Gamma Radiation Field Reconstruction Method Based on Inversion of Non-uniform Source Activity Distribution

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Abstract

Accurate reconstruction of gamma radiation fields is fundamental to the digitalization of radiation protection and a prerequisite for radiation dose assessment and visualization simulation. To effectively address the pressing challenge of large reconstruction deviations in three-dimensional gamma radiation fields at nuclear facility sites caused by high-gradient non-uniform radiation source term distributions, this paper proposes an innovative non-uniform source activity distribution model selection method based on the Bayesian Information Criterion, and utilizes the optimal model for three-dimensional gamma radiation field reconstruction. The results demonstrate that this method significantly outperforms Kriging interpolation in radiation field reconstruction accuracy under high activity gradient and high dose rate gradient conditions; in a case study of three-dimensional radiation field reconstruction for an actual nuclear facility scenario, the mean relative deviation between the reconstructed dose rates of the test set and the measured values is only 12.69%, which is significantly lower than the 85.40% obtained by Kriging interpolation. This research provides advanced technical support for dynamic data-driven digital radiation protection simulation at nuclear facility sites, enhancing its practical applicability.

Full Text

Preamble

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Radiation Field Reconstruction Method Based on Non-uniform Source Activity Distribution Inversion

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Abstract

[**Background**] The accurate reconstruction of γ radiation fields is fundamental to digitizing radiation protection and serves as a prerequisite for radiation dose assessment and visualization simulation. Traditional interpolation methods and uniform source activity inversion approaches struggle to accurately reconstruct three-dimensional γ radiation fields with high dose-rate gradients, particularly in nuclear facilities where large-volume source terms exhibit high-gradient, non-uniform activity distributions. [**Purpose**] This study aims to develop an inversion method for non-uniform source activity distribution and apply it to accurately reconstruct 3D gamma radiation fields under such challenging conditions. [**Methods**] We propose an innovative inversion method for non-uniform source activity distribution based on multi-objective source activity inversion and the Bayesian Information Criterion (BIC). In a pipeline simulation case, we compare the accuracy of radiation field reconstruction between our method and ordinary Kriging interpolation under various source activity distribution conditions. The effectiveness of the proposed method is further validated using measured data from an actual nuclear facility. [**Results**] Under four different activity distribution conditions in the pipeline simulation, the proposed method achieved an average relative deviation (ARD) of less than 5% for radiation field reconstruction across all regions, significantly outperforming ordinary Kriging interpolation, especially in high dose-rate gradient areas. In the real nuclear facility scenario, the ARD between 77 reconstructed dose-rate values (calculated from 30 measured values) and actual measurements was only 12.69%, substantially lower than the 85.40% obtained by ordinary Kriging interpolation. [**Conclusions**] This study introduces the Bayesian Information Criterion to achieve inversion of non-uniform source activity distribution, providing a valuable supplement to gamma radiation field reconstruction methods under complex source term conditions. The approach offers advanced technical support for dynamic data-driven digitalization and simulation of radiation protection in nuclear facilities, enhancing its practical applicability.

Keywords: 3D radiation field, source activity inversion, non-uniform distribution, information criterion, radiation protection

1. Radiation Field Reconstruction Method Based on Non-uniform Source Activity Distribution Inversion

The proposed method comprises four sequential steps: (1) source term region segmentation, (2) multi-objective source activity inversion, (3) model selection, and (4) forward radiation field calculation. In step 1, the unknown non-uniform source term is divided into regions based on source term survey databases, partial radiation field measurement distributions, and field experience. The minimum segmentation unit is assumed to have uniform activity distribution and is not further subdivided. Multiple source term segmentation schemes are generated by varying the size and position of these minimum units. Step 2 involves establishing multi-objective source activity inversion equations for each segmentation scheme based on dose-rate measurements and scene models, then iteratively solving for the activity values in each region using the Maximum Likelihood Expectation-Maximization algorithm to obtain multiple source activity distribution models corresponding to different segmentation schemes. Step 3 applies the Bayesian Information Criterion to calculate BIC values for each inverted source activity distribution model, selecting the model with the minimum BIC as optimal. Finally, step 4 forward-calculates dose-rate values at target positions based on this optimal source activity distribution model to complete the radiation field reconstruction. The following sections elaborate on the methods employed in steps 2 and 3.

1.1 Multi-objective Source Activity Inversion

The dose-rate at each measurement location represents the cumulative contribution from all radiation sources. Theoretically, the calculated value should equal the measured value at the same position, leading to the following equation:

$$D = HS$$

where D is the dose-rate measurement vector of length m, S is the unknown source activity vector of length n, and H is an $m \times n$ response matrix. Each element h_{ji} represents the dose-rate contribution to measurement point j from unit activity of source i. Based on point-kernel integration theory, each matrix element is calculated as:

$$h_{ji} = \frac{1}{V_i} \int_{V_i} K_{ij}(E_i, \mathbf{r}_i \to \mathbf{r}_j) F(E_i) \, dV_i$$

where V_i is the volume of source i, $\mathbf{r}_i$ is the position of source i, $\mathbf{r}_j$ is the position of dose-rate measurement point j, E_i is the gamma particle energy, $K_{ij}(E_i, \mathbf{r}_i \to \mathbf{r}_j)$ is the point-kernel function equal to the fluence rate at $\mathbf{r}_j$ from a unit-activity point source at $\mathbf{r}_i$, and $F(E_i)$ is a conversion factor for different radiation quantities. The point-kernel integration calculations in this study were performed using CIRPDose, a proprietary software developed by the China Institute for Radiation Protection. This software optimizes both accuracy and efficiency through implementation of the latest buildup factor databases and weighted discretization techniques.

The number of dose-rate measurements typically exceeds the number of radiation sources, making equation (1) an overdetermined system. We transform this into a least-squares problem, equivalent to solving the corresponding normal equations:

$$H^T H S = H^T D$$

Direct solution is impractical due to computational complexity and the non-negativity constraint. We employ the Maximum-Likelihood Expectation-Maximization (ML-EM) algorithm with non-negative constraints to iteratively solve equation (3). The iteration formula at step $(q+1)$ is:

$$S^{(q+1)} = S^{(q)} \cdot \frac{H^T}{H^T H S^{(q)}} \cdot D$$

where $\mathbf{1}$ is a column vector of length m with all elements equal to 1, and $\cdot$ and / denote element-wise multiplication and division, respectively. $S^{(0)}$ is typically initialized as a matrix of ones. Convergence is achieved when the change in S falls below a specified threshold, at which point $S^{(q+1)}$ represents the solution vector for source activities.

1.2 Model Selection Based on Information Criterion

For non-uniform source term distributions, we generate multiple candidate models through reasonable segmentation based on prior information. The optimal model must balance fidelity to the actual non-uniform distribution with minimal overfitting. This model selection problem is common in machine learning, where information criterion algorithms provide classical solutions. The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are the most prominent methods. Given that non-uniform source activity distribution inversion involves parametric model selection, we employ BIC, which is better suited for such problems. BIC selects the model that maximizes the sum of the log-likelihood of the optimal solution and the product of parameter count and evidence, expressed mathematically as:

$$\mathrm{BIC} = m \ln\left(\frac{\mathrm{RSS}}{m}\right) + n \ln(m)$$

where $\mathrm{RSS} = \|D - H\hat{S}\|^2$ represents the sum of squared residuals between measured and calculated dose-rate values, n is the number of source term segmentation regions, and m is the number of dose-rate measurements used in the inversion.

The BIC formula comprises two components: the first term, positively correlated with RSS, characterizes how well the activity solution $\hat{S}$ fits the measurement data D (better fit yields smaller values); the second term penalizes model complexity proportionally to the number of source term regions. For each segmentation scheme, after solving for source activities $\hat{S}$, we calculate its BIC using equation (5). The model with the minimum BIC is selected as optimal. This ensures that when multiple models fit the data equally well, BIC favors the simpler model with fewer source regions, thereby mitigating overfitting.

2. Comparison with Interpolation Methods for Radiation Field Reconstruction

We conducted numerical simulation experiments to compare the accuracy of our non-uniform source activity distribution inversion method against ordinary Kriging interpolation for gamma radiation field reconstruction under various source activity distributions and dose-rate gradient conditions. The computational workflow is illustrated in [Figure 1: see original paper].

2.1 Simulation Case Setup and Evaluation Methods

In domestic nuclear power plants, collective doses primarily arise from external exposure to corrosion and activation products deposited in pipelines during maintenance outages. To maintain consistency with actual nuclear facility conditions, we developed a numerical simulation based on a pipeline scenario. As shown in [Figure 2: see original paper], the pipeline is 300 cm long with a diameter of 40 cm and wall thickness of 2 cm, composed of elemental iron. We assume the source term releases one 1 MeV gamma photon per decay, distributed along the inner pipe wall with activity varying only along the axial direction and uniformly around the circumference. The axial activity distribution follows an exponential profile with total activity of 1×10^9 Bq and probability density function:

$$f(z) = ce^{-\lambda z}$$

where z is the distance from the pipe bottom in cm, c is a normalization parameter, and λ is the exponential distribution parameter. Larger λ values indicate greater non-uniformity in activity distribution. Exploiting circumferential symmetry, dose-rate measurement points were arranged only on one side plane containing the pipe central axis. A total of 900 measurement points were positioned in a 30×30 array as shown in [Figure 2: see original paper].

Monte Carlo methods simulated dose-rate values at all measurement points. These 900 points constitute the test set for evaluating reconstruction accuracy. From this dataset, 55 points were selected as the training set after adding 10% random Gaussian error to simulate measurement uncertainty. The training set distribution accommodates both reconstruction methods: 30 points positioned near the pipe and distributed along the activity variation direction, plus 25 points uniformly distributed throughout the test region, as shown in [Figure 3: see original paper].

We define two evaluation metrics. First, Relative Deviation (RD) for each measurement point quantifies the directional and magnitude deviation between reconstructed and Monte Carlo-simulated dose-rate values:

$$\mathrm{RD}_k = \frac{d_k^{(r)} - d_k^{(m)}}{d_k^{(m)}} \times 100\%$$

where $d_k^{(r)}$ and $d_k^{(m)}$ are the reconstructed and Monte Carlo-simulated dose-rate values at point k, respectively. Second, Average Relative Deviation (ARD) assesses overall reconstruction accuracy:

$$\mathrm{ARD} = \frac{\sum_{k=1}^{l} |d_k^{(r)} - d_k^{(m)}|}{\sum_{k=1}^{l} d_k^{(m)}} \times 100\%$$

where l is the number of reconstructed dose-rate points (900 in this case). ARD represents a weighted average of absolute RD values, with higher dose-rate points carrying greater weight.

2.2 Analysis of Results

Setting λ values from 1 to 4 in equation (6) creates four scenarios with varying degrees of activity distribution non-uniformity. Monte Carlo simulations generated training and test datasets for each case. Both our inversion method and ordinary Kriging interpolation reconstructed test set dose-rates from the training data, with RD values computed for comparison. Ordinary Kriging was implemented using the PyKrige Python library with the 10 nearest training points for interpolation.

Non-uniform Source Activity Distribution Inversion

For $\lambda = 1$, we demonstrate the inversion process and overfitting effects. The pipe inner wall source was uniformly divided into 1 to 20 axial segments, creating 20 segmentation schemes. Point-kernel integration calculated response matrices for each scheme, equation (4) iteratively solved for source activity distributions, and equation (5) computed BIC values. As shown in Figure 4: see original paper, the 7-segment division yielded the optimal model. Figure 4: see original paper compares the actual, optimal inverted, and 20-segment activity distributions. The optimal model matches the actual distribution well, showing decreasing activity along the z-axis, though total activity is lower at 0.751×10^9 Bq. This underestimation occurs because point-kernel integration conservatively accounts for photon scattering through buildup factors (yielding higher calculated dose-rates), which propagates to lower inverted activity values. However, using the same point-kernel method for forward radiation field calculation largely cancels this conservative bias. The 20-segment distribution shows similar overall trends but exhibits large fluctuations at certain positions—manifesting overfitting to the error-containing training data. The BIC-based model selection effectively prevents this overfitting.

Table 1 lists the optimal segmentation counts and total activities for all four distribution scenarios. The optimal number of segments increases with λ, as more non-uniform distributions require finer models. Total activities remain relatively stable across cases, all approximately 0.75 times the actual activity.

