as well as for solar polar orbit satellites.

References

- [1] Deng Y Y, Gan W Q, Yan Y H, et al. *Infrared and Laser Engineering*, 2020, 49: 230
- [2] Li W X, Tian H. *Chinese Journal of Nature*, 2022, 44: 19
- [3] Stenflo J O. *SSRv*, 2017, 210: 5
- [4] Wiegelmann T. *SoPh*, 2004, 219: 87
- [5] Wiegelmann T, Sakurai T. *LRSP*, 2012, 9: 5
- [6] Feng X S, Jiang C W, Xiang C Q, et al. *ApJ*, 2012, 758:
- [7] Jiang C W, Wu S T, Feng X S, et al. *ApJ*, 2014, 786: L16
- [8] Zeeman. *Nature*, 1897, 55: 347
- [9] Borrero J M, Tomczyk S, Kubo M, et al. *SoPh*, 2011, 273: 267
- [10] Metcalf T R, Leka K D, Barnes G, et al. *SoPh*, 2006, 237:
- [11] Leka K D, Barnes G, Crouch A D, et al. *SoPh*, 2009, 260:
- [12] Zhou R Y, Wang Y M, Su Y N, et al. *Acta Astronomica Sinica*, 2021, 62: 41
- [13] Zhou R Y, Wang Y M, Su Y N, et al. *ChA&A*, 2021, 45:
- [14] Müller D, Cyr O C S, Zouganelis I, et al. *A&A*, 2020, 642: A1
- [15] Gherardo V, Philipp L, David S, et al. *SoPh*, 2022, 297:
- [16] Wei Q G, Cheng Z, Yuan Y, et al. *RAA*, 2019, 19: 5
- [17] Ji H S, Wang Y M, Wang J X. *Scientia Sinica: Physica, Mechanica &*

Astronomica, 2019, 49: 44

[18] Deng Y Y, Zhou G P, Dai S W, et al. Chinese Science Bulletin, 2023, 68: 298

[19] Wang Y M, Bai X Y, Chen C Y, et al. AdSpR, 2023, 71:

[20] Wang Y M, Ji H S, Wang Y M, et al. ScChE, 2020, 63:

[21] Bobra M G, Sun X, Hoeksema J T, et al. SoPh, 2014, 289: 3549

[22] Freeland S L, Handy B N. SoPh, 1998, 182: 497

[23] Hagyard M J. SoPh, 1987, 107: 239

[24] Gary G A, Hagyard M J. SoPh, 1990, 126: 21

[25] Sun X. arXiv preprint, 2013, arXiv:1309.2392

[26] Guo Y, Cheng X, Ding M D. ScChD, 2017, 60: 1408

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