A Model for the Susceptibility Assessment of Glacial Lake Outburst Floods Based on Physical Processes and the Analytic Hierarchy Process

Authors: Jieqing Hou, Jieqing Hou

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Abstract

Objective: The study aims to develop a susceptibility assessment model for glacial lake outbursts based on physical processes and the Analytic Hierarchy Process (AHP), to quantitatively and objectively evaluate the threats of glacial lake outbursts. This is in response to the challenges posed by the expansion of glacial lake areas and the increase in their numbers under global warming.

Methods: The research introduces the concept of "instantaneous triggering body mechanical energy transfer efficiency" from the perspective of mechanical energy transfer and transformation. Impact indicators are categorized into three major groups, with mechanical energy used to decouple and assign values to each indicator. An E/F calculation model is constructed, aligning with the safety factor concept in reliability theory, and the model is validated as a susceptibility assessment tool using the AHP method. The study also summarizes the Analytic Hierarchy Process Based on Physical Processes from the model construction and applies it to build a landslide susceptibility model, thereby verifying its generalization capability.

Results: The study finds that the new model can effectively assess the susceptibility of glacial lake outbursts, offering a novel perspective to understand the dynamic changes of such events. Key indicators include the mechanical energy of instantaneous triggering bodies, the mechanical energy of lake water, and the critical failure Newton force of the ice dam. Analysis indicates that the model is highly universal in assessing susceptibility and provides a more comprehensive and systematic framework compared to traditional methods. The Analytic Hierarchy Process Based on Physical Processes demonstrates a certain level of generalization ability.

Limitations: Potential limitations of the study include the difficulty in accurately obtaining some parameters and the need for further validation of the generalization ability of the Analytic Hierarchy Process Based on Physical Processes. Additionally, the accuracy of the model may be constrained by data quality and monitoring technology.

Conclusions: The model proposed in this study has good applicability and universal significance, capable of providing a scientific basis for disaster prevention and mitigation. Compared to existing research, the uniqueness of this work lies in its combination of quantitative methods based on physical processes with AHP, offering a new tool and analytical method for more accurately identifying and assessing disaster risks.

Full Text

Preamble

A Model for the Susceptibility Assessment of Glacial Lake Outburst Floods Based on Physical Processes and the Analytic Hierarchy Process Based on Physical Processes

Jieqing Hou[1],[2]*

Abstract

Objective: This study develops a susceptibility assessment model for Glacial Lake Outburst Floods (GLOFs) that integrates physical processes with the Analytic Hierarchy Process Based on Physical Processes (AHPBPP). The model aims to quantitatively and objectively evaluate GLOF threats, shifting errors from subjective expert judgment to objective data quality limitations. Inspired by this approach, we propose the Law of Relative Quantities with Uncancelled Dimensions.

Methods: We introduce the concept of “instantaneous triggering body mechanical energy transfer efficiency” from the perspective of mechanical energy transfer and conversion. Impact indicators are categorized into three major classes and decoupled/valued using mechanical energy for each indicator. An E/F calculation model is constructed, consistent with the safety factor concept in reliability theory, and validated using the AHPBPP method. The study also summarizes the AHPBPP methodology and applies it to construct a landslide susceptibility model, thereby verifying its generalization capability.

Results: The new model effectively assesses GLOF susceptibility while minimizing subjective influence, offering a novel perspective for understanding the dynamic changes of such events. Key indicators include the mechanical energy of instantaneous triggering bodies, the mechanical energy of lake water, and the critical failure Newton’ s force of the dam. The analysis demonstrates high universality in susceptibility assessment and provides a more comprehensive framework compared to traditional methods. The AHPBPP shows promising generalization capability.

Limitations: Challenges include difficulties in accurately obtaining some parameters and the need for further validation of the AHPBPP’ s generalization capability. Model accuracy may also be constrained by data quality and monitoring technology.

Conclusion: The proposed model demonstrates good applicability and universal significance, providing a scientific basis for disaster prevention and mitigation. Its uniqueness lies in combining quantitative physical process-based methods with AHPBPP, offering new tools for more accurate disaster risk identification and assessment. The AHPBPP enables traditional AHP to overcome limitations of subjective experience.

Keywords: Glacial Lake; Outburst; Analytic Hierarchy Process; Susceptibility; Quantitative

*Corresponding author: Jieqing Hou (E-mail: houjieqing@imde.ac.cn)

1 Introduction

As sensitive indicators of global change, cryospheric systems are not only undergoing rapid and significant transformations but also responding directly and sensitively to climate system dynamics (Qin et al., 2018). Glaciers, as crucial components of the cryosphere, are experiencing accelerated melting rates under global warming (Marta, S. et al., 2021; Lee, E. et al., 2021; Carrivick, J. L. et al., 2023). This phenomenon leads to substantial convergence of glacial meltwater, promoting further expansion of existing glacial lakes. Suitable topographic conditions also facilitate formation of new unstable glacial lakes, a trend that has persisted in recent years (Nie et al., 2017; Harrison et al., 2018; Shugar, D. H et al., 2020).

Recent research indicates there are currently over 110,000 glacial lakes globally, covering approximately 15,000 square kilometers, with lake areas increasing by about 22% between 1990 and 2020 (Zhang et al., 2024). Additionally, global warming promotes frequent extreme climate events, which may further exacerbate GLOF triggers such as ice/snow avalanches (Richardson et al., 2000; Nie et al., 2018), debris flows (Richardson et al., 2000; Haeberli et al., 2017), and floods (Emmer, 2013; Emmer, 2017). GLOF disasters have become significant obstacles to economic and social development in high-altitude cold regions, historically causing tens of thousands of deaths and massive infrastructure damage (Carey, 2005; Liu, J et al., 2014; Allen et al., 2016; Carrivick and Tweed, 2016; Nie et al., 2018).

The high-altitude location of glacial lakes means that flood releases carry enormous potential energy and can transport substantial material, forming destructive hyperconcentrated floods or debris flows (Cui et al., 2003; Worni et al., 2014; McKillop and Clague, 2007). Currently, over 10 million people worldwide live under potential GLOF threats, posing severe challenges to infrastructure construction and planning in High Mountain Asia and constraining sustainable regional economic development (Xu, 1988; