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Abstract

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Full Text

Static or dynamic pear-shapes in radioactive nucleus ^{224}Rn?*

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We report a comprehensive study on the low-lying parity-doublet states of ^{224}Rn with the mixing of both quadrupole and octupole-shaped configurations in a multireference covariant density functional theory, where the symmetries broken in the configurations are restored with projection techniques. The low-lying energy spectrum is reasonably reproduced. The electric octupole transition strength in ^{224}Rn is found to be $B(E3; 3^-_1 \to 0^+_1) = 43$ W.u., comparable to that in ^{224}Ra, whose data are 42(3) W.u. Our results indicate that ^{224}Rn shares a similar low-energy structure with ^{224}Ra, despite the first 3^- state of the former nucleus being higher in energy than that of the latter. This study suggests ^{225}Rn to be another candidate to search for a permanent electric dipole moment.

Keywords: Covariant density functional theory, Parity-doublet bands, Octupole correlations, Electric transition strengths

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I. INTRODUCTION

The majority of nuclei on the nuclear chart are characterized by reflection-symmetric shapes, either spherical or quadrupole, in their ground states. However, atomic nuclei with proton number Z or neutron number $N \simeq 34, 56, 88,$ and 134 possess strong octupole correlations, exhibiting dynamical or static octupole deformation [1, 2]. The signatures of nuclei with a static octupole-deformed shape include the presence of low-lying positive- and negative-parity doublets, as well as strong electric dipole ($E1$) and octupole ($E3$) transition strengths between them. According to this criterion, the observation of low-lying 3^- states and enhanced $E3$ transitions in ^{144,146}Ba [3–5], ^{222}Ra [6], ^{224}Ra [7], and ^{226}Ra [8] suggests that these nuclei have a rather stable octupole deformation. In contrast, some atomic nuclei with slightly weaker $E3$ transitions but with a relatively larger excitation energy for the 3^- state are usually interpreted as octupole vibrators, such as ^{228}Ra [6], as well as ^{208}Pb and ^{220}Rn [7]. Additionally, low-lying negative-parity states or enhanced $E1$ transitions have been measured, indicating that $^{222-226}$Rn are vibrators [9] and that ^{228}Th is pear-shaped [10]. So far, however, in suggested octupole-deformed nuclei, a strict alternation between positive- and negative-parity energy levels has not been observed. To draw a solid conclusion on whether these nuclei belong to octupole rotors or vibrators, more measurements and comprehensive theoretical studies on these nuclei are needed.

Atomic nuclei with octupole correlations have been extensively studied with a variety of nuclear models [11–16], including self-consistent mean-field (SCMF) methods based on different energy density functionals (EDFs)[17–30] and beyond, in combination with the interacting boson models[31–34] or collective Hamiltonians [35–38]. In particular, the generator coordinate method (GCM) implemented with quantum-number projection techniques, including parity, particle-number, and angular-momentum projections, has been developed for the low-lying states of atomic nuclei with octupole correlations based on different EDFs [39–43]. Within this multi-reference density functional theory (MR-DFT), it has been demonstrated that the low-lying states of ^{208}Pb are multioctupole-phonon excitations [42]. In contrast, for ^{144,146}Ba, and ^{224}Ra, the octupole shapes of positive-parity states are stabilized rapidly with the increase of spin, gradually drifting toward those of negative-parity ones [40, 41]. In view of this success, in this work, we extend the MR-DFT framework to study the low-lying states of ^{224}Rn, including both the energy spectrum and electric multipole transition strengths, based on a relativistic energy density functional (EDF), shedding light on whether this nucleus belongs to octupole rotors or vibrators.

The paper is arranged as follows. In Sec. II, a brief introduction to the MR-DFT based on a relativistic EDF is presented. In Sec. III, we discuss the results of calculations on the low-lying states of ^{224}Rn, in comparison with those of ^{224}Ra. Finally, a brief summary is given in Sec. IV.

II. THEORETICAL FRAMEWORK

In the MR-DFT, wave functions of low-lying parity-doublet states are constructed as a linear combination of a set of quantum-number projected nonorthogonal mean-field states $|\mathbf{q}\rangle$ around the equilibrium shape

$$|\Psi_\alpha^{J\pi}\rangle = \sum_{\mathbf{q}} f_\alpha^{J\pi}(\mathbf{q}) \hat{P}^J_{MK} \hat{P}^N \hat{P}^Z \hat{P}^\pi |\mathbf{q}(\beta_2, \beta_3)\rangle, \tag{1}$$

where α labels the different quantum states corresponding to a given J^π. The symbol $\mathbf{q}$ stand for quadrupole and octupole

deformation parameters of each mean-field state. The operators $\hat{P}^J_{MK}$, $\hat{P}^{N,Z}$, and $\hat{P}^\pi$ are used to select components of configurations with specific quantum numbers, namely angular momentum J, neutron (proton) number $N(Z)$, and parity $\pi = \pm$, respectively [44].

The mean-field states $|\mathbf{q}\rangle$ are generated from the point coupling relativistic mean-field plus BCS (PC-RMF+BCS) calculations with constraints on average nucleon numbers and quadrupole-octupole moments through the variational principle,

$$\delta \left\langle \mathbf{q} \middle| \hat{H} - \sum_{\tau=n,p} \lambda_\tau \hat{N}_\tau - \sum_{\lambda=1,2,3} C_\lambda \left(\hat{Q}_{\lambda 0} - q_\lambda\right)^2 \middle| \mathbf{q} \right\rangle = 0. \tag{2}$$

with the Lagrange multipliers λ_τ being determined by the constraints $\langle \mathbf{q}|\hat{N}_\tau|\mathbf{q}\rangle = N(Z)$. The position of the center-of-mass coordinate is fixed at the origin to decouple the spurious states by the constraint $\langle \mathbf{q}|\hat{Q}_{10}|\mathbf{q}\rangle = 0$. The $\hat{N}_\tau$ and $\hat{Q}_{\lambda 0} = r^\lambda Y_{\lambda 0}$ are particle number and multipole moment operators. q_λ is the constrained value of the multipole moment, and C_λ is corresponding stiffness constant [44]. The deformation parameters β_λ $(\lambda = 2, 3)$ are defined as

$$\beta_\lambda \equiv \frac{4\pi}{3AR_0^\lambda} \langle \mathbf{q}|\hat{Q}_{\lambda 0}|\mathbf{q}\rangle \tag{3}$$

with $R_0 = r_0 A^{1/3}$ which A represents the mass number of the nucleus and $r_0 = 1.2$ fm.

The weight function $f_\alpha^{J\pi}(\mathbf{q})$ in Eq. (1) is determined by solving the Hill-Wheeler-Griffin (HWG) equation [45, 46],

$$\sum_{\mathbf{q}_b} \left[H^{J\pi}(\mathbf{q}_a, \mathbf{q}_b) - E_\alpha^{J\pi} \mathcal{N}^{J\pi}(\mathbf{q}_a, \mathbf{q}_b) \right] f_\alpha^{J\pi}(\mathbf{q}_b) = 0, \tag{4}$$

where the kernels are given by

$$O^{J\pi}(\mathbf{q}_a, \mathbf{q}_b) = \langle \mathbf{q}_a | \hat{O} \hat{P}^J_{KM} \hat{P}^\pi \hat{P}^N \hat{P}^Z | \mathbf{q}_b \rangle \tag{5}$$

with the operator $\hat{O}$ representing $\hat{H}$ and 1 for the Hamiltonian kernel $H^{J\pi}(\mathbf{q}_a, \mathbf{q}_b)$ and the norm kernel $N^{J\pi}(\mathbf{q}_a, \mathbf{q}_b)$, respectively.

