Corrected Proof of Sub-picosecond Propagation Delay and Discontinuous Measurement Allan Variance in Inter-satellite Time Transfer

Authors: He Keliang, He Yuling, Wang Guoyong

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Abstract

Inter-satellite signals traversing asymmetric propagation paths induce unequal delays in the signal reception epochs at satellites, necessitating precise correction in the clock offset solution process for time transfer. Through analysis of clock offset observation methodologies, the asymmetric propagation delay arising from satellite motion is rigorously calculated; employing the time transfer function approach, the propagation delay attributable to relativistic light deflection is evaluated term-wise, and the supplementary stability of residual delay corrections is assessed—under current orbit determination conditions, the propagation delay correction precision attains sub-picosecond levels. To address the impact of missing inter-satellite time-frequency comparison data, the autocorrelation function representation of Allan variance is extended, and a statistical methodology is proposed for separately computing stability correction coefficients for distinct power-law noise types, thereby analyzing the influence of data gaps on stability calculations for various power-law noises. Collectively, a systematic methodology for correcting propagation delays and discontinuous-measurement Allan variance in inter-satellite time-frequency comparison is presented, applicable to clock offset correction in picosecond-level inter-satellite time transfer.

Full Text

The Corrections of Sub-picosecond Propagation Time Delays and the Allan Variance of Discontinuous Measurement for Inter-satellite Time Transfer

HE Ke-liang, HE Yu-ling, WANG Guo-yong
China Academy of Space Technology (Xi'an), Xi'an 710100

Abstract

Unequal time delays at the receiving signal epoch are caused by asymmetric signal propagation paths between two satellites, which must be accurately corrected when solving the clock error in inter-satellite time transfer. By analyzing the clock difference observation method, we derive and reduce the asymmetric propagation delay caused by satellite motion, evaluate the propagation delay due to relativistic light deflection using the time transfer function method, and assess the additional stability of the delay correction residuals. Under current orbit determination conditions, the propagation delay correction accuracy can reach the sub-picosecond level. To address the impact of missing inter-satellite time-frequency comparison data, we extend the autocorrelation function representation of Allan variance and propose a statistical method to calculate stability correction coefficients for different power-law noises separately, analyzing how missing comparison data affects stability calculations for various noise types. Overall, we present a systematic method for correcting propagation delays and Allan variance under discontinuous measurement conditions for inter-satellite time-frequency comparison, which can be applied to clock error correction in picosecond-level inter-satellite time transfer.

Key words: time: time transfer; propagation delay; reference systems: atomic time; methods: statistical

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Long-term, high-precision satellite-to-ground and inter-satellite time-frequency transfer links are crucial for improving the accuracy of timekeeping, timing, orbit determination, navigation, and fundamental physics measurements. Inter-satellite time synchronization involves transmitting and receiving signals between two satellites according to a specific timing sequence, measuring two-way pseudoranges and local or remote transmit-receive time intervals, correcting equipment and propagation delays, and thereby solving the clock difference (time offset) between the two satellites' local times under the same coordinate time. Asymmetric signal propagation paths between satellites cause unequal delays at the receiving signal epochs, generating asymmetric propagation delays that can lead to additional observation errors in pseudoranges and transmit-receive time intervals if left uncorrected. The two-way asymmetric propagation paths are primarily caused by relative satellite motion and are also affected by relativistic light deflection and asynchronous signal transmission epochs. For high-precision time transfer between widely separated satellites, asymmetric propagation delay correction error has become one of the major error sources.

With the demand for further improvement in inter-satellite time-frequency transfer accuracy, asymmetric propagation delays must be corrected with higher precision. Current satellite navigation system satellite-to-ground and inter-satellite time-frequency comparisons, intercontinental two-way satellite time and frequency transfer (TWSTFT), and satellite-to-ground two-way time comparison have demonstrated that existing satellite-to-ground and inter-satellite time-frequency transfer links achieve frequency stability at the 10^{-15}–10^{-16} day^{-1} level, with long-term stability indicators degrading. Correspondingly, current time measurement errors in satellite-to-ground and inter-satellite time-frequency transfer links are at the 0.1 ns level, with asymmetric propagation delay corrections considering only the simple case of satellite relative motion. Establishing inter-satellite and satellite-to-ground time-frequency transfer links at the 10^{-17}–10^{-18} day^{-1} level is a current research focus, requiring asymmetric propagation delay corrections to reach the 0.1 ps level or even higher precision. For example, the European Space Agency's Atomic Clock Ensemble in Space (ACES) microwave link plans to achieve satellite-to-ground time-frequency transfer with time stability of 0.4 ps/300 s and 8 ps/day. Current asymmetric propagation delay correction methods cannot support correction requirements at the 0.1 ps level and beyond.

