Investigation of System Grid-Forming Requirements and Device Grid-Forming Capabilities under Small-Signal Disturbances

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Abstract

As synchronous generators are replaced by renewable energy sources, the voltage magnitude and frequency support strength of power systems continuously decline, increasing the risk of system instability. Deploying grid-forming devices with voltage magnitude/frequency support capabilities can reduce the risk of instability. However, the current definition of grid-forming is based on the analysis of existing control structures, such as droop control or virtual synchronous machine control, lacking quantitative criteria for analyzing the potential grid-forming capabilities of other control structures. To this end, this paper starts from the closed-loop dynamics of the system, employs the concept of system strength to characterize the small-signal stability and robustness of power systems, and describes the system-level grid-forming requirement as the system strength meeting specified criteria. Based on this, devices that can enhance system strength are defined as grid-forming devices, and methods are proposed to quantify the magnitude grid-forming capability and frequency grid-forming capability of devices separately. The proposed method can also achieve frequency-band-specific quantification for quantitatively analyzing the grid-forming capability of devices at different frequency points.

Full Text

Preamble

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Article Number, Classification Codes, and Subject Index: Full-width punctuation, small five-point font, Chinese in bold, numbers and English in Times New Roman.

Discussion on System Grid-Forming Requirement and Device Grid-Forming Capability From Small-Signal Dynamic Perspective

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ABSTRACT: As synchronous generators are replaced by renewable energy sources, the support strength of voltage and frequency in the power system continues to decline, increasing the risk of system instability. Grid-Forming (GFM) devices, characterized by voltage source behavior, provide support for the system voltage and frequency. However, current understanding of GFM characteristics is mostly from the device perspective, lacking research from the system perspective on what GFM is, why it is needed, and how to evaluate GFM capabilities. Moreover, the current definition of GFM is based on qualitative analysis of existing control structures, such as droop control or virtual synchronous machine control, lacking quantitative criteria to analyze potential GFM capabilities of other control structures. To address this, this paper starts from the closed-loop interaction between devices and the grid, using the concept of system strength to represent the stability and robustness of the power system. It explains the relationship between GFM capabilities and system strength, exploring the GFM requirements and evaluation methods from the perspective of system dynamic. On this basis, devices that can enhance system voltage strength/frequency strength are defined as voltage GFM/frequency GFM devices. Corresponding mathematical criteria and methods for quantitatively analyzing voltage/frequency GFM capabilities are proposed. The proposed definition and quantification methods of GFM can also achieve frequency-band-specific quantification, enabling quantitative analysis of the GFM capabilities of devices at different frequency points.

KEY WORDS: System strength; Amplitude Grid-Forming; Frequency Grid-Forming; Robustness.

Abstract

As synchronous generators are replaced by renewable energy sources, the voltage amplitude and frequency support strength (referred to as system strength) of power systems continues to decline, increasing the risk of instability. Deploying grid-forming devices with voltage amplitude/frequency support capabilities can reduce this risk. However, current definitions of grid-forming are based on qualitative analysis of existing control structures, such as droop control or virtual synchronous machine control, lacking quantitative criteria to analyze the potential grid-forming capabilities of other device control structures. To address this, this paper starts from the closed-loop dynamics of the system, characterizing system strength through the small-signal stability and robustness of power systems, and describes the capability requirements of grid-forming from a system strength perspective. On this basis, devices that can enhance system strength are defined as grid-forming devices, and methods for quantifying device amplitude grid-forming capability and frequency grid-forming capability are proposed. The proposed methods can also achieve frequency-band-specific quantification

for analyzing device grid-forming capabilities at different frequency points.

Keywords: System strength; Amplitude grid-forming; Frequency grid-forming; Robustness

0 Introduction

Establishing stable voltage amplitude and frequency is a core requirement of power systems. With the strategic goals of "carbon peak and carbon neutrality," the proportion of renewable energy in China's power system is gradually increasing. It is projected that by 2060, renewable energy installed capacity will exceed 90%, and generation will account for over 65% of the total [1]. How to build a high-proportion renewable energy power system has become an urgent problem to solve [2].

After large-scale renewable energy integration through Voltage Source Converters (VSCs), some VSCs must possess grid-building capabilities to provide voltage amplitude/frequency support, such as those using Virtual Synchronous Generator (VSG) control or droop control. Otherwise, the system faces significant voltage amplitude/frequency deviation or even instability risks after disturbances [4]. The industry has thus defined these controls as grid-forming control [5]. In contrast, current vector control based on Phase-Locked Loop (PLL) synchronization, which typically does not actively participate in system voltage and frequency regulation, is defined as grid-following control [3]. To standardize VSC grid-forming characteristics, many publications and reports have further established functional requirements [6]-[9]: for example, the ability to operate in islanded mode, actively establish internal voltage, provide inertia support, exhibit voltage source characteristics, achieve self-synchronization with the grid without requiring a PLL, and not be limited to specific implementation methods.

Although the industry gradually recognizes the need to standardize grid-forming definitions from functional requirements, quantitative description of device grid-forming capabilities from a system perspective remains in its preliminary exploration stage. It is well known that the necessary condition for building a power system is ensuring good dynamic performance of system voltage and frequency. Therefore, analyzing system-level grid-forming requirements and quantifying device grid-forming capabilities must focus on the fundamental dynamic performance requirements of power systems. As power systems are a special class of dynamic systems, we can draw on several fundamental aspects for analysis: existence of equilibrium points, small-signal dynamic performance in the neighborhood of equilibrium points, and large-signal dynamic performance far from equilibrium points. Among these, small-signal-based grid-forming analysis is a fundamental issue (referring to grid-forming analysis scenarios where linearized models are applicable), but it remains an unresolved problem. In fact, controversies have emerged in the industry regarding whether certain devices possess grid-forming functions. For example, both VSG and PLL-VSC with PV outer loop control can regulate the point of common coupling bus as a PV node, suggesting they both have grid-forming capabilities; however, according to current control structure-based definitions, only VSG is considered a grid-forming device.

Given the current lack of deep understanding of grid-forming connotations, we believe it is necessary to propose quantitative evaluation methods for device grid-forming capabilities from a system perspective. Otherwise, planning new power systems may lead to overly aggressive or conservative allocation of grid-forming resources. However, due to the high-dimensional complexity of power systems, accurately measuring a device's contribution to system building is extremely difficult, presenting two main problems:

Problem 1: How to cognitively understand grid-building requirements in high-proportion power electronics-based systems from a system perspective, and characterize and quantify them?

