

## Anisotropic Propagation Speed of Light in a Uniform Linear Motion System Relative to Earth-Centered Inertial Frame

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### Abstract

Based on the Global Positioning System (GPS) range measurement equation whose correctness has been fully proven by GPS practices, we found that in an inertial system which is moving relative to the Earth Centered Inertial (ECI) frame, the propagation speed of light is neither constant nor isotropic, but  $c\sqrt{1 - \frac{v^2}{c^2}}$ , where  $v$  is the velocity of the system relative to the ECI frame and  $\hat{n}$  is the unit vector of the direction of light propagation. Utilizing an interferometer of two independent ultrastable lasers, a crucial experiment examining this important scientific problem with a low translational speed of the interferometer is proposed; and its comparison with an existing experiment of the generalized Sagnac effect is also presented. Besides, such an interferometer can be utilized to examine another important scientific problem: whether the speed of light is isotropic or not on rotating Earth's surface. Because the vast majority of optical laboratories on the surface of the Earth have high linear velocities of the Earth's rotation, only a small change of the orientation of the interferometer is sufficient.

### Full Text

## Anisotropic Propagation Speed of Light in a System with Uniform Linear Motion Relative to the Earth-Centered Inertial Frame

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## Abstract

Based on the Global Positioning System (GPS) range measurement equation, whose correctness has been fully proven by GPS practices, we found that in an inertial system moving relative to the Earth-Centered Inertial (ECI) frame, the propagation speed of light is neither constant nor isotropic, but rather follows the relation  $c = c_0 \sqrt{1 - \frac{v^2}{c_0^2}}$ , where  $v$  is the velocity of the system relative to the ECI frame and  $\hat{d}$  is the unit vector of the light propagation direction. Utilizing an interferometer with two independent ultrastable lasers, we propose a crucial experiment to examine this important scientific problem at low translational speeds, and we present a comparison with existing experiments on the generalized Sagnac effect. Furthermore, such an interferometer can be used to examine another important question: whether the speed of light is isotropic on the rotating Earth's surface. Since the vast majority of optical laboratories on Earth's surface have high linear velocities due to Earth's rotation, only a small change in the interferometer's orientation is sufficient to test this.

**Keywords:** Propagation speed of light; System moving relative to ECI; Theoretical derivation; Crucial experiments; Interferometer of lasers

## I. Introduction

The first principle of Special Relativity is the principle of relativity, which states that all physical laws are the same in all coordinate systems in uniform linear motion [1]. Accordingly, this also means that the propagation speed of light should be isotropic in all inertial systems. In 1980, it was pointed out that the principle of relativity had not been verified by experiments in systems moving relative to the Earth, and a new Michelson-Morley experiment in Space Lab was proposed [2]. Popper considered conducting such an experiment to examine the principle of relativity a worthwhile idea [3]. Although the experiment has been repeated many times using new technologies with increasing accuracy [4-7], all of them share the same limitation: the experimental devices are not in uniform linear motion relative to Earth-based laboratories. This means that the principle of relativity of Special Relativity and the isotropy of the speed of light in all inertial systems have not been truly examined experimentally.

In this paper, we analyze the propagation of light between two adjacent points in an inertial system moving relative to the ECI frame using the GPS range measurement equation, which is fully validated by GPS practices. We found that the propagation speed of light in that system is neither constant nor isotropic, but rather follows the relation  $c = c_0 \sqrt{1 - \frac{v^2}{c_0^2}}$ , where  $v$  is the linear velocity of the system relative to the ECI frame and  $\hat{d}$  is the unit vector of the light propagation direction. The time interval for a light beam to travel the light path  $d$  in that system is  $Dt$ , where  $dv \times +$ . To examine these findings, we should first recall the key results from previous generalized Sagnac effect experiments [8,

9]: linear motion causes a first-order propagation time difference between two counter-propagating light beams when the optical path and direction of motion are aligned, and more generally when they form an angle. However, since these experiments used only one laser and required a looped optical path, they cannot be regarded as direct tests of the principle of relativity.

This raises the question: how can we create a non-looped optical path to enable first-order experiments that examine the principle of relativity? Clearly, we need two independent coherent light sources. With the development of ultrastable lasers, modern systems achieve linewidths of 5 mHz and coherence times of 200 s [10-12], enabling interference of light beams from two independent lasers [13, 14]. This arrangement eliminates the looped optical path, making experiments with two independent lasers in uniform linear motion excellent candidates for crucially testing the theoretical predictions presented in this paper.

## II. Theoretical Derivation for Anisotropic Propagation Speed of Light in an Inertial System with Uniform Linear Motion Relative to ECI Frame

Let us examine the propagation of light in a system in uniform linear motion with respect to the ECI frame. Consider two inertial systems: the ECI frame  $\Sigma_0$  and another frame  $\Sigma_1$  moving relative to it with velocity  $v$ , as shown in Fig. 1 [Figure 1: see original paper]. Fixed in  $\Sigma_1$  are two points A and B, separated by vector  $d$ , which makes an angle  $\phi$  with  $v$ . A light beam travels from A to B, and we want to find the time interval  $Dt$  for this propagation and determine the speed of light in this moving system.

GPS is a timing-ranging system whose operations are based on the range measurement equation in an Earth-centered inertial system, ECI [15-17]: the transmission time of the signal from the source is  $t_s$ , the reception time at the receiver is  $t_r$ , the position vector of the source at transmission time is  $r_s(t_s)$ , and the position vector of the receiver at reception time is  $r_r(t_r)$ . Highly successful GPS practices have proved the validity of this range measurement equation. We now use it to analyze the propagation of the light beam.

Since the GPS range measurement equation is applied in the ECI frame, the problem should be studied in  $\Sigma_0$ . The light propagation from A to B is shown in Fig. 1: the light starts from point A when the vector from the origin  $O_0$  of  $\Sigma_0$  to point A is  $r_A(t_s)$ . After a time interval  $Dt$ , when the light reaches the receiving point, B has moved in  $\Sigma_0$  to point B', and the vector from  $O_0$  to B' is  $r_{B'}(t_r)$ . Apparently,  $Dt$  in the GPS range measurement equation is the time interval we are seeking.

In Fig. 1 we have a triangle of vectors  $d$ ,  $vDt$ , and  $r_{B'} - r_A$ , and  $r_{B'} - r_A$ . The relationship among the vectors  $d$ ,  $vDt$ , and  $r_{B'} - r_A$  gives us  $r_{B'} - r_A = d + vDt$ . Using the GPS range measurement equation (1), we have  $cDt = r_{B'} - r_A$ . Thus we have  $cDt = d + vDt$ . Because of  $c > v$ , then  $Dt > 0$ . Solving this equation for  $Dt$ , we have  $Dt = \frac{d}{c - v \cos \phi}$ .  $Dt$  is the time interval of the light beam to

travel the path  $d$ .

Now let us calculate the speed of light  $c'$  of this beam in inertial system  $\Sigma_1$ , namely  $c' = (\text{distance of propagation})/(\text{time interval}) = d/Dt$ . Thus we have , neglecting quantities of second and higher orders, where  $v$  is the constant velocity of the system relative to the ECI frame and  $\hat{d}$  is the unit vector of the light propagation direction. Alternatively, we can express it as . The difference between this result and the conventional belief that the propagation speed of light is isotropic in any inertial system, i.e.,  $c' = c$ , is a first-order quantity proportional to  $v \cdot \hat{d}/c$ . Therefore, it is clear that this difference would not disappear even when considering Lorentz contraction and relativistic time dilation, which are second-order effects.