Inter-satellite time-frequency comparison data suffers from periodic or non-periodic gaps due to Earth occlusion or equipment failure, affecting satellite clock difference measurement, on-board atomic clock performance evaluation, and clock error prediction. For periodically missing comparison data, if the averaging time τ is much smaller than the comparison period T, the Allan variance calculation is essentially unaffected. When τ is of the same order as T but smaller than T, Barnes et al. and Wang et al. introduced correction coefficients $B_2(T;\ \tau)$ for correction. However, for cases where τ is larger than T or for non-periodic missing data with high missing data rates, it becomes difficult to accurately calculate Allan variance. The inter-satellite time-frequency comparison data period T is on the order of 10,000 s, and missing comparison data affects stability calculations at averaging times of 1 day, with correction methods for this scenario lacking relevant research.

To address the correction of asymmetric propagation delays and stability under discontinuous measurement conditions for inter-satellite time transfer, we propose a systematic correction method. First, we analyze the clock difference observation method. Second, we reduce the asymmetric propagation delay caused by satellite relative motion and evaluate the propagation delay caused by relativistic light deflection using the time transfer function method. Then we assess the additional stability of the correction residuals and noise types. Finally, we extend the autocorrelation function representation of Allan variance, propose using statistical methods to calculate stability correction coefficients for different power-law noises separately, and analyze the impact of missing comparison data on stability calculations for various noise types.

2.1 Clock Difference Model for Inter-satellite Two-way Time Transfer

General relativity shows that standard clocks at different gravitational potentials have different rates. Clock synchronization at different locations requires adjusting clock rates according to a specified reference surface. For example, the definition of the International Atomic Time (TAI) second uses the geoid as reference, and satellite navigation systems adjust on-board atomic clock rates to align their paper time with ground atomic clocks. We establish the inter-satellite time transfer clock difference model in the geocentric non-rotating reference frame, using the metric form specified by the International Astronomical Union (IAU) 2000 resolution for the geocentric reference frame. The time coordinate uses Terrestrial Time (TT) defined on the geoid, with spacetime coordinates (cT, X), where c, T, and X represent the speed of light, Terrestrial Time, and spatial coordinates respectively, with i taking values 1, 2, 3. The inter-satellite time transfer link measures the local time difference between satellites at corresponding coordinate times. In addition to time conversion, relativistic light deflection-induced signal propagation delays and satellite orbit corrections introduced by relativistic celestial mechanics must also be considered as relativistic effects in clock difference correction. Inter-satellite time asynchrony and unequal signal propagation and equipment delays cause the two-way pseudorange and local or remote transmit-receive time interval observations to be unequal—this very effect is utilized to construct various forms of clock difference observation equations. Figure 1 [Figure 1: see original paper] shows a schematic diagram of inter-satellite two-way time transfer, where X represents the satellite position vector, subscripts A or B denote satellite labels, t_1 and t_3 represent the coordinate times of signal transmission from satellites A and B respectively, t_2 and t_4 represent the coordinate times of signal reception at satellites A and B respectively, and T_{12} and T_{34} are the signal propagation times from satellite A to B and from B to A respectively.

Neglecting relativistic effects and assuming the clock difference between satellite B and satellite A at the same moment in the spacetime reference frame is $\Delta\tau$, the clock difference equation based on satellite local transmit-receive epoch observations (temporarily ignoring equipment delays, ionospheric delays, etc.) is approximately:

$\Delta\tau$ [τ_B^r(t_2) + τ_B^e(t_3) - τ_A^r(t_4)] + (T_{34} - T_{12}) (1)

where τ represents local time, superscripts e and r denote transmission and reception respectively, and subscripts A or B denote satellites. The observable in the above equation can be understood either as the transmit and receive signal epochs of each satellite or as the remote transmit-receive time intervals [τ_A^r(t_2) - τ_A^e(t_1)] and [τ_B^r(t_4) - τ_B^e(t_3)]. In particular, when the two satellites agree to transmit signals at the same local time, i.e., τ_B^e(t_1) = τ_A^e(t_1), the observable can be understood as [τ_B^r(t_2) - τ_A^r(t_4)], which is the time interval between local receive signal epochs. Regardless of which form of observable is used, the asymmetric propagation delay (T_{34} - T_{12}) must be precisely corrected.

Considering relativistic effects, when the two satellites agree to transmit signals at the same local time, the theoretical derivation yields the clock difference equation at the signal transmission epochs:

$\Delta\tau$_c = f$\Delta\tau$_B - ($\Delta\tau$_A)_B + [$\Delta\tau$_A(t_4) + $\Delta\tau$_B(t_3) + T_{34} + $\Delta\tau$_A^d(t_4)]_B - [$\Delta\tau$_B(t_2) + $\Delta\tau$_A(t_1) + T_{12} + $\Delta\tau$_B^d(t_2)]_B

(2)

where $\Delta\tau$_c = τ_B(t_1) - τ_A(t_1), τ_N(t_n) (n taking values 1, 2, 3, 4) represents the satellite local time at coordinate time t_n, the observables $\Delta\tau$_A and $\Delta\tau$_B represent the time intervals between transmit and receive signal epochs observed locally at satellites A and B respectively; the correction terms $\Delta\tau$_N(t_n) represent equipment delays when satellite N transmits or receives signals at time t_n; the correction terms $\Delta\tau$_N^d(t_n) represent time measurement delays caused by Doppler frequency shift when satellite N receives signals at time t_n; ($\Delta\tau$_N)_M represents the time interval conversion from satellite N's local reference frame to satellite M's local reference frame; and (T_{34} - T_{12})_B is the asymmetric propagation delay.

High-precision Earth coordinate time establishment and satellite navigation systems have thoroughly studied the time conversion effects required, including time conversions between ground/satellite local time and TT/TCG (Geocentric Coordinate Time). Establishing high-precision Earth coordinate time and evaluating long-term autonomous operation of on-board clocks requires considering the cumulative effect of time conversion and maximizing time conversion accuracy. Wolf et al. achieved 10^{-18} conversion accuracy between Earth surface/satellite local time and TCG, considering Earth's gravitational potential, second-order Doppler effects, external celestial bodies, and solid tides. Since a single time comparison can be completed within several seconds, the time interval observable is corrected by multiplying by the current time conversion coefficient, and 0.1 ps-level clock difference correction only requires time conversion accuracy at the 1×10^{-13} level. According to the geocentric celestial reference system metric specified by the IAU 2000 resolution, for time conversion between Earth surface and high-orbit satellite local time and TCG, estimation shows that contributions from the scalar potential squared W^2 and vector potential W (a taking values 1, 2, 3) are less than 10^{-18}. Therefore, clock difference correction time conversion mainly considers contributions from W and second-order Doppler effects to the metric. The time conversion formula between satellite local time and TT is:

dτ [1 - W - v^2] (1 - L_g) dT (3)

where v is satellite velocity, 1 - L_g is the conversion coefficient between TT and TCG, L_g = U_g/c^2, and U_g is the equivalent gravitational potential of the geoid including Earth's rotation effects. Using high-precision Earth potential models, satellite local time conversion to TCG and TT can achieve 10^{-17} accuracy under current precise orbit determination and velocity measurement precision, including Earth's gravitational potential and second-order Doppler effects, making the time conversion accuracy for inter-satellite time transfer observables better than the femtosecond level. This far exceeds the requirements for picosecond-level clock difference correction, so relativistic time conversion is not the key point for clock difference correction and will not be discussed further. As mentioned, besides time conversion, signal propagation delays caused by relativistic light deflection and satellite orbit corrections introduced by relativistic celestial mechanics affect clock difference correction. Since satellite orbits are obtained through orbit determination and prediction, the primary relativistic effect still requiring consideration in clock difference correction is the signal propagation delay caused by light deflection.

2.2 Correction of Asymmetric Propagation Delay Caused by Satellite Motion

Asymmetric propagation delay correction requires classification, reduction, and simplification based on expressions for one-way signal propagation time. Without relativistic effects, signal propagation time can be calculated using ordinary pursuit problem methods. Under relativistic theory, electromagnetic wave propagation trajectories follow null geodesic equations that extremize proper time. When considering higher-order terms of the metric and its gravitational potential, exact solutions become difficult, affecting propagation delay calculation. Shapiro delay is the simplest relativistic light deflection delay, obtained under the condition of a general static isotropic metric by taking the first post-Newtonian approximation of the metric and considering only the mass monopole potential to simplify the null geodesic equation, yielding propagation delay through integration of motion integrals and time differential expressions. Below we first consider the reduction of asymmetric propagation delay caused by satellite motion, then address the more complex relativistic light deflection effects.

Including Shapiro delay, the expression for reducing one-way signal propagation time to the emission epoch in the geocentric non-rotating reference frame is:

$$T_{12} = D_{AB}(t_1)/c + [D_{AB}(t_1) \cdot v_B(t_1)]/c^2 + [|v_B(t_1)|^2 - D_{AB}(t_1) \cdot a_B(t_1) - (D_{AB}(t_1) \cdot v_B(t_1))^2]/c^3 + (2GM_E/c^3) \ln[(r_A(t_1) + r_B(t_1) + D_{AB}(t_1))/(r_A(t_1) + r_B(t_1) - D_{AB}(t_1))] + T_{og}^{AB} + T_{iono}^{AB} + T_{tropo}^{AB} \quad (4)$$

where $D_{AB}(t_n) = X_B(t_n) - X_A(t_n)$ represents the relative position vector between satellites B and A at time t_n, $D_{AB}(t_n) = |D_{AB}(t_n)|$, $v_N(t_n)$ represents the velocity vector of satellite N at time t_n, $a_N(t_n)$ represents the acceleration vector of satellite N at time t_n, and G and M_E represent the gravitational constant and Earth's mass respectively. The second and third terms in the above equation are second-order and third-order delay terms caused by relative motion, the fourth term is Shapiro delay, T_{og}^{AB} represents other relativistic light deflection delays, and T_{iono}^{AB} and T_{tropo}^{AB} represent delay terms caused by possible passage of electromagnetic waves through the ionosphere and troposphere respectively.

As mentioned, the asymmetric propagation delay $(T_{34} - T_{12})_B$ is the difference in transmission times of signals propagating between the two satellites. Direct calculation using the one-way propagation time expression would be complicated as it contains parameters from both emission epochs. Theoretical calculations show that by reducing the c^{-1} and c^{-2} terms in the one-way propagation time to parameters at one emission epoch, the asymmetric propagation delay can be simplified. After reduction, some c^{-1} terms are eliminated, and the asymmetric delay can be expressed as:

$$T_{34} - T_{12} = N_{AB}(t_1) \cdot v_{AB}(t_1)\Delta t + [N_{AB}(t_1) \cdot a_{AB}(t_1)\Delta t^2]/2 + [v_{AB}(t_1)^2 - (D_{AB}(t_1) \cdot [v_A(t_1) + v_B(t_1)])/D_{AB}(t_1)]\Delta t^2/(2D_{AB}(t_1)c) - [D_{AB}(t_1) \cdot a_A(t_1)\Delta t]/c^2 + [v_{AB}(t_1) \cdot v_A(t_1)\Delta t]/c^2 + [D_{AB}(t_1) \cdot j_A(t_1)\Delta t^2]/c^2 - [v_{AB}(t_1) \cdot a_A(t_1)\Delta t^2]/c^2 + \ldots \quad (5)$$

where $N_{AB}(t_1) = D_{AB}(t_1)/|D_{AB}(t_1)|$, $\Delta t = t_3 - t_1 = -\Delta\tau c/k_B$, *$k_B$ represents the time conversion coefficient from coordinate time to satellite B's local reference frame,* $v_{AB}(t_1) = v_B(t_1) - v_A(t_1)$, $a_{AB}(t_1) = a_B(t_1) - a_A(t_1)$, $j_A(t_1)$ represents the jerk of satellite A at time t_1, and the ellipsis represents negligible small terms. The above equation shows that correcting the asymmetric propagation delay term in clock differences requires first substituting the clock difference, making clock error calculation an iterative process. Considering equipment delay corrections, after a few simple iterations, when the clock error reaches 10 ns accuracy, its impact on asymmetric propagation delay correction can be reduced to the 0.1 ps level. Asymmetric propagation delay correction errors mainly originate from orbit determination and velocity measurement errors. The asymmetric propagation delay caused by satellite motion is the largest term in propagation delay correction, and its magnitude, correction error, and additional stability are quantitatively evaluated by substituting simulated satellite ephemerides.

We developed a program to simulate ephemerides for Low Earth Orbit, Medium Earth Orbit, Geostationary Orbit, and Inclined Geosynchronous Orbit satellites (labeled LEO, MEO, GEO, and IGSO respectively). The coordinate system selected was the geocentric non-rotating reference frame, with the x-axis pointing to the vernal equinox, the x-y plane in the equatorial plane, and the z-axis pointing to Earth's north pole. Orbit calculations mainly considered Earth's central force and applied Binet's formula for orbit integration. The orbital altitudes for LEO, MEO, GEO, and IGSO were set to 400 km, 21,528 km, 35,786 km, and 35,786 km respectively, with eccentricities of 0.0005 and inclinations of 42°, 55°, 0°, and 55° respectively. At the initial epoch, the right ascension of ascending node for LEO, MEO, GEO, and IGSO were set to 109°, 120°, 110.5°, and 118° respectively, and the argument of perigee and mean anomaly for all four satellites were set to $\omega = 2$ and $M = 3\pi/2$. Figure 2 [Figure 2: see original paper] shows the motion trajectories of the LEO, MEO, GEO, and IGSO satellites and their link establishment status at a certain moment. Figure 3 [Figure 3: see original paper] shows the asymmetric delays caused by relative motion between LEO, MEO, GEO, and IGSO, assuming initial synchronization accuracy of 10 ns for two-way time transfer, which is readily achievable in engineering practice. The discontinuous portions of the curves in Figure 3 indicate periods when the two satellites cannot establish links due to Earth occlusion. Figure 3 demonstrates that asymmetric delays caused by satellite motion range within $\pm$3 s.