Problem 2: How to quantitatively evaluate grid-forming capabilities from a device perspective to provide a foundation for control design?

The main contributions of this paper are:

(1) Describing system-level grid-forming requirements as meeting voltage amplitude strength and frequency strength needs—i.e., maintaining voltage amplitude/frequency stability within safe bounds after disturbances—and proposing a quantification method based on characteristic subsystems.

(2) At the device level, defining devices that can enhance system amplitude strength/frequency strength as amplitude grid-forming/frequency grid-forming devices, and proposing a sensitivity-based method for evaluating device grid-forming capabilities.

1.1 Overview of Power System Strength

To facilitate power system planning and operation, organizations such as CIGRE have introduced the concept of strength in public reports [13], using short-circuit ratio as a core quantification metric. Furthermore, literature [23] has supplemented the connotation of strength from both stability and robustness perspectives. However, an intuitive definition of strength is still lacking. To clarify power system building requirements, we borrow ideas from structural dynamics [11] to provide definitions and physical interpretations of strength-related terms as follows:

Voltage/Frequency Instability or Limit Violation: System voltage/frequency becomes unstable or deviates beyond safe ranges.

System Voltage/Frequency Support Strength (System Strength): The ability of key buses (generally multiple buses) in the system to resist deviation and instability of voltage amplitude/frequency under disturbances, comprising voltage amplitude strength and frequency strength. Low system strength means voltage amplitude/frequency are prone to instability or limit violations under the same disturbance. Hereinafter, system strength refers to both voltage amplitude strength and frequency strength unless otherwise specified.

Corresponding to system strength, **Bus Voltage/Frequency Support Strength (Bus Strength)** describes the ability of a single bus to resist deviation and instability of voltage amplitude/frequency under disturbances, which is not deeply explored in this paper.

Grid Voltage/Frequency Support Strength (Grid Strength): The ability of the grid subsystem to resist deviation and instability under disturbances after removing device dynamics. Here, the power system is divided into connected device subsystems and grid subsystems. After removing device dynamics (i.e., removing the device subsystem), the remaining part is the grid subsystem. For example, when grid-following converters are selected as the device subsystem, virtual synchronous machines and synchronous machines belong to the grid subsystem, as shown in [Figure 1: see original paper]. In some cases, grid strength can approximately characterize system strength.

Note 1: Strength encompasses both stability and robustness, which can be quantified by stability margin and wideband gain from disturbance to voltage, respectively. Stability refers to whether the system is stable under fixed parameters [12]; robustness includes robust stability and robust performance [12], i.e., whether the system can remain stable under parameter uncertainties and whether the gain from disturbance to voltage amplitude/frequency output (i.e., amplitude/frequency deviation under disturbance, also called disturbance rejection) meets requirements.

Note 2: System strength focuses on the dynamic response performance of voltage amplitude/frequency at system-critical buses, involving the stability and robustness of system-wide critical modes, whereas bus strength only focuses on the voltage/frequency response of a single bus. For example, if voltage instability at a certain bus belongs only to device-level stability rather than system-level stability (the distinction between device-level and system-level stability is discussed in literature [25]), then the voltage support strength of that bus does not fall within the scope of system strength. The characteristics of system strength are detailed in Section 3.1. Hereinafter, strength refers to system strength unless otherwise specified.

Note 3: For linear systems, stability performance and robust performance analysis models can be transformed into each other and unified in methodology [12]. Under the premise of a stable nominal system (without considering uncertainties), both stability performance and robust performance can be measured by the singular values (wideband gain) of the system transfer function matrix [12]. When focusing on unstable modes, strength is determined solely by stability and can be measured by eigenvalues (stability margin). Therefore, system strength is both related to and distinct from stability: strength focuses on characteristics of system-level critical modes, with results related to both zeros and poles of transfer functions, whereas stability generally focuses only on poles. For narrative convenience, system strength is primarily described from the perspective of robust performance and singular values hereinafter; when the mode of interest is unstable, one simply switches from the singular value perspective to the eigenvalue perspective.

1.2 Power System Grid-Forming Requirements

Grid-forming devices with AC voltage source characteristics help maintain voltage amplitude/frequency within safe ranges after disturbances, which aligns with the system strength requirements described above. Therefore, the basic function of grid-forming is to macroscopically meet system strength requirements. For other improvement functions, such as harmonic suppression, some literature also considers them as grid-forming functions, but we categorize them as extended functions of grid-forming, which are not included in the discussion of this paper and are addressed in Section 6.1. For convenience of discussion, grid-forming hereinafter refers to the basic function.

Amplitude and frequency are two important dimensions of AC system voltage vectors. Amplitude strength and frequency strength describe the system's ability to resist deviation and instability of voltage amplitude/frequency after disturbances, and building voltage amplitude and frequency are two functions of grid-forming [16][28]. Therefore, simultaneously ensuring amplitude strength and frequency strength to meet system strength requirements is the basic need for grid building. Example response characteristics under different amplitude and frequency strengths are shown in [Figure 2: see original paper].

1.3 Definition of Grid-Forming Devices

The core requirement of system grid-forming is to meet system strength requirements and ensure strong disturbance rejection of system bus voltage amplitude/frequency. Based on this, devices with the following characteristics are called grid-forming devices.

Grid-Forming Device: A device capable of enhancing system strength, i.e., system strength increases as the device's capacity increases. When a device only has the ability to enhance amplitude strength or frequency strength, it is called an amplitude grid-forming device or frequency grid-forming device, respectively.

Frequency-Band-Specific Grid-Forming: The multiple critical modes of concern for system strength exhibit multi-frequency-band characteristics, so device grid-forming capabilities can be characterized and quantified according to critical modes (here, frequency band refers to the range of complex frequency $s = j\omega$).

Note 4: Electrical quantities include amplitude, phase, and frequency, corresponding to voltage stability, synchronization stability, and frequency stability [25]. Since phase is the integral of frequency, their deviation quantifications can be converted to each other when modeling in the power-frequency rotating coordinate system. Existing literature has also proven that differential-mode frequency components and common-mode frequency components can characterize synchronization stability and frequency stability, respectively [15]. Therefore, this paper adopts the concepts of frequency strength and frequency grid-forming to uniformly describe both, and uses phase deviation of multiple device buses (including VSG, PLL-VSC, and other multi-type devices) for unified characterization, as modeling phase output based on the Jacobian matrix is convenient.