### III. Propagation Speed of Light in a System with Uniform Linear Motion on Rotating Earth's Surface

The vast majority of us live on Earth's surface, where most events and experiments related to the propagation speed of light occur. We therefore further investigate the propagation speed of light in a system with uniform linear motion on Earth's rotating surface. We first mention our previous work [18], which gives the propagation speed on rotating Earth's surface as , where  $v$  is the local linear velocity of Earth's rotation and  $\hat{d}$  is the unit vector of the light propagation direction.

Based on that, we now examine the scenario shown in Fig. 2 [Figure 2: see original paper]: an ECI frame  $\Sigma_0$  and a system  $\Sigma_1$  in uniform linear motion  $v$  with respect to a location on Earth's rotating surface. Fixed in  $\Sigma_1$  are two points A and B separated by vector  $d$ , with a light beam traveling from A to B. We want to find the time interval  $Dt$  for this propagation.

Unlike the scenario in Fig. 1, here when a light beam is emitted from A, A and B have two simultaneous motions: one is the uniform linear motion with speed  $v$ , and the other is motion caused by Earth's rotation with speed  $v$ . That is to say, compared with the previous section, vector  $v$  there is replaced by the sum of two vectors  $v + v$ . We could solve the problem step by step as before; however, due to the similarity between these scenarios, we can also directly modify the previous results.

The vector triangle now involves  $d$ ,  $(v + v)Dt$ , and  $r_D$ . Using the GPS range measurement equation in this case, we obtain the time interval of light propagation:

Obviously, if there is no uniform motion  $v$ , this result becomes the propagation speed of light on rotating Earth's surface, ; and if there is no effect caused by Earth's rotation, it reduces to the result from the previous section, .

## IV. Crucial Interferometric Experiments with Two Independent Ultrastable Lasers

### 1. Examining the Anisotropic Propagation Speed of Light in a System with Uniform Linear Motion

As previously mentioned, an experimental apparatus with two independent lasers in uniform linear motion can be used to examine our theoretical prediction. However, since experiments are generally carried out in Earth-based laboratories, we must address and exclude the effect of Earth's rotation discussed in the previous section. For this reason, the direction of light propagation should be perpendicular to the direction of Earth's rotation—that is, the propagation direction should be north-south.

Fig. 3 [Figure 3: see original paper]-1 shows an interferometric experiment using two independent ultrastable lasers mounted on a movable optical platform with air bearings or magnetic levitation. Let us investigate the detector reading when the platform moves southward and then northward. The observable change arises from the motion of the optical path of length  $L$  from the mirror to the beam splitter (i.e., the difference between the two optical paths aligned with the motion). The two optical paths perpendicular to the direction of motion generate no change at the detector because they are very short, can be made equal in length  $L_1$ , and their light propagation is not affected by the motion.

When the optical platform is stationary, the propagation time difference of the two independent coherent beams at the detector is  $\Delta t$ . When the platform moves with uniform speed  $v$  to the south, according to the analysis in Section II, we have  $\Delta t_s$  and  $\Delta t_n$ . This means the change observed at the detector due to southward motion is  $\Delta t_s - \Delta t$ . When the whole platform moves northward with speed  $v$ , we have  $\Delta t_n$ . Therefore, the propagation time difference between these two states of motion is  $\Delta t_s - \Delta t_n = \Delta t$ .

An optically stable configuration is shown in Fig. 3-2 [Figure 3: see original paper], with two detectors. Detector  $D_1$  measures the propagation time difference in the southward light paths, while  $D_2$  measures that in the northward light paths, and then we find the difference between the two measurements with a differentiator (wired or wireless). This differencing process evidently eliminates the influence of vibrations in the light path and the influence of the propagation medium, because these effects are identical in both directions, leaving only the difference caused by the motion of the light path that we are interested in. Therefore, this configuration inevitably yields a very stable experimental result, as demonstrated in generalized Sagnac experiments for linear motion.

When the platform moves southward, compared with the stationary state, on  $D_1$  we have  $\Delta t_s$  and on  $D_2$   $\Delta t_n$ . This means  $\Delta t_s - \Delta t_n$ . Finally, we have  $\Delta t$ . For northward motion,  $\Delta t_n - \Delta t_s = -\Delta t$ , and as mentioned before, the influences of vibration and propagation medium are automatically cancelled.