By adding white noise and periodic noise to the simulated ephemerides to model satellite orbit determination and velocity measurement errors, the asymmetric propagation delay correction errors are quantitatively evaluated. For orbital white noise, the variance of radial, along-track, and cross-track orbit determination errors was set to 1 m, and the variance of radial, along-track, and cross-track velocity measurement errors was set to 0.0001 $\mathrm{m \cdot s^{-1}}$. The correction residual sequence is obtained by differencing the asymmetric propagation delay correction value sequences calculated from the simulated and noise-added ephemerides. Figure 4 [Figure 4: see original paper] shows the correction residuals of asymmetric propagation delay. It can be seen that under the above orbit determination and velocity measurement accuracy conditions, the fluctuation of propagation delay correction residuals is already less than 1 ps. The magnitude of delay correction residuals is independent of noise type; the results for orbital white noise also apply to orbital periodic noise, though the noise type affects the additional stability of delay correction residuals. The additional stability is obtained by calculating the time deviation (TDEV, the square root of time Allan variance) of the propagation delay correction residual sequence, which quantitatively characterizes the impact of propagation delay correction errors on time-frequency transfer link stability. Figure 5 [Figure 5: see original paper] shows the TDEV of asymmetric propagation delay correction residuals between GEO and IGSO under both random and periodic orbit noise conditions. The periodic orbit noise terms were set to three components with periods of 1, 0.5, and 0.33 times the orbital period, orbit noise amplitudes of 0.5 m, 0.4 m, and 0.3 m, and velocity noise amplitudes of 0.00005 $\mathrm{m \cdot s^{-1}}$, 0.00004 $\mathrm{m \cdot s^{-1}}$, and 0.00003 $\mathrm{m \cdot s^{-1}}$ respectively.

The results show that under orbital random noise conditions, the TDEV of asymmetric propagation delay correction residuals $\sigma\ (\tau)$ (where subscript x specifically indicates normalized phase or time observables) is better than 0.1 ps/s and follows $\sigma\ (\tau)\quad \tau^{-1}/^{2}$, with noise type approximating white phase noise that decreases faster with averaging time than the white frequency, flicker frequency, and random walk frequency noise typical of atomic clocks. Orbital and velocity periodic noise cause the TDEV of asymmetric propagation delay correction residuals for 1–1000 s to be smaller than that from random noise, while for 10,000–100,000 s it is slightly larger than from random noise, but the maximum amplitude is about 10^{-14} s.

In addition to quantitatively analyzing propagation delay correction accuracy, inter-satellite time-frequency transfer link design also requires deriving constraints on satellite orbit determination and velocity measurement precision based on accuracy requirements. Since asymmetric propagation delay caused by satellite motion is most sensitive to orbital and velocity variations, it is most suitable for deriving constraints on orbit determination and velocity measurement. Atomic clock phase noise typically manifests as white frequency noise, with TDEV proportional to $\tau^{1}/^{2}$. Considering the noise type of delay correction residuals—specifically, how the $\sigma\ (\tau)$ curve varies with averaging time τ—when $\sigma\ (\tau)$ is shifted to be entirely below the atomic clock TDEV curve, we identify the closest point between the curves as σ (τ_c). For example, based on Figure 5, when the atomic clock $\sigma(\tau) = 1.0 \times 10^{-13}\ \tau^{1}/^{2}$ s, the closest points between the atomic clock and delay correction residual TDEV curves under orbital white noise and periodic noise conditions are at σ (1 s) and σ (10,000 s) respectively. The relationship between σ (τ_c) and delay correction residual magnitude is then used to sequentially evaluate the delay correction residual magnitude and the maximum allowable values for orbit determination and velocity measurement. If the delay correction residual stability curve is only below the atomic clock stability curve in the medium-to-long-term τ region, the closest point between the two curves is their intersection, which relaxes the requirements for orbit determination and velocity measurement precision. Taking orbital white noise as an example, based on Figures 4 and 5, the propagation delay correction error δT_c $\leq 4\sigma$ (1 s), while σ (1 s) $< k_1\sigma$ (1 s), thus δT_c $\leq 4k_1\sigma$ (1 s), where σ (1 s) is the atomic clock time stability at 1 s and $0 < k_1 < 1$ is a proportionality coefficient derived from time-frequency transfer link index decomposition, determined by comprehensive equipment delay correction and time measurement error conditions. For detailed inter-satellite time-frequency transfer link design, the above approach can be combined with specific orbit determination and velocity measurement noise types to derive requirements for orbit determination and velocity measurement precision. Table 1 provides example constraints on orbit determination and velocity measurement for each satellite to meet 0.1 ps delay correction residuals, where δR, δT, and δN represent radial, along-track, and cross-track orbit determination errors respectively, and δv_R, δv_T, and δv_N represent radial, along-track, and cross-track velocity measurement errors respectively. The two values before and after the hyphen represent constraints for the two satellites being compared. Table 1 shows that 0.1 ps asymmetric propagation delay correction requires orbit determination precision of 0.7–6 m radial, 0.5–1 m along-track, and 1–60 m cross-track, and velocity measurement precision of 0.07–0.3 mm $\cdot$ s^{-1} radial, 0.1–0.8 mm $\cdot$ s^{-1} along-track, and 0.1–7 mm $\cdot$ s^{-1} cross-track.