Radiation Field Reconstruction Comparison
Using the optimal inverted activity distributions, we forward-calculated test set dose-rates via point-kernel integration and computed RD values. Ordinary Kriging interpolation was similarly applied. [Figure 5: see original paper] shows RD distributions across the test set for both methods. As λ increases, our method gradually overestimates low-activity regions near the pipe end, increasing deviation in those areas, but maintains low overall error (maximum RD of 25.68% for $\lambda = 4$). Kriging interpolation shows less variation with λ, with large RD values concentrated in high dose-rate gradient regions within 50 cm of the pipe; deviations are smaller beyond 50 cm. Visual comparison clearly favors our inversion method.

Table 2 quantifies ARD values for the entire test set (ARD_0), within 50 cm of the pipe (ARD_1), and beyond 50 cm (ARD_2). Consistent with [Figure 5: see original paper], our method maintains stable ARD values below 5% across all regions and λ values. Kriging interpolation shows substantially higher ARD_1 in high dose-rate gradient regions, which elevates overall ARD_0, with all three ARD metrics increasing with λ. Our method outperforms Kriging in all regions, though the advantage is less pronounced beyond 50 cm.

These results demonstrate that our method accurately reconstructs radiation fields across all four activity distribution scenarios, outperforming ordinary Kriging interpolation particularly in high activity-gradient and high dose-rate gradient regions, while showing more modest advantages in low gradient areas.

3. Validation with Field Measurement Data from a Nuclear Facility

We further validated our method using measured gamma dose-rate data from within a nuclear facility. The measurement scenario and point distribution are shown in [Figure 6: see original paper], featuring a pipeline geometry with 107 measurement points. The pipeline material was simplified as elemental iron, with source terms assumed uniformly distributed around each pipe circumference and composed of Co-60.

Thirty points were selected from the 107 measurements as the training set, with the remaining 77 as the test set. All pipeline sources were segmented axially into schemes of whole pipe, 100 cm, 90 cm, 80 cm, 70 cm, 60 cm, 50 cm, 40 cm, and 30 cm segments (9 schemes total). Pipes shorter than the segment length were not divided. Point-kernel integration calculated response matrices for each scheme, enabling multi-objective source activity inversion. [Figure 7: see original paper] shows BIC values for each resulting activity distribution model, identifying the 80 cm segmentation (Scheme 4) as optimal.

Using this optimal model, we forward-calculated test set dose-rates and computed RD values relative to measurements, with ARD calculated via equation (8). For comparison, Kriging interpolation was also performed. [Figure 8: see original paper] and [Figure 9: see original paper] show the spatial RD distributions and frequency histograms for both methods. While our method exhibits a pattern of underestimation on the left side and overestimation on the right side—possibly due to unmodeled left-side sources in the room geometry—it still significantly outperforms Kriging interpolation. Most RD values fall within $±$25%, with test set ARD of only 12.69% compared to Kriging's 85.40%. Overall, our method shows good agreement with measurements, meeting radiation protection requirements.

Based on the optimal source activity distribution model, the CIRPDose point-kernel integration software conveniently calculates the full 3D gamma radiation field, shown in [Figure 10: see original paper]. The distribution indicates source terms concentrated at bends in narrower pipes, consistent with expected deposition patterns.

Conclusion

High-gradient, non-uniform activity distributions in large-volume source terms pose significant challenges for accurate 3D radiation field reconstruction near sources in nuclear facilities. This paper proposes a non-uniform source activity distribution inversion method based on the Bayesian Information Criterion for accurate gamma radiation field reconstruction. Numerical simulations demonstrate superior performance over ordinary Kriging interpolation in high activity-gradient and high dose-rate gradient scenarios. Field validation further confirms effectiveness, achieving 12.69% ARD compared to 85.40% for Kriging interpolation. This research enhances the practical application of digital radiation protection systems, providing robust data foundations for dose assessment and visualization simulation while improving dynamic data-driven capabilities for field operations in nuclear facilities.

Author Contributions

Liye Liu: Overall research design guidance and manuscript framework development. Qing Fan: Model development, computational analysis, data processing, figure generation, and results section writing. Hua Li: Technical roadmap formulation and manuscript review. Hui Li: Software usage guidance and computational resources. Haijing Jin: Literature search and compilation. Yuan Zhao: Experimental data collection and organization.

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Note: Figure translations are in progress. See original paper for figures.