1.4 Approach to Quantifying Grid-Forming Capability

For ease of understanding, the characterization and quantification process of grid-forming capability is briefly summarized below, with detailed explanations provided in Sections 2-4.

Step 1: Using characteristic subsystem theory [19], the problem of quantifying system grid-forming requirements is transformed into a strength quantification problem for low-dimensional systems. Strength quantification must first evaluate voltage/frequency deviations based on norms. The voltage deviation of a single node is characterized by absolute value; the deviation degree of the vector composed of voltages/frequencies of all buses is characterized by the vector norm. For ease of analysis, a low-dimensional characteristic subsystem is obtained based on modal decoupling methods [19][21], whose stability and robustness are approximately equivalent to those of the original system. Thus, the stability and robustness of critical modes in the characteristic subsystem serve as strength indicators, enabling simplified analysis.

Step 2: System dynamics exhibit multi-timescale characteristics [20], and strength can be extended to multiple frequency bands where critical modes reside. Both eigenvalue and singular value functions of transfer function matrices can quantify system grid-forming requirements in different frequency bands. Generally, eigenvalue functions of transfer function matrices describe stability; singular value functions characterize matrix gain and describe robustness. Since system stability is a prerequisite for robustness study, if an unstable oscillation mode exists in a certain frequency band, strength in that band is characterized solely by its stability margin, and strength is considered negative. When the system is stable, singular value functions can uniformly characterize both stability and robustness, with strength being positive. For simplified presentation, this paper uses singular value analysis for grid-forming capability only under the premise of system stability, but the approach is equally applicable to eigenvalues [23].

Step 3: From a system perspective, quantify device amplitude grid-forming/frequency grid-forming capabilities separately. Devices that increase amplitude strength/frequency strength are defined as amplitude grid-forming/frequency grid-forming devices, with capability quantification indicators being the sensitivity of strength indicators to device capacity.

2 Mathematical Characterization Method for System Strength

This section first derives the closed-loop transfer function from disturbance to voltage output and quantifies system strength based on the norm of the transfer function.

2.1 Frequency-Domain Modeling of System Dynamics

Consider the renewable energy power system shown in [Figure 1: see original paper], assuming an inductive network connection (extendable to resistive-inductive networks, see [23]), with n devices (when focusing on load buses, loads are also modeled as devices) and m passive nodes. The Jacobian transfer function matrix of the i-th device is (positive power direction is out of the device):

$$\begin{bmatrix} \Delta P_i \\ \Delta Q_i \end{bmatrix} = -S_i \cdot J_{S,i}(s) \cdot \begin{bmatrix} \Delta U_i \\ \Delta \theta_i \end{bmatrix}$$

where ΔU_i and $\Delta\theta_i$ represent small-signal increments of the output voltage and phase of the i-th device; S_i represents the capacity of the i-th device; $J_{S,i}(s)$ represents the Jacobian transfer function matrix of the i-th device.

Further, if considering other loads, line capacitive effects, and modeling errors in networks and devices, they can be modeled as uncertainties $\Delta(s)$, as shown in [Figure 3: see original paper]. The device nodes are represented as:

$$\Delta \mathbf{U} = \mathbf{C}(s) \cdot \Delta \mathbf{S}$$

where $\mathbf{S}$ is a diagonal matrix composed of device capacities; $\mathbf{I}$ is a 2$×$2 identity matrix; $\mathbf{J}_S(s)$ is a diagonal block matrix composed of Jacobian transfer function matrices.

The voltage response characteristics of the power system are characterized by the closed-loop transfer function matrix after coupling device dynamics and grid dynamics. The network Jacobian transfer function matrix is further written as [14]:

$$\mathbf{J}_{net}(s) = \mathbf{M}(s) + \mathbf{N}(s)$$

where $\mathbf{M}(s)$ and $\mathbf{N}(s)$ are network Jacobian matrices, with specific forms available in literature [14].

To further characterize the response of the node voltage vector $\mathbf{U}$ under disturbances, consider unit power disturbances occurring in arbitrary directions at all device grid-connection buses (or focus on some critical buses) $\Delta \mathbf{P}_L, \Delta \mathbf{Q}_L$. Under disturbances, the closed-loop response of the system can be expressed as:

$$\Delta\mathbf{U} = \mathbf{G}(s) \cdot \Delta\mathbf{S}$$

where $\mathbf{G}(s) = (\mathbf{I} + \mathbf{J}_S(s) \cdot \mathbf{J}_{net}(s))^{-1}$ is the closed-loop transfer function matrix.

2.2 Norm-Based System Strength Characterization Method

Since the above equation reflects the deviation degree of system voltage amplitude/frequency under unit disturbance, and strength is the system's ability to resist such deviation, the strength characterization method can be derived as follows.

Criterion 1 (Strength Characterization): When the system is stable, at complex frequency $s = j\omega$, system strength, amplitude strength, and frequency strength can be characterized by the norm of the transfer function matrix from disturbance to voltage vector, voltage, or frequency (the smaller the value, the higher the corresponding strength).

Note 5: System strength characterizes the "gap" between the column vector composed of voltages of numerous buses and the ideal voltage source, and the norm of the aforementioned transfer function matrix reflects the magnitude of this "gap." Frequency-band-specific explanation shows that this "gap" magnitude varies across different timescales.

Note 6: Criterion 1 is a universal characterization method for system strength. The numerous buses considered can include load buses: simply model the load as a device as shown in Equation (1). However, due to the high-dimensional complexity of power systems, different technical means must be employed to approximate the quantification of Equation (5) under different scenario requirements. The main purpose of this paper is to explore and characterize the connotation of grid-forming from a system requirements perspective. Therefore, limited by space, subsequent sections only propose quantification methods for multi-power-electronic-device feed-in systems in large-scale renewable energy base delivery scenarios, temporarily excluding loads and line capacitive effects. System strength quantification for different scenarios will be detailed in future research and publications.

2.3 Robustness Connotation of Strength Under D-Norm Characterization

This paper uses the D-norm to quantify strength. The rationale for norm selection is provided in Appendix A.

Definition 4 (Matrix D-Norm) [18]: For any matrix $\mathbf{A} \in \mathbb{C}^{n\times n}$, there exists a positive definite matrix $\mathbf{D}$, and its D-norm is defined as:

$$\|\mathbf{A}\|_D = \|\mathbf{D}^{1/2}\mathbf{A}\mathbf{D}^{-1/2}\|_2$$

where $\|\cdot\|_2$ denotes the matrix 2-norm, i.e., the maximum singular value of the matrix. It is worth noting that in some scenarios, eigenvalue magnitude can also be used to approximate singular values, mainly depending on computational convenience. This paper only uses singular value solving as an example; the relationship between the two can be found in literature [23].

When the system is stable, the smaller the maximum deviation of system voltage amplitude/frequency under unit-sized, arbitrary-direction disturbances, the higher the system strength. Mathematically, the voltage amplitude deviation in Equation (4) can be denoted as $\|\Delta\mathbf{U}\|$, and the frequency deviation $\|\Delta\mathbf{f}\|$ (frequency differs from phase by a factor of s) can be used to characterize amplitude strength and frequency strength, respectively. When both voltage/frequency strengths are high, system strength is also high. Under unit disturbance, i.e., $\|\Delta\mathbf{S}\| = 1$, the deviation can be expressed as:

$$\|\Delta\mathbf{U}\| = \|\mathbf{G}(s)\|$$

Further simplification can be achieved by dividing the analysis into multiple frequency bands and ensuring each band meets requirements. For example, the minimum singular value σ_{min} in the frequency band where critical modes reside is:

$$\sigma_{min}(j\omega) = \min_{\omega\in\Omega} \sigma(\mathbf{G}(j\omega))$$

Combined with the meaning of norms, σ_{max} measures the maximum voltage/frequency deviation, characterizing system disturbance rejection; the larger σ_{min}, the higher the system strength. On the other hand, from control theory [12], as shown in [Figure 4: see original paper], when the system is stable, the minimum singular value of the sensitivity function $\mathbf{S}(s) = (\mathbf{I} + \mathbf{G}(s))^{-1}$ can measure the system's robust stability margin—the smaller the value, the smaller the robust stability margin. σ_{min} can also measure the minimum singular value of the inverse matrix $\mathbf{G}^{-1}(s)$ in the frequency band of interest, characterizing the stability margin of system critical modes, with proof provided in Appendix B.

When using robustness indicators to characterize strength, according to the small-gain theorem [12], the deterministic model and uncertainty satisfy:

$$\|\Delta(s)\|_\infty < \frac{1}{\|\mathbf{S}(s)\|_\infty}$$

This ensures robust stability. Therefore, σ_{min} can characterize system strength, requiring sufficient strength to maintain good dynamic performance after considering uncertainty effects. In practice, uncertainty modeling should be minimized. For example, if $\Delta(s)$ is a load model, its uncertainty function shape is generally specified. Limited by space, this paper uses solving σ_{min} as an example for quantification.

3 Quantifying System Grid-Forming Requirements Based on System Strength

This section uses multi-power-electronic-device feed-in systems as an example. Based on modal decoupling, the original high-order system is decomposed into low-order characteristic subsystems, whose robustness and stability of low-frequency critical modes are approximately equivalent to those of the original system. System grid-forming capability is quantified by solving singular values/eigenvalues of characteristic subsystems. This quantification approach can be extended to other scenarios but requires different matrix transformation methods, which are not expanded upon here due to space limitations and will be discussed separately in future papers.

3.1 Modal Characteristics of Strength

For large-scale power systems, the number of state variables and modes may reach hundreds of thousands, while the system is concerned with wide-area global critical modes, mainly including system-level synchronization stability, voltage stability, and frequency stability-related modes [25]. These modes have low oscillation frequencies and greater oscillation energy, involving large-scale energy balance, causing severe system voltage fluctuations and reflecting overall system response characteristics. For example, with a system inertia time constant of 60s, a 0.1Hz frequency offset results in an energy deficit (surplus) of 11.8 p.u. The aforementioned modes of concern for system strength cannot be ensured solely by local control improvements; they require system-wide coordination.

Some local high-frequency modes typically have higher oscillation frequencies and correspondingly smaller oscillation energy. For example, typical LC resonance problems physically manifest as periodic charging/discharging between inductors and capacitors, with oscillation energy within 0.01 p.u., which is small compared to the oscillation energy involved in voltage stability and synchronization/frequency stability. Such issues are usually limited to a few units and can be solved through local damping control without requiring system-wide coordination.

Voltage stability modes can be described by amplitude strength, while synchronization stability and frequency stability modes are further subdivisions within frequency strength and can be uniformly measured by frequency strength. Additionally, the modes involved in strength reside in different frequency bands, enabling frequency-band-specific strength quantification results to facilitate reinforcement in targeted bands.

3.2 Solving Characteristic Subsystems and Key Characteristic Subsystems

The singular value σ_{min} can measure system strength magnitude—the larger the value, the stronger the system strength. Since $\mathbf{D}$ is a positive definite matrix, its eigenvalues and singular values are identical and positive. Through decomposition, the singular value decomposition of $\mathbf{G}(s)$ is:

$$\mathbf{G}(s) = \mathbf{V}(s) \ (s)\mathbf{W}^H(s)$$

where $\mathbf{V}(s)$ and $\mathbf{W}(s)$ are left and right singular vector matrices.

When devices are completely homogeneous (with per-unit control structures and operating parameters identical based on their own capacity ratings), we obtain:

$$\mathbf{J}_S(s) = \mathbf{J}_{S,0}(s) \otimes \mathbf{I}_n$$

On the other hand, the Jacobian transfer function matrix of a single-machine system is $\mathbf{J}_{single}(s)$. Comparing with the above, the strength analysis problem can be simplified to the strength problem of n single-machine systems with σ_i, and the corresponding decoupled single-machine systems are called characteristic subsystems, with detailed explanations in literature [19].

When devices have weak heterogeneity differences, according to the singular value perturbation theorem [23], characteristic subsystems approximating the stability and robustness of the original system can be constructed, as illustrated in [Figure 5: see original paper]. The i-th singular value perturbation approximation of the system closed-loop singular value is:

$$\sigma_i(\mathbf{G}(s)) = \sigma_i(\mathbf{G}_{sub,i}(s)) + O(\epsilon^2)$$

where $\mathbf{G}_{sub,i}(s)$ is the closed-loop transfer function matrix of the low-dimensional characteristic subsystem; $O(\epsilon^2)$ represents second-order infinitesimal terms.

Note 7: A total of n characteristic subsystems are obtained. When analyzing strength, typically only the characteristic subsystems whose dominant modes are the global modes described in Section 3.1 need to be considered, usually the low-order characteristic subsystems corresponding to small singular values, called key characteristic subsystems. Compared with other modes, these modes have larger oscillation energy at the same amplitude, higher oscillation energy proportion under random disturbances, and wider involvement, belonging to system-level concerns.

It is worth mentioning that traditional power systems analyzing low-frequency oscillations have reduction methods such as the SMA method [24] that reduce to “2nd-order” dynamics. The SMA method analyzes system stability based on equivalent second-order systems, similar to the approach used in this paper with equivalent characteristic subsystems. The characteristic subsystem method is a further extension of the SMA method and can be used to analyze the dynamic characteristics of high-order device-interconnected systems.

3.3 Direct Quantification Method and Indicators for System Grid-Forming Requirements

As shown in Equation (12), based on the decoupling method above, the characteristic subsystem retains the physical meaning of an equivalent single machine, expressed as:

$$\mathbf{G}_{sub,i}(s) = \frac{1}{S_i}\mathbf{J}_{S,i}(s) + \mathbf{J}_{net,i}(s)$$

This encompasses both amplitude strength and frequency strength. Furthermore, according to the unitary invariance property of the D-norm, the maximum singular value of the characteristic subsystem transfer function is strictly equal under homogeneous conditions and approximately equal under heterogeneous conditions. Therefore, the norm of characteristic subsystems can be used to quantify strength.

Indicator 1 (Direct Quantification Indicator for System Grid-Forming Requirements): When the system is stable, at complex frequency $s = j\omega$, indicators α_{am} and α_f are used to measure system amplitude strength and frequency strength, respectively, which are also indicators of system amplitude grid-forming and frequency grid-forming requirements, expressed as:

$$\alpha_{am}(j\omega) = \frac{1}{\sigma_{max}(\mathbf{G}_U(j\omega))}, \quad \alpha_f(j\omega) = \frac{1}{\sigma_{max}(\mathbf{G}_\theta(j\omega))}$$

α_{am} and α_f only require operations on 2$\times$2 matrices, making computation simple. When both amplitude strength and frequency strength are high, the system strength indicator κ is also high. For systems with mixed device types, characteristic subsystems can also be constructed to utilize the above equation for quantification.

3.4 Indirect Quantification Method and Indicators for System Grid-Forming Requirements

Although the direct quantification method can evaluate system strength, it couples devices and networks during computation, which is inconvenient for practical engineering responsibility allocation between grid companies and equipment manufacturers. Therefore, indirect indicators that can separate grid and device quantification are also needed, such as the following indicator.

Indicator 2 (Indirect Quantification Indicator for System Grid-Forming Requirements): When focusing primarily on the voltage response performance of multi-PLL-VSC grid-connection buses, i.e., when the system's dominant modes are PLL-VSC voltage stability modes or small-signal synchronization stability modes, the smaller the SCR in the characteristic subsystem, the smaller the system singular value. At this time, system strength depends on the strength of the equivalent single-machine system with the minimum singular value (denoted as σ_{min}) [23]. For descriptive convenience, the short-circuit ratio of this equivalent single-machine system is defined as the "Generalized Short-Circuit Ratio" (gSCR) [23], whose physical meaning is the maximum sensitivity of the multi-port voltage vector to multi-port current vector (or voltage to power) of the AC network connected by multiple PLL-VSCs, and is an indicator describing grid strength [23]. At this point, regardless of system stability, the following can be used as an indirect quantification indicator of strength from the stability margin perspective:

$$\beta = \frac{gSCR - SCR_{crit}}{SCR_{crit}}$$

where SCR_{crit} represents the critical short-circuit ratio of traditional grid-following PLL-VSCs, whose physical meaning is the short-circuit ratio corresponding to PLL-VSC critical stability. The advantage of using the above equation to quantify system strength is that it enables source-grid separation during strength evaluation, facilitating grid planning and equipment retrofit guidance. Additionally, when the device's critical short-circuit ratio is fixed, grid strength can also approximately characterize system strength.

The indirect indicator based on generalized short-circuit ratio essentially measures system grid-forming requirements in the medium-low frequency band (the frequency band where PLL-VSC dominant modes reside, including PLL-dominated small-signal synchronization stability and static voltage stability in the zero-frequency band). Furthermore, when focusing on system frequency stability, indirect indicators such as frequency nadir and rate of change of frequency can be used to measure low-frequency band grid-forming requirements, with details available in literature [15]. Future research should further investigate indirect quantification methods for more scenarios.

4 Quantifying Device Grid-Forming Capability Based on System Strength

This section derives quantification indicators from Equation (14) in Section 3.3, evaluates device grid-forming capability through singular value sensitivity to device capacity changes, and provides engineering practical criteria.

4.1 Separated Voltage Amplitude/Frequency Grid-Forming Capability Characterization for Devices

The connotation of grid-forming devices is: device integration can increase system amplitude strength/frequency strength and enhance the system's ability to resist amplitude/frequency deviation under disturbances. From this, the characterization method for device grid-forming can be derived.

Criterion 2 (Mathematical Characterization of Device Grid-Forming): Given complex frequency $s = j\omega$, if the capacity of the i-th device increases from S_i to $S_i + \Delta S_i$, causing system amplitude strength/frequency strength to increase, then the device is an amplitude/frequency grid-forming device at s, mathematically represented as the partial derivative of strength indicators with respect to capacity, with specific calculation methods provided in Section 4.2.

If both conditions in the equation are satisfied, the device is an amplitude-frequency grid-forming device at complex frequency s. Traditional grid-forming concepts typically require devices to simultaneously possess both amplitude and frequency grid-forming capabilities, belonging to a narrow definition of grid-forming. Criterion 2's characterization is universal, but limited by space, the quantification method below also only proposes a device grid-forming capability quantification method for multi-power-electronic-device feed-in systems as an example. Therefore, based on separated amplitude/frequency grid-forming capability characterization, we believe future devices will be divided into four types: amplitude grid-forming devices, frequency grid-forming devices, amplitude-frequency grid-forming devices, and grid-following devices, as shown in Figure 6: see original paper. Similarly, it can be inferred that device types under large disturbances also follow this classification, which will be further studied in the future.

4.2 Direct Quantification Method for Device Grid-Forming Capability

When evaluating device grid-forming capability in systems with mixed device types, characteristic subsystems can also be constructed for computation, with specific construction methods available in literature [19][23]. In the k-th characteristic subsystem, the equivalent device dynamics are expressed as:

$$\mathbf{J}_{sub,k}(s) = \sum_{i=1}^{n} p_{k,i} \mathbf{J}_{S,i}(s)$$

where $\mathbf{J}_{S,i}(s)$ represents the Jacobian matrix of the i-th device, and $p_{k,i}$ is the weighting coefficient of that device in the k-th characteristic subsystem, with different calculation methods for different device types [19][23].

Representing the equivalent system with the minimum singular value as $\mathbf{G}_{min}(s)$, this characteristic subsystem is:

$$\mathbf{G}_{min}(s) = \frac{1}{\sigma_{min}}\mathbf{v}_{min}\mathbf{w}_{min}^H$$

where $\mathbf{v}_{min}$ and $\mathbf{w}_{min}$ represent disturbance injection direction and voltage output direction, respectively. As shown in [Figure 7: see original paper], the voltage output vector under power input disturbance $\Delta\mathbf{S}$ is $\Delta\mathbf{U} = \mathbf{G}_{min}(s)\Delta\mathbf{S}$. The voltage deviation generated by the i-th device is $\Delta\mathbf{U}_i = \mathbf{J}_{S,i}(s)\Delta\mathbf{S}_i$. If the projection of $\Delta\mathbf{U}_i$ on the output direction is negative, it can reduce voltage/frequency deviation, thus playing a grid-forming role, i.e., the angle difference between the two output directions $\Delta\theta_i > 90^\circ$.

Based on Equations (13)-(16), the grid-forming criterion for devices in hybrid systems is: the i-th weighted device transfer function in $\mathbf{G}_{min}(s)$ is $p_{k,i}\mathbf{J}_{S,i}(s)$. If increasing S_i increases the minimum singular value of $\mathbf{G}_{min}(s)$, then the device is an amplitude-frequency grid-forming device. According to singular value perturbation theory, taking the partial derivative of both sides of the equation with respect to S_i yields:

$$\frac{\partial\sigma_{min}}{\partial S_i} = \mathrm{Re}\left(\mathbf{v}_{min}^H\frac{\partial\mathbf{G}_{min}}{\partial S_i}\mathbf{w}_{min}\right)$$

where $\mathrm{Re}(\cdot)$ denotes the real part. This provides the sensitivity of singular values to parameters [22], from which the quantification indicators for voltage/frequency grid-forming capabilities are derived as follows.

Indicator 3 (Direct Quantification Indicator for Device Grid-Forming Capability): When the system is stable, at given complex frequency $s = j\omega$, the quantification indicators for the amplitude grid-forming capability and frequency grid-forming capability of the i-th device are:

$$\alpha_{am,i} = \mathrm{Re}\left(\mathbf{v}_U^H\frac{\partial\mathbf{J}_{S,i}}{\partial S_i}\mathbf{w}_U\right), \quad \alpha_{f,i} = \mathrm{Re}\left(\mathbf{v}_\theta^H\frac{\partial\mathbf{J}_{S,i}}{\partial S_i}\mathbf{w}_\theta\right)$$

where $\mathbf{v}_U$ and $\mathbf{w}_U$ are the first row vectors of $\mathbf{v}_{min}$ and $\mathbf{w}_{min}$, respectively; $\mathbf{v}_\theta$ and $\mathbf{w}_\theta$ are the second row vectors; $\mathbf{v}_{min}$ and $\mathbf{w}_{min}$ are the left and right singular vectors corresponding to σ_{min}. Larger calculated values in the equation indicate stronger device amplitude/frequency grid-forming capabilities.

Understanding these indicators from a geometric perspective is more intuitive. At complex frequency $s = j\omega$, let the left and right singular vectors corresponding to the minimum singular value be $\mathbf{v}_{min}(j\omega)$ and $\mathbf{w}_{min}(j\omega)$. As shown in [Figure 7: see original paper], the voltage deviation generated by the device is $\Delta\mathbf{U}_i = \mathbf{J}_{S,i}(j\omega)\Delta\mathbf{S}_i$. The projection of the voltage deviation generated by the i-th device on the output direction is $\mathbf{v}_{min}^H\Delta\mathbf{U}_i$. If this projection is negative, the voltage/frequency deviation can be reduced, thus playing a grid-forming role, i.e., the angle difference between the two output directions $\Delta\theta_i > 90^\circ$.

4.3 Engineering Quantification Method for Device Grid-Forming Capability

Defining grid-forming from strength requirements necessitates obtaining the system closed-loop transfer function matrix. However, devices are coupled with each other and with network structure factors, which is not conducive to practical application of device grid-forming capability evaluation.

Most converters at engineering sites are still PLL-based devices with outer-loop power control (PQ-VSC), while grid-forming devices remain in the research stage. For a considerable period in the future, even with large-scale installation of grid-forming devices, power systems will still be in a hybrid form of PQ-VSC and grid-forming devices. Therefore, as shown in [Figure 8: see original paper], using a grid-connected system with a single PQ-VSC and a single additional device in parallel as a reference system for device control design is reasonable. This system is a special case of characteristic subsystems: when the multi-PQ-VSC and additional device in the actual system are homogeneous, the characteristic subsystem shown in Equation (18) is the system shown in [Figure 8: see original paper].

The PQ-VSC capacity at Node 1 is constant at 1 p.u., with Jacobian matrix $\mathbf{J}_{S,1}(s)$; the additional device capacity at Node 2 is variable S, with Jacobian matrix $\mathbf{J}_{S,2}(s)$. The system closed-loop transfer function matrix is:

$$\mathbf{G}(s) = \left(\mathbf{I} + \mathbf{J}_{S,1}(s) + S \cdot \mathbf{J}_{S,2}(s) + \mathbf{J}_{net}(s)\right)^{-1}$$

Assume the minimum singular value of the above equation is σ_{min}, with corresponding left and right singular vectors $\mathbf{v}$ and $\mathbf{w}$. Then the amplitude-frequency grid-forming capability of the additional device is $\partial\sigma_{min}/\partial S$. Similarly, based on Equation (20), the device's amplitude/frequency grid-forming capabilities can also be separately quantified in this system.