2.3 Correction of Relativistic Light Deflection Delay

Light propagation trajectories follow null geodesic equations. Under the assumptions of axisymmetric and non-rotating Earth gravitational potential, based on first-order post-Newtonian approximation and considering only Earth's mass monopole potential, electromagnetic wave propagation trajectories in near-Earth space approximate hyperbolas. However, when considering Earth's multipole moments and rotation effects, it becomes difficult to obtain motion integrals or solve null geodesic equations to calculate light propagation trajectories and delays. The time transfer function can calculate delays caused by various gravitational potentials to c^{-4} order without precisely solving the null geodesic equation. Taking the metric to first-order post-Newtonian approximation, G_ = _ + h_ , where _ is the Minkowski metric and h_ represents metric perturbation terms, with , taking values 0, 1, 2, 3. Based on the zero proper time condition ds^2 = c^{2}dτ^2 = -G_ dX^ dX^ =

0, substituting the metric and approximating to first-order Taylor expansion yields:

dt (1/c) [1 - (h_{00} + 2h_0 n + h n n)/2] dl (6)

where j takes values 1, 2, 3, n = v /c, the equation uses Einstein summation convention, and separates time and spatial coordinates. Integration along the light propagation trajectory yields the propagation delay. The time transfer function is defined as the integral of G_ (dX $^{/\mathrm{d}\lambda)(\mathrm{dX}}$ /dλ) along the light propagation trajectory, where λ is an affine parameter. Since proper time takes the extremal value of zero, the integral is zero. Based on variational problems with fixed endpoints, the Euler-Lagrange equation can be applied. Linet et al. derived an alternative form of the light propagation geodesic equation and proved that if the integration path is expressed as a straight line between emission and reception sites plus a small perturbation, this perturbation is a c^{-1} second-order term whose contributions to the integrand and propagation delay are c^{-4} and c^{-5} order respectively, and can be neglected when calculating propagation delay to c^{-4} precision. Integrating dt from equation (6) along a Euclidean space straight-line propagation trajectory, the time transfer function (propagation delay) accurate to c^{-4} order is:

T_s(t_A, X_A, X_B) = (1/c) _A^B [h (X'(λ)) N N + 2h_0 (X'(λ)) N + h_{00}(X'(λ))] dλ (7)

where X'(λ) represents the straight line connecting A and B, X' (λ) are components of X'(λ), N = [X' (B) - X' (A)]/R_{AB}, R_{AB} is the straight-line distance between A and B, and X' (λ) = X' (A) + (X' (B) - X' (A))λ with $0 \leq \lambda \leq 1$. Soffel et al. also derived signal propagation time expressions using the time transfer function method. Overall, the time transfer function method expresses propagation delay as an integral of h_ , and since h_   is expressed as multiple c^{-1} terms containing various gravitational potentials, the light deflection delays caused by multiple gravities can be calculated separately.

Directly applying the signal propagation delay expression derived by Linet et al. and substituting satellite ephemerides, we evaluate the light deflection delay caused by gravity. Figure 6 [Figure 6: see original paper] shows the variation of light deflection delay during inter-satellite one-way transfer with satellite motion, mainly including delays caused by Earth's mass monopole, quadrupole moment, and rotation. Figure 7 [Figure 7: see original paper] shows the asymmetric light deflection delay, with a maximum of about 10 fs, meeting the 0.1 ps correction requirement.

In addition to the above light deflection delays, light deflection delays caused by smaller gravitational potentials such as Earth tidal potential, solid Earth tides, and solar or lunar gravitational potentials can also be estimated using the time transfer function method. Han et al. evaluated satellite-to-ground light deflection delays caused by tidal and inertial potentials at the 10^{-16} s level or smaller. This method can be extended to inter-satellite light deflection delays, and these smaller gravitational potential-induced relativistic light deflection delays do not affect 0.1 ps-level time comparison.

In summary, asymmetric propagation delay is caused by the asymmetry of inter-satellite signal propagation paths and is one of the main correction terms in the clock difference equation. Satellite relative motion is the primary factor generating asymmetric propagation delay, which also couples with asynchronous signal transmission epochs and relativistic light deflection effects to cause other asymmetric propagation delay terms that must be corrected separately. By establishing an asymmetric propagation delay correction method, we simplify the asymmetric propagation delay caused by satellite motion, calculate separately the light deflection delays caused by Earth's mass monopole, quadrupole moment, and spin, and evaluate the asymmetric propagation delay correction values, residuals, and additional stability. The results demonstrate that under current precise orbit determination conditions, asymmetric propagation delay can be corrected to the 0.1 ps level.

3 Impact of Missing Inter-satellite Comparison Data on Allan Variance Calculation and Correction Method

Atomic clocks on different satellites obtain clock difference or frequency difference sequences through inter-satellite time-frequency comparison links. Since the variance of clock difference or frequency difference sequences cannot accurately characterize frequency source drift characteristics and may diverge with increasing statistical data volume, making it difficult to accurately predict the accumulated time error after a clock source operates autonomously for some time, time-frequency statistical theory calculates Allan variance of clock difference or frequency difference sequences to realize atomic clock timekeeping deviation prediction. Frequency Allan variance is expressed as:

$\sigma y^2(\tau) = 1/(M-1)\ \Sigma\{p=1\}^{\wedge}\{M-1\}\ (\bar{y}\_\{p+1\} - \bar{y}\_p)^2$ (8)

where subscript y specifically indicates the observable is relative frequency, the frequency difference sequence is divided into M sampling intervals according to averaging time τ, and ȳ_p is the average relative frequency difference in the p-th sampling interval. Allan variance can also be expressed as an integral form of frequency autocorrelation function or power spectral density, facilitating power-law noise-based analysis of frequency stability performance.

As mentioned, for periodically missing comparison data, when averaging time τ is much smaller than comparison period T, Allan variance calculation is essentially unaffected. When τ is of the same order as T but smaller than T, Barnes et al. and Wang et al. used correction coefficient $B_2(T; \tau)$ to correct Allan variance. For another case of periodically missing comparison data, where τ is larger than T, or for non-periodic missing data, the relative frequency average ȳ_p in equation (8) cannot be accurately calculated due to missing sampling points, making accurate Allan variance calculation difficult. Theoretical calculations show that by extending the autocorrelation function representation of

Allan variance, the impact of missing comparison data can be attributed to changes in the statistical weights of the autocorrelation function. Combined with analytical forms of power-law noise autocorrelation functions, deviations in Allan variance for different noises can be calculated separately.

Under discrete sampling conditions, the frequency Allan variance calculation formula is:

$\sigma y^2(\tau)$ = Σ\{m_1=1\}^\{M-m\} Σ_\{m_2=m_1\}^\{m_1+m\} R_y[(m_1 - m_2)τ_0] (9)

where R_y represents the relative frequency autocorrelation function, subscript y specifically indicates the observable is relative frequency, m = τ/τ_0, and τ_0 is the sampling time interval. Letting m_1 - m_2 = k and q = |k| + 1, the frequency Allan variance calculation formula can be rewritten as:

$\sigma y^2(\tau)$ = Σ\{q=1\}^\{2m-1\} N_\{pq=m^2\} R_y[(q - 1)τ_0] (10)

where N_\{pq=m^2\} represents the normalized probability of R_y[(q - 1)τ_0] in the (ȳ_\{p+1\} - ȳ_p)^2/2 term of equation (8). This equation shows that Allan variance sequentially calculates the probability of autocorrelation function R_y[(q - 1)τ_0] in each sampling interval's (ȳ_\{p+1\} - ȳ_p)^2/2 term, averages the results M-1 times, and finally expresses them as a weighted sum of R_y[(q - 1)τ_0].

The autocorrelation function is the Fourier transform of power spectral density, and power-law noise power spectral density in time-frequency statistical theory is commonly expressed as S_y(f) = h_α f^α, where f represents noise frequency and α = 2, 1, 0, -1, -2 correspond to white phase, flicker phase, white frequency, flicker frequency, and random walk frequency noise respectively. When α is not an integer, applying the integral formula based on Γ functions:

_0^∞ x^\{-z\} cos(bx) dx = (π/2)[b^\{z-1\}/Γ(z) cos(πz/2)] (11)

yields R_y(τ_1) |τ_1|^\{-α-1\}. When α is 0, positive integer, or negative integer, R_y(τ_1) calculation is more difficult, and the above power-law relationship between R_y(τ_1) and τ_1 is approximately continued. Due to finite bandwidth limitations, R_y(0) = R_y(1/w_B), where w_B is the sampling instrument bandwidth.

Under data missing conditions, if sampling points are still divided according to averaging time τ, missing data points are directly discarded, relative frequency averages are calculated for each sampling interval, and frequency stability is then calculated, this will cause stability calculation errors. This is equivalent to changing the weights participating in the weighted average of R_y(τ_1) (where τ_1 takes values 0, τ_0, 2τ_0, …, (2m-1)τ_0 under discrete sampling conditions), no longer conforming to the Allan variance definition. Figures 8 [Figure 8: see original paper] and 9 [Figure 9: see original paper] simulate the impact of periodic data missing on stability calculation, where the simulated observation data period is the orbital period of low Earth orbit (about 400 km altitude, 5714 s), and the original data comes from three active hydrogen masers (labeled HM1, HM2, and HM3). Figure 8 shows frequency Allan deviation under conditions of 95% missing comparison data (denoted DCC) and no missing data (denoted CC). Figure 9 shows the relationship between HM3 frequency Allan deviation and periodic effective data proportion, where percentages in the legend represent effective data proportion. These figures demonstrate that under high missing data proportion conditions, medium-term stability offsets can be as large as three orders of magnitude, with the main noise type being white frequency noise, while long-term stability offsets are smaller, with main noise types being flicker and random walk frequency noise.