The electric multipole transition probabilities $B(E\lambda)$ from the initial state $(J_i \pi_i \alpha_i)$ to the final state $(J_f \pi_f \alpha_f)$ are calculated according to the Wigner-Eckart theorem,

$$\begin{aligned} & B(E\lambda; J_i\pi_i\alpha_i \to J_f\pi_f\alpha_f) \\ &= \frac{e^2}{2J_i+1} \left| \sum_{\mathbf{q}_i, \mathbf{q}_f} \left[f_{\alpha_i}^{J_i\pi_i}(\mathbf{q}_i) \right]^* \left[f_{\alpha_f}^{J_f\pi_f}(\mathbf{q}_f) \right] \right. \\ & \qquad \times \left. \left\langle \Phi_{\alpha_f}^{J_f\pi_f}(\mathbf{q}_f) \left\| \hat{Q}_\lambda \right\| \Phi_{\alpha_i}^{J_i\pi_i}(\mathbf{q}_i) \right\rangle \right|^2 . \end{aligned} \tag{6}$$

with the reduced transition matrix element

$$\begin{aligned} & \left\langle \Phi_{\alpha_f}^{J_f\pi_f}(\mathbf{q}_f) \left\| \hat{Q}_\lambda \right\| \Phi_{\alpha_i}^{J_i\pi_i}(\mathbf{q}_i) \right\rangle \\ &= \delta_{\pi_i\pi_f,(-1)^\lambda} \frac{(2J_f+1)(2J_i+1)}{2} \sum_M \begin{pmatrix} J_f & \lambda & J_i \\ 0 & M & -M \end{pmatrix} \\ & \times \int_0^\pi d\theta \, \sin(\theta) \, d^{J_i *}_{-M0}(\theta) \left\langle \Phi(\mathbf{q}_f) \left| \hat{Q}_{\lambda M} e^{i\theta \hat{J}_y} \hat{P}^\pi \hat{P}^N \hat{P}^Z \right| \Phi(\mathbf{q}_i) \right\rangle , \end{aligned} \tag{7}$$

where $\hat{Q}_{\lambda M} \equiv e r^\lambda Y_{\lambda M}$ is the electric multipole moment operator of rank λ. More details to the MR-DFT for quadrupole-octupole nuclei can be found in Refs. [40–43, 47, 48].

III. RESULTS AND DISCUSSIONS

The Dirac spinors of nucleons are expanded in a set of harmonic oscillator basis with fourteen major shells. In the PC-RMF+BCS calculation, the relativistic EDF PC-PK1 [49] is employed. Only the axial symmetry deformation degree of freedom is considered in the current calculations. Pairing correlations between nucleons are treated within the BCS approximation by using a density-independent δ-force with a smooth cutoff [50]. The strength parameters of the pairing force are set to $V_n = -349.5$ MeV fm^3 for neutrons and $V_p = -330.0$ MeV fm^3 for protons. In the calculation of the projected kernels, $N_\beta = 16$ mesh points are used for the rotation angle β, and $N_\varphi = 7$ for the gauge angle φ, both within the interval $[0, \pi]$. Since the low-lying parity-doublet bands of ^{224}Rn are primarily characterized by prolate deformed configurations with $\beta_2 > 0$, the oblate deformed configurations with $\beta_2 < 0$ are excluded in the final configuration-mixing GCM calculation to reduce computational costs. The Pfaffian method [51, 52] is implemented to avoid the sign problem when calculating the norm kernel overlap.

Fig. 1. (Color online) The mean-field energy surface of ^{224}Rn in β_2-β_3 deformation plane normalized to the energy minimum. Two neighboring contour lines are separated by 0.4 MeV.

Figure 1 displays the energies of mean-field states for ^{224}Rn normalized to the energy minimum in β_2-β_3 deformation plane. It is shown that although the energy minimum is located at $\beta_3 = 0$, the energy surface is soft along the β_3 direction around the minimum, similar to the finding in study using the relativistic Hartree-Bogoliubov (RHB) method [25]. The softness of the energy surface in ^{224}Rn is attributed to the coupling of proton orbitals $i_{13/2} - f_{7/2}$ and neutron orbital

$j_{13/2} - g_{7/2}$ around the Fermi surface, respectively [53]. The soft behavior indicates that dynamical correlation effects, including symmetry restoration and quadrupole-octupole shape fluctuations, could be significant in the low-lying states of ^{224}Rn.

Fig. 2. (Color online) The energies of states in ^{224}Rn with projections onto good nucleon numbers, different spin parities with (a) $J^\pi = 0^+$, (b) $J^\pi = 2^+$, (c) $J^\pi = 1^-$, and (d) $J^\pi = 3^-$ in the β_2-β_3 deformation plane, normalized to the energy minimum of each J state.

Figure 2 displays the energy surfaces of ^{224}Rn with projections onto good nucleon numbers and spin-parity $J^\pi = 0^+, 1^-, 2^+$, and 3^-. Since the mean-field configurations with a very small value of β_3 are predominated by the components with positive parity, the energies of negative-parity states projected from these mean-field configurations are not shown. It is seen that the energy minimum of the 0^+ state is shifted to the octupole-deformed shape with $\beta_2 = 0.1, \beta_3 = \pm 0.05$. The energy gained from the restoration of broken symmetries for the energy-minimum state is ~ 4.23 MeV. The energy surface of the 2^+ states is similar to that of 0^+ states, except that it is softer along the quadrupole deformation $\beta_2 \simeq [0.10, 0.20]$ with octupole deformation $\beta_3 \simeq [-0.10, +0.10]$. A similar feature is shown in the potential energy surfaces (PESs) with $J = 1$, cf. Fig. 2(c), and $J = 3$, cf. Fig. 2(d), where the absolute minima are well separated along the β_3 direction. The projected PESs of $J = 1$ and $J = 3$ display soft structure in octupole $\beta_3 \simeq \pm 0.15$ with the quadrupole deformation ranging from $\beta_2 \simeq 0.10$ to 0.25.

The quadrupole-octupole deformed configurations with good quantum numbers serve as a set of basis states to expand the wave functions of low-lying states within the GCM. Figure 3 displays excitation energies of positive-parity and negative-parity bands calculated by solving the HWG equations (4) in three different configuration-mixing schemes. The results of calculations are compared to the data from Ref. [9]. One can see that the calculation with the mixing of configurations with different β_2 and fixed $\beta_3 = 0.05$ gives a very spread energy spectrum. Especially, the negative-parity states are very high in energy. In contrast, with the mixing of configurations with different β_3, but fixed $\beta_2 = 0.15$, the energy spectrum becomes significantly compressed. In the full quadrupole-octupole configuration mixing calculation, negative-parity states are shifted down further and become close to the data.

Fig. 3. (Color online) The energy spectra of low-lying states in ^{224}Rn obtained from the GCM calculations with (b) (β_2, β_3), (c) $(\beta_2 = 0.15, \beta_3)$, and (d) $(\beta_2, \beta_3 = 0.05)$ as generating coordinates, respectively. The data from Ref. [9] are shown in (a).

Fig. 4. (Color online) Low-lying energy spectra for ^{224}Rn. The available data are collected from Ref. [9] and the results calculated from configuration-mixing GCM and single energy-minimum configuration are shown in (a), (b), and (c) columns, respectively. The numbers on the arrows are intraband $E2$ (blue color for positive-parity and red color for negative-parity bands) and interband $E1$ (green color) or $E3$ (violet color) transition strengths connecting to the ground-state band. All transition strengths are in Weisskopf units.

Figure 4 displays a detailed comparison of the low-lying parity-doublet states, including electric multipole transition.

strengths $B(E\lambda)$. One can see that the results calculated from configuration-mixing GCM [Fig. 4(b)] and the single energy-minimum configuration [Fig. 4(c)] show similar parity-doublet bands with a rotational character. In contrast to the results of the GCM calculation, the positive-parity band is more compressed in the single-configuration calculation, whereas the negative-parity band becomes somewhat lower than that obtained from the GCM calculation. Quantitatively, the excitation energy $E(1^-)$ of the negative-parity band head is 0.47 MeV and 0.39 MeV from the GCM and single-configuration calculations, respectively. For the 3^- state, the calculated excitation energy $E(3^-)$ from the GCM is 0.63 MeV, which is in good agreement with the data of $E(3^-) = 0.65$ MeV. The electric octupole transition strength in ^{224}Rn is $B(E3; 3^- \to 0^+) = 43$

W.u., comparable to the data for ^{224}Ra, $B(E3; 3^- \to 0^+) = 42(3)$ W.u. This provides evidence for an increased strength of octupole correlations in the area surrounding $A = 224$ mass nuclei, even though the negative-parity 3^- state of ^{224}Rn has a much higher excitation energy. These results have been found for ^{224}Ra; however, the transition strengths are reproduced perfectly [40]. For future experiments, it is important to further verify the calculated intraband $E2$ transitions in the same parity band, as well as the interband $E1$ and $E3$ transitions connecting to the ground-state band in ^{224}Rn. This investigation indicates that the calculated energies for the excited states are somewhat higher than the expected data. The discrepancy is likely due to the omission of triaxial and time-reversal symmetry-breaking components in this model calculations, as discussed in previous studies for other nuclei with different GCM approaches [54, 55]. However, taking into account these symmetry breaks would require the consideration of GCM calculations with cranking configurations or particle-hole excitation configurations, as well as the inclusion of three-dimensional angular-momentum projection (3DAMP), which is beyond the framework of our current model.

Figure 5 displays nuclear collective wave functions $\left|g_\alpha^{J\pi}\right|^2$ for the low-lying parity-doublet states of each angular momentum and parity in ^{224}Rn, where the orthonormal collective wave function $g_\alpha^{J\pi}$ is constructed as

$$g_\alpha^{J\pi}(\mathbf{q}_a) = \sum_{\mathbf{q}b} \left[N^{J\pi}\right]^{1/2}_{\mathbf{q}a,\mathbf{q}_b} f_\alpha^{J\pi}(\mathbf{q}_b). \tag{8}$$

Fig. 5. (Color online) The collective wave functions of the parity-doublet states (a) with $J^\pi = 0^+, 2^+, \cdots, 8^+$ and (b) with $J^\pi = 1^-, 3^-, \cdots, 9^-$ in the β_2-β_3 deformation plane. See text for details.

The distribution of their collective wave functions in the β_2-β_3 plane is usually adopted to analyze the staggering behaviors in the low-lying parity-doublet states. One can see that the wave functions of the states become more and more concentrated to a quadrupole-octupole deformed configuration with the increase of angular momentum, demonstrating the picture of rotation-induced shape stabilization. Similar to ^{224}Ra [40], from the view of nuclear collective wave functions, the radioactive nucleus ^{224}Rn exhibits a transition picture from gentle octupole deformation to stable pear-shaped. We have checked the nuclear wave functions of ^{224}Rn and found that the calculations involving configurations with different β_3 and a fixed β_2 yield results similar to those of ^{224}Ra (refer to Figs. 4(e) and 4(f) in Ref. [40]). As the spin increases, the dominant configuration for positive-parity states gradually shifts from weak octupole configurations to those with large octupole shapes. Conversely, for negative-parity states, their collective wave functions are zero at $\beta_3 = 0$, becoming concentrated around large octupole deformed configurations. This corresponds with the trend in the evolution of collective wave functions with spin from the full GCM calculation, as depicted in Fig. 5.

Figure 6 displays the energy ratio $R_{J/2}$ of excitation energy of each state with angular momentum J to that of positive-parity 2^+ state for ^{224}Rn. For comparison, experimental data of ^{224}Ra are also provided. The ratio is defined as $R_{J/2} = E_x(J^\pi)/E_x(2^+)$, where $\pi = +$ and $-$ indicate positive- and negative-parity, respectively. The phenomenon of interleaving positive- and negative-parity bands is a common method used to study the nuclei with octupole correla-

Fig. 6. (Color online) The ratio $R_{J/2}$ of the excitation energy of each $J^\pm$ state to that of the positive-parity 2^+ state as a function of the angular momentum $J(\hbar)$ for ^{224}Rn. The inset panel is the normalized staggering amplitude $S_{J/2}$ as a function of the angular momentum $J(\hbar)$. The results are calculated from configuration-mixing GCM and single energy-minimum configuration. Experimental data of ^{224}Ra [7] are also given for comparison.

tion. In idealized interleaving parity doublet bands, the ratio $R_{J/2}$ shows a quadratic-like function as angular momentum J increases. Firstly, the energy levels of positive- and negative-parity bands remain decoupled resulting in an odd-even staggering and the staggering amplitude globally decreases with increasing J in ^{224}Rn. This feature is similar to that obtained from the GCM calculations for ^{224}Ra [40] and Ba isotopes [41]. The tendency of staggering amplitude of $R_{J/2}$ as a function of angular momentum is reproduced qualitatively in ^{224}Rn. Nevertheless, the staggering amplitude deviates from experimental data. The single energy-minimum configuration overestimates the staggering amplitude of ratio $R_{J/2}$ resulting from the lower $E(2^+)$ value of positive-parity 2^+ state obtained [cf. Fig. 4(c)]. However, after considering configuration-mixing effects, as shown in Fig. 4(b), an overestimation of the excitation energy 2_1^+ state results in underestimating $R_{J/2}$ staggering amplitude in the GCM calculation. In addition, the inset panel of Fig. 6 displays the normalized staggering $S_{J/2}$ between the positive- and negative-parity bands. It is defined as [56]

$$S_{J/2} = \left| E_x(J^\pm) - \frac{(J+1)E_x(J-1)^\mp - JE_x(J+1)^\mp}{2J+1} \right| / E_x(2^+). \tag{9}$$

Here the superscripts denote the parities of the two bands. This quantity reflects the octupole deformation stability changing with angular momentum $J(\hbar)$. It is obvious that the normalized staggering $S_{J/2}$ decreases as angular momentum $J(\hbar)$ grows up. In short, the variations of the staggering $R_{J/2}$ and $S_{J/2}$ with increasing angular momentum $J(\hbar)$ show a behavior characteristic from octupole vibration at lower $J(\hbar)$ to octupole rotation at higher $J(\hbar)$ in ^{224}Rn.

Fig. 7. (Color online) (a) Theoretical and experimental excitation energies of positive-parity states in ^{224}Ra against the values in ^{224}Rn. (b) The same as (a) but for negative-parity states. The results of calculations for ^{224}Ra are taken from Ref. [40] and the available data are taken from Refs. [7, 9].

Fig. 8. (Color online) (a) The same as Fig. 7, but for electric multipole transition strengths. (a) The intraband transitions $B(E2; L^\pm \to (L-2)^\pm)$ with

even $L = 2, 4, 6, 8$ and odd $L = 3, 5, 7$ stand for positive- and negative-parity bands, respectively. (b) The interband transition $B(E3; L^- \to (L-3)^+)$ or $B(E1; L^- \to (L-1)^+)$ with $L = 3, 5, 7$ connect to parity-doublet bands.

Figure 7 shows the correlation between the excitation energies of ^{224}Rn and ^{224}Ra. Both calculated values and experimental data slightly deviate from the diagonal line. As the spin increases, the excitation energies of positive- and negative-parity states in ^{224}Rn and ^{224}Ra grow at a similar rate. In Fig. 7(a) it shows clearly that our calculations overestimate the excitation energies of positive-parity states, however, the linear relationship of excitation energies between ^{224}Rn and ^{224}Ra is in line with that of experimental data. This phenomenon is also exhibited in negative-parity bands in Fig. 7(b). Furthermore, we plot Fig. 8 to show the relationship of electric transition strengths between ^{224}Ra and ^{224}Rn. A linear increasing relationship is also found in the intraband transitions $B(E2; L^\pm \to (L-2)^\pm)$ or the interband transition $B(E3; L^- \to (L-3)^+)$ and $B(E1; L^- \to (L-1)^+)$ in parity-doublet bands. Moreover, for the interband $B(E1)$ and $B(E3)$ transitions between positive- and negative-parity doublet bands, the tendency of linearity gradually deviates from the diagonal line as spin increases in Fig. 8(b). However, the

intraband transitions $B(E2)$ of positive- and negative-parity bands tend to the diagonal distribution with increasing spin in Fig. 8(a). It appears that the fundamental structure associated with quadrupole-octupole correlations in ^{224}Rn exhibits a similar behavior to that of ^{224}Ra.

Fig. 9. (Color online) The relationship between the transition octupole moment $Q_3(3^- \to 0^+)$ and the energy $E(3^-)$ of the negative-parity state in ^{224}Rn, denoted by an open square. The solid squares indicate the measured Q_3 values with error bars for nuclei with mass $A > 200$ reported in [6, 7]. The calculated Q_3 value for ^{224}Ra [40] (open square) have been included for comparison. The horizontal dashed line represents the Q_3 value of octupole vibrator ^{208}Pb.

The transition octupole moment $Q_3(3^- \to 0^+)$ is derived from the transition matrix elements $\langle 3^- || \hat{M}(E3) || 0^+ \rangle$ corresponding to the $3^- \to 0^+$ transitions. We plot Fig. 9 to show the relationship between transition octupole moment $Q_3(3^- \to 0^+)$ and the energy $E(3^-)$ of the negative-parity state in ^{224}Rn, denoted by an open square. The solid squares indicate the measured Q_3 values with error bars for nuclei with mass $A > 200$ in the $Z \simeq 88$ and $N \simeq 134$ region, as reported in [6, 7]. For comparison, we have also included the calculated Q_3 value for ^{224}Ra as given in Ref. [40]. The ^{208}Pb as a typical dynamic pear-shaped octupole vibrator is characterized by the largest $E(3^-)$ energy among them with a minor Q_3 value. For the nuclei such as ^{220}Rn, ^{230,232}Th, and ^{234}U with similar Q_3 values to that of ^{208}Pb are considered to be octupole vibrators. Larger Q_3 and smaller $E(3^-)$ values for ^{222}Ra, ^{224}Ra and ^{226}Ra indicate an enhancement in octupole collectivity that is consistent with an onset of octupole deformation in this mass region. Although a more stretched negative-parity band is obtained, our results show that the $E(3^-)$ energy of experimental data is reproduced very well for ^{224}Rn [cf. Fig. 4]. The predicted transition octupole moment Q_3 value of ^{224}Rn displays as larger as that of ^{224}Ra, indicating that ^{224}Rn is likely to exhibit strong octupole correlations. In other words, ^{224}Rn has a large probability of being a rotor from our theoretical calculations. Further measurements of $Q_\lambda(\lambda = 1$ or $3)$ are required to confirm the possible existence of enhanced octupole collectivity for ^{224}Rn.

IV. SUMMARY

In this work, we have presented a beyond-mean-field study of low-lying parity-doublet bands in ^{224}Rn with a multi-reference covariant density functional theory, where dynamical correlations related to symmetry restoration and quadrupole-octupole shape fluctuation have been treated with a generator coordinate method combined with parity, particle-number, and angular-momentum projections. The low-lying energy spectrum can be reasonably reproduced when the shape fluctuations in both quadrupole and octupole shapes are considered. The nuclear collective wave functions and the low-lying spectra related energy ratio $R_{J/2}$ and the normalized staggering $S_{J/2}$ suggest a transition picture from gentle octupole deformation to stable pear-shaped. The results of ^{224}Rn were compared with those of ^{224}Ra. We have found that these two nuclei have a similar value of electric octupole ($E3$) transition strength. Specifically, $B(E3; 3^- \to 0^+) = 43$ W.u. for ^{224}Rn, comparable to the data of ^{224}Ra with its value 42(3) W.u. This result indicates that ^{224}Rn may possess a similar strong octupole correlation to that in ^{224}Ra, even though the excitation energy of 3^- in ^{224}Rn is about twice that of ^{224}Ra. Of course, a more solid conclusion can only be drawn after other related octupole criteria such as electric dipole $E1$ and octupole $E3$ transition probabilities quantities are measured in the future. This study suggests that ^{225}Rn atom, similar to ^{225}Ra atom [57, 58], could serve as another candidate for the measurement of permanent atomic electric dipole moment.

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Figure 1

Figure 1: Figure 1

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Figures