Similar to the indirect quantification method for system grid-forming requirements, when analyzing PLL-VSC voltage stability or synchronization stability modes, the generalized short-circuit ratio method can also be used for indirect quantification of device grid-forming capability, which is more computationally convenient. This yields the following indicator:

Indicator 4 (Indirect Quantification Indicator for Device Grid-Forming Capability): The indirect quantification indicator for the grid-forming capability of the i-th device can be expressed as:

$$\beta_i = \frac{\partial gSCR}{\partial S_i}$$

Further, according to Equation (15), when the critical short-circuit ratio of PQ-VSC remains unchanged, the above equation can be transformed into the sensitivity of the generalized short-circuit ratio:

$$\beta_i = \frac{\partial gSCR}{\partial S_i} = \frac{\partial}{\partial S_i}\left(\frac{1}{\sigma_{min}(\mathbf{J}_{net})}\right)$$

For example, literature [27] has proven that virtual synchronous machine integration can increase gSCR, thereby increasing the system strength indicator κ. Therefore, virtual synchronous machines can be considered grid-forming devices.

Note 8: In addition to analyzing gSCR for PLL-VSC dominant modes, other common indirect indicators exist in engineering. For example, when focusing on frequency stability, indicators such as damping/inertia coefficients and primary frequency regulation coefficients essentially indirectly quantify device low-frequency band grid-forming capabilities. It is worth noting that increasing inertia is one means to ensure frequency stability, but besides this, enhancing low-frequency band frequency strength through fast primary and secondary frequency regulation can also ensure frequency stability.

5 Case Study

The case study validates the proposed direct quantification indicators. For validation of indirect quantification indicators based on generalized short-circuit ratio, refer to literature [23][27].

5.1 Validation of Device Grid-Forming Criterion Effectiveness

First, the effectiveness of strength indicators and grid-forming capability indicators is validated using the simple system shown in [Figure 8: see original paper]. Node 1 is a PQ-controlled PLL-VSC (PQ-VSC), and the additional device at Node 2 is exemplified by PQ-VSC, PV-VSC, and weak/strong damping VSG. Where $L_1 = L_2 = 0.1$ p.u., $L_3 = 0.4$ p.u. In the complex frequency range of 0.1-100 Hz, the relationship curves between strength indicators and grid-forming capability indicators for different device types are shown in [Figure 9: see original paper]. The strong/weak damping of VSG ($D = 20/100$) is relative to parameters in this paper, aiming to explore the impact of damping coefficients on VSG grid-forming capability.

According to strength indicator 1, when weak-damping VSG is integrated, the system has lower strength in the frequency band around 2 Hz; when strong-damping VSG is integrated, the system has high strength across all frequency bands; when PV-VSC and PQ-VSC are integrated, the system has lower strength in the frequency band around 8 Hz.

According to grid-forming capability indicators, typical control structures can also be functionally classified from two dimensions of support and timescale: PQ-VSC is a grid-following device in 0.1-100 Hz; PV-VSC is an amplitude grid-forming and frequency grid-following device in 0.1-10 Hz, and an amplitude-frequency grid-forming device in 10-100 Hz; strong-damping VSG is an amplitude-frequency grid-forming device in 0.1-100 Hz; weak-damping VSG is an amplitude-frequency grid-forming device in 3-100 Hz, and an amplitude grid-forming and frequency grid-following device in 0.1-3 Hz.

Taking the integration of 1.0 p.u. and 1.1 p.u. capacity weak-damping VSG and 1 p.u. and 1.1 p.u. capacity PV-VSC as examples, time-domain simulations are performed to validate criterion effectiveness. At $t = 2.0$ s, power disturbances with amplitude 0.1 p.u. and oscillation frequencies of 2 Hz and 8 Hz are injected, respectively. The voltage amplitude-frequency deviation at the device convergence point is compared, with time-domain responses shown in [Figure 10: see original paper].

When PV-VSC is integrated and the system is injected with 8 Hz disturbance, the voltage amplitude-frequency deviation is large, validating the effectiveness of strength indicator calculation results: the system has lower strength in the frequency band around 8 Hz. Conversely, when weak-damping VSG is integrated and the system is injected with 2 Hz disturbance, the frequency deviation is large, proving that the system's frequency strength is lower in the frequency band around 2 Hz.

Comparing voltage amplitude-frequency deviation under capacities of 1 p.u. and 1.1 p.u. shows that when a device's voltage/frequency grid-forming capability in a certain frequency band is greater than 0, integrating a larger capacity device reduces voltage/frequency deviation under disturbances in that band, and vice versa. Time-domain response results are consistent with theoretical analysis conclusions, validating the correctness of grid-forming capability theoretical analysis. It is worth noting that strong device grid-forming capability means strong ability to reduce amplitude-frequency deviation, i.e., large deviation difference before and after integration; however, the magnitude of amplitude-frequency deviation is not directly related to device grid-forming capability but only to strength.

5.2 Validation of Separated Amplitude/Frequency Grid-Forming Effectiveness

In the system shown in [Figure 8: see original paper], Node 3 is replaced with an amplitude grid-forming device using PV-VSC; Node 2 is a frequency grid-forming device using inertia PLL-controlled VSC (see literature [26]). A load disturbance of $\Delta P_{load} = 0.2$ p.u. is applied at $t = 4$ s. The power and voltage waveforms of the three devices are shown in [Figure 11: see original paper]. It can be observed that after the power disturbance occurs, the frequency grid-forming device actively undertakes the power disturbance, providing inertia and frequency regulation; while the amplitude grid-forming device provides voltage amplitude support to maintain constant voltage amplitude. This demonstrates that separated amplitude/frequency grid-forming through coordination between amplitude grid-forming and frequency grid-forming devices is a feasible grid-building solution.

5.3 Validation of Characteristic Subsystem Effectiveness

A two-area four-machine system is used as an example, with specific parameters available in literature [27]. Nodes 1-3 are PQ-VSCs, and Node 4 is a strong-damping VSG. [Figure 12: see original paper] shows the system strength indicators for three systems. It can be found that the quantification method based on characteristic subsystems has high accuracy, with indicators almost matching the original system, only showing slight differences below 1 Hz.

Further, considering 5% modeling error in the network (i.e., perturbing the network model $\mathbf{J}_{net}$ to $\mathbf{J}_{net} + \Delta\mathbf{J}_{net}$ with $\|\Delta\mathbf{J}_{net}\|/\|\mathbf{J}_{net}\| = 5\%$), the system strength indicators are shown as purple lines in [Figure 12: see original paper]. It can be observed that after considering model uncertainty, the strength calculation results change only slightly.

6.1 Discussion on Basic and Extended Functions of Grid-Forming

The most basic function of grid-forming is to help the system build a stable power-frequency fundamental voltage, with related functions shown in [FIGURE:6(b). This requires devices to have voltage/frequency grid-forming functions in the medium-low frequency band near the power frequency to ensure the stability and robustness of system-level critical modes. For example, devices are required to maintain fundamental voltage/frequency approximately unchanged during 0.02 s-10 s after a disturbance (i.e., possessing amplitude-frequency grid-forming functions in approximately the 0.1-50 Hz band) to minimize changes in voltage amplitude/frequency of multiple critical buses after disturbances.

High-frequency circuit resonance and harmonic distortion with limited scope do not affect the overall stability of system voltage amplitude/frequency. After meeting basic grid-forming functions, devices can also treat improving higher-frequency band voltage response performance (resonance suppression, harmonic compensation, etc.) as extended functions, as shown in [FIGURE:6(b).

6.2 Discussion on Separated Amplitude/Frequency Grid-Forming

Current requirements for grid-forming devices typically demand that a single device possesses all basic functions described in Section 6.1, i.e., achieving amplitude grid-forming and frequency grid-forming across a wide frequency band. However, from the perspectives of controllability [12] (waterbed effect constraints, where wide frequency bands involve trade-offs) and economics [7] (simultaneously requiring energy reserve and overcurrent capability), such requirements are extremely stringent.

Building voltage amplitude and frequency are two different functions of grid-forming [28]. Supporting amplitude strength and frequency strength separately, and achieving system-level grid-forming through coordinated cooperation between devices, can also ensure stable operation of all-power-electronic systems. Separated grid-forming provides more control degrees of freedom, facilitating coordination of multi-frequency-band system performance; and can fully exploit the grid-forming potential of devices such as energy storage and SVG, also offering economic advantages.

6.3 Discussion on Correlation Between Device Grid-Forming and Control Structure

Some studies impose requirements on grid-forming from the control structure perspective, such as VSG, etc. We believe this conclusion is debatable: imitating synchronous machines is one way to achieve grid-forming, but it does not mean grid-forming is equivalent to synchronous machine-like behavior. On the one hand, device grid-forming should not only consider device characteristics but also whether the device meets system requirements; on the other hand, from a small-signal perspective, virtual synchronous control and PLL vector control have similar effects. With different outer loop control types (PQ/PV) and different control bandwidths, both can achieve grid-forming or grid-following [28], depending on whether they satisfy Equation (16). For example, literature [17] demonstrates that physically, both can be unified as a composition of a virtual "rotor" and virtual "winding," with the grid-forming/grid-following distinction lying in the different response speeds of the "rotor" and dynamic characteristics of the "winding."

Additionally, according to control theory, when device power control/synchronization control bandwidth is lowered, the system generally has higher robustness and usually satisfies the requirements of Equation (20). However, under low power control bandwidth, devices cannot track quickly, requiring greater hardware investment to provide sufficient energy reserve to support power deficits caused by tracking delays. Therefore, device grid-forming implementation often involves sacrificing flexibility and economy, requiring a balance between grid-forming and flexibility/economy when considering device grid-forming. In practical engineering, device grid-forming characteristics can be preliminarily judged simply by device bandwidth levels.

7 Conclusions and Outlook

(1) The basic requirement of system-level grid-forming is to ensure that system voltage/frequency do not become unstable or exceed safe ranges during dynamic processes after disturbances, which is equivalent to meeting system strength requirements, including voltage amplitude strength and frequency strength.

(2) The basic grid-forming requirements of the system can be measured by the norm of the transfer function matrix from disturbance to voltage amplitude/frequency output; to simplify solving, the matrix D-norm can

be introduced to decouple the complex high-dimensional system into low-dimensional characteristic subsystems, facilitating frequency-band-specific quantification of voltage/frequency strength through singular values/eigenvalues.

(3) The basic characteristic of grid-forming devices can be viewed as devices that augment amplitude or frequency strength, quantifiable using the sensitivity of strength indicators to device capacity. Even if devices do not simultaneously possess amplitude strength and frequency strength support capabilities, complementary combinations of amplitude grid-forming, frequency grid-forming, amplitude-frequency grid-forming, and grid-following devices can achieve system grid-forming. Therefore, the combination of inertia and voltage source is not a necessary condition for grid-forming.

Future research should further investigate how to characterize grid-forming characteristics and capabilities under large disturbances.

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Appendix A: Basis for Norm Selection

Common vector and matrix norms include 1-norm, 2-norm, ∞-norm, and D-norm [21]. Theoretically, any norm can describe the magnitude of vectors and matrices to a certain extent. Considering that norms of high-dimensional matrices are difficult to solve analytically, in practice, we hope to achieve both interpretability and accuracy through mathematical transformation. When using the D-norm, matrix transformation and order reduction do not introduce additional errors due to scaling. Furthermore, based on the properties of system transfer function matrices, high-dimensional multi-machine systems can be reduced to a series of equivalent single-machine systems under the D-norm, i.e., the characteristic subsystems described in the paper, thereby extending single-machine analysis and control theories and methods to multi-machine systems. Therefore, this paper adopts the D-norm to quantify system strength.

Appendix B: Robust Stability Margin

The system closed-loop characteristic equation is:

$$\det\left(\mathbf{I}+\mathbf{G}(s)\right)=0$$

The system's sensitivity function is $\mathbf{S}(s) = (\mathbf{I} + \mathbf{G}(s))^{-1}$, and:

$$\|\mathbf{S}(s)\|_\infty = \sup_\omega \sigma_{max}(\mathbf{S}(j\omega))$$

The above shows that the magnitude of $\|\mathbf{S}(s)\|_\infty$ reflects the system's robust stability margin—the larger the value, the larger the margin. σ_{min} can also measure the minimum singular value of $\mathbf{G}^{-1}(s)$ in the frequency band of interest, characterizing the stability margin of system critical modes.

Note: Figure translations are in progress. See original paper for figures.

Source: ChinaXiv — Machine translation. Verify with original.