Noise type and data missing proportion are the two main factors affecting stability calculation. Atomic clocks exhibit different noise types at different averaging times or show composite noise. On-board atomic clocks can obtain noise types at various averaging times or their weight distributions from ground test results. After calculating correction coefficients for common power-law noise stability under data missing conditions, they can be applied to correct stability at different averaging times. Combining the above autocorrelation function representation of Allan variance, we define the proportionality coefficient:

k_2 = Σ_{τ_1=0} P_R(τ_1)R_y(τ_1) / Σ_{τ_1=0} P_0(τ_1)R_y(τ_1) (12)

where P_R(τ_1) and P_0(τ_1) are the normalized probabilities of R_y(τ_1) under missing and non-missing comparison data conditions respectively, calculated through programming. The coefficient k_2 characterizes the stability deviation of a single noise type under data missing conditions.

Figure 10 [Figure 10: see original paper] shows, for example, the probability of R_y(τ_1) under different effective data proportion conditions when calculating 10,000 s frequency Allan deviation, where percentages in the legend represent effective data proportion and the data period is 5714 s. According to equations (9) and (10), R_y(τ_1) probabilities can become negative, but this does not affect analysis and calculation. Figure 10 reveals that when comparison data is significantly missing, R_y(τ_1) probabilities exhibit extrema at different τ_1 values. Example values of k_2 for α = 2, 1, 0, -0.5, -1.5 at τ = 10,000 s are shown in Table 2 , where percentages represent effective data proportion. The simulation uses periodic data missing but can be extended to non-periodic missing. When α is negative, -0.5 and -1.5 are selected instead of -1 and -2 because the Γ function becomes infinite for negative integers, preventing calculation of analytical autocorrelation function forms.

Table 2 shows that when α = 2, 1, 0 and effective data proportions are 5%, 32%, 63%, and 95%, the 10,000 s Allan deviation calculation values increase by 3.52, 0.83, 0.27, and 0.04 times respectively. Figure 8 shows that under 5% data proportion conditions, the average increase factor for 10,000 s stability of HM1 and HM3 is 3.4, which approximately matches the 3.52 result in Table 2. HM2 is not used for comparative analysis because it exhibits multiple noise types at 10,000 s. Figure 9 shows that under data proportions of 5%, 32%, 63%, and 95%, the increase factors for HM3 10,000 s stability are 4.13, 0.87, 0.20, and 0.07 respectively, which also approximately match Table 2 results. The table also indicates that under data missing conditions, when -(α+1) < 0 (α > -1), stability calculation results show larger deviations, while when -(α+1) > 0 (α < -1), deviations are smaller. This trend is consistent with the results in Figures 8 and 9. Figures 8 and 9 demonstrate that for active hydrogen masers, stability at averaging times of 100–10,000 s is mainly determined by white frequency noise ($\alpha = 0$), showing larger calculation deviations under data missing conditions, while 1-day stability is mainly determined by flicker frequency (α = -1) and random walk frequency noise (α = -2), showing smaller deviations. Therefore, high-proportion missing comparison data mainly affects stability calculations for white phase noise, flicker phase noise, and white frequency noise, while having less impact on flicker frequency and random walk frequency noise. Since medium-term stability of atomic clocks is mostly determined by white frequency noise, high-proportion missing comparison data severely affects stability calculation. To ensure small Allan variance result deviations, ground or on-board relay methods should be used to reduce data missing proportion or apply corrections.

Based on the above analysis, stability correction can first skip missing frequency difference or time difference data points, directly calculate averages within each sampling interval, compute an initial Allan deviation value, then statistically calculate the correction coefficient k_2 according to data missing conditions, and finally obtain the corrected Allan deviation by multiplying the initial value by k_2.

In addition to equipment delays, asymmetric inter-satellite signal propagation delay and data missing are the two main factors affecting inter-satellite time transfer clock difference correction. For asymmetric propagation delay correction, we start by establishing the clock difference model, analyze relativistic time conversion effects, reduce asymmetric propagation delays, apply the time transfer function method to separately calculate relativistic light deflection delays, evaluate the additional stability of delay correction residuals, and finally derive constraints on orbit determination and velocity measurement from link accuracy requirements, proposing a systematic method for inter-satellite time-frequency comparison propagation delay correction and evaluation. Under current orbit determination precision conditions (satellite orbit determination and velocity measurement accuracies of 1 m and 0.1 $mm \cdot s^{-1}$ respectively), inter-satellite asymmetric propagation delay correction accuracy can reach the 0.1 ps level. For missing inter-satellite comparison data, we extend the autocorrelation function representation of Allan variance, reveal the impact of missing comparison data on stability of different noise types, and show that high-proportion missing data significantly affects stability calculations for white phase noise, flicker phase noise, and white frequency noise, while having less impact on flicker frequency and random walk frequency noise. We propose calculating time-frequency comparison stability deviations from the perspective of autocorrelation function weighted statistics, introducing the Allan deviation correction coefficient k_2 for inter-satellite time-frequency transfer stability correction.

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Note: Figure translations are in progress. See original paper for figures.