If the time difference is measured as a shift of interference fringes, it is  $\Delta \phi$  and the

differentiator used is a fringe shift differentiator. Its corresponding phase shift is and the differentiator used is a phase shift differentiator. These are first-order effects; therefore, compared to second-order Michelson-Morley experiments, the required speed of motion is considerably lower. For example, if we choose  $v = 0.1$  m/s,  $L = 3$  m, and  $\lambda = 1.5$  m, according to eq. (8) we have . For an optically stable interferometer, such an interference fringe shift is not difficult to detect. By comparison, Lorentz contraction and relativistic time dilation are second-order effects.

Now let us compare the experimental configuration in Fig. 3-2 with the fiber optic parallelogram configuration used in generalized Sagnac effect experiments [9], as shown together in Fig. 4 [Figure 4: see original paper]. The similarities are that both have moving top optical paths whose motion generates phase differences across the optical paths, and both have short side paths that contribute no phase difference. The only difference is that the stationary bottom side of the fiber optic parallelogram is part of the entire loop, whereas the present configuration with two independent lasers has no optical path at the bottom, avoiding a looped structure.

In the fiber optic parallelogram experiment, there is a phase difference when the top side moves at speed  $v$ , and a phase difference when it moves to the other side at speed  $v$ ; therefore the difference between the two is  $=D$  as described above. Thus we can expect that the experiment in Fig. 3-2 should yield the result . This comparison highlights once again the importance of using two independent ultrastable lasers in interferometric experiments, as they eliminate the looped structure of the optical path, making this a crucial experiment.

## 2. Examining the Anisotropy of the Propagation Speed of Light on the Rotating Earth's Surface

Previous research [18] proposed a crucial experiment examining the anisotropy of the propagation speed of light on Earth's rotating surface using stable pulsed lasers and ultrafast imaging techniques to compare pulse spacing in different directions. We have now designed an experiment using an interferometer with two independent lasers for this purpose, which offers several advantages. First, examining light speeds in different directions requires only a change in the optical platform's orientation, which is much easier than changing its translational speed. Second, the local linear velocities of Earth's rotation at most optical laboratories exceed 300 m/s, so even a small change in the platform's orientation causes a significant change in the local propagation speed of light. Fig. 5 [Figure 5: see original paper] shows such experimental setups.

For Fig. 5-1a, there is no translational speed  $v$ , and  $= 0$ ; therefore we have . For Fig. 5-1b, we have  $rEv_c = -$  , and . Therefore, at the detector we can find:

Or, we have:  $= 300$  m/s, and  $L = 3$  m,  $\lambda = 1.5$  m and degrees, according to eq. (10) we have .

Fig. 5-2 [Figure 5: see original paper] shows an optically stable configuration. By the same principle, if we conduct the experiment in Fig. 5-2, we will . This confirms that any small change in the optical platform's orientation causes a large change in the local propagation speed of light, so in practice we need only a very small angular change to conduct this experiment.

## V. Conclusions

According to the principle of relativity, the propagation speed of light is constant and isotropic in all inertial systems. However, utilizing the GPS range measurement equation, we find that in an inertial system moving with respect to the ECI frame, the propagation velocity of light is neither constant nor isotropic, but rather follows the relation  $c' = c \sqrt{1 - \frac{v^2}{c^2}}$ , where  $v$  is the velocity of the system relative to the ECI frame and  $\hat{d}$  is the unit vector in the light propagation direction. We propose a crucial experiment using an interferometer with two independent ultrastable lasers to examine this important scientific problem. Additionally, we present a comparison between the configuration of this crucial experiment and that of completed fiber optic parallelogram experiments on the generalized Sagnac effect. The main similarity is that both have moving top optical paths whose motion generates phase differences across the optical paths. Therefore, from the results of fiber optic parallelogram experiments, we can expect that the theoretically deduced crucial experiment would show the result .

Furthermore, previous research has indicated that the propagation speed of light on Earth's rotating surface is  $c' = c \sqrt{1 - \frac{v^2}{c^2}}$ , where  $v$  is the local linear velocity of Earth's rotation and  $\hat{d}$  is the unit vector of the light propagation direction. The interferometer with two independent ultrastable lasers can also be used to study this important problem. Since the vast majority of optical laboratories on the ground have high linear velocities from Earth's rotation, this interferometer can be used especially efficiently, requiring only a small change in orientation.

## References

- [1] A. Einstein, "Zur Elektrodynamik bewegter Körper", *Annalen der Physik*, 17, 891 (1905).
- [2] R. Wang, Z. Chen, X. Dong, "Has the Relativity Principle in the Special Theory of Relativity Been Fully Verified by Experiments?", *Phys. Lett.* 75A, 176 (1980).
- [3] K. Popper, private communication, Karl Popper Archive, 1981.
- [4] H. Müller, S. Herrmann, C. Braxmaier, S. Schiller, and A. Peters, "Modern Michelson-Morley experiment using cryogenic optical resonators," *Phys. Rev. Lett.* 91, 020401 (2003).
- [5] P. Stanwix, M. Tobar, P. Wolf, M. Susli, C. Ivanov, J. Winterflood, and F. Kann, "Test of Lorentz invariance in electrodynamics using rotating cryogenic sapphire microwave oscillators," *Phys. Rev. Lett.* 95, 040404 (2005).
- [6] C. Eisele, A. Nevsky, S. Schillerv, "Laboratory test of the isotropy of light

- propagation at the 10–17 level,” Phys. Rev. Lett. 103, 090401 (2009).
- [7] Herrmann, A. Senger, K. Möhle, M. Nagel, E. Kovalchuk, and A. Peters, “Rotating optical cavity experiment testing Lorentz invariance at the 10–17 level,” Phys. Rev. D. 80, 105011 (2009).
- [8] R. Wang, Y. Zheng, A. Yao, D. Langley, “Modified Sagnac Experiment for Measuring Travel-Time Difference between Counter-Propagating Light Beams in a Uniformly Moving Fiber”, Phys. Lett. A, 312, 7 (2003).
- [9] R. Wang, Y. Zheng, A. Yao, “Generalized Sagnac Effect”, Phys. Rev. Lett. 93, 143901 (2004).
- [10] Matei D., et al., “1.5 [11] Kessler, T., et al. “A sub-40-mHz-linewidth laser based on a silicon single-crystal optical cavity.” Nature Photon., 6, 687 m Lasers with Sub-10 mHz Linewidth”, Phys. Rev. Lett., 118, 263202 (2017). (2012).
- [12] Schioppo, M., et al. “Comparing ultrastable lasers at  $7 \times 10^{-17}$  fractional frequency instability through a 2220 km optical fibre network.” Nature Commun., 13, 212 (2022).
- [13] Magyar, G., Mandel, L. “Interference Fringes Produced by Superposition of Two Independent Maser Light Beams,” Nature 198, 255 (1963).
- [14] Kawalec T. and Sowa P., “Observation of two truly independent laser interference made easy” Eur. J. Phys., 42, 055305 (2021).
- [15] ICD-GPS-200C, “Navstar GPS Space Segment/Navigation User Interfaces”, Revision 4, 12 April 2000.
- [16] Ashby, “Relativity in the Future of Engineering,” IEEE Trans. Inst. and Meas. 43(4), 505-514 (1994).
- [17] B. Parkinson, J. Spilker, Global Positioning System: Theory and Applications, Vol. I (AIAA Inc.1996).
- [18] R. Wang, L. Zhan, “Anisotropic propagation speed of light on rotating Earth’s surface - theoretical derivation, implications for definition of meter, and crucial experiment” chinaXiv:202307.00036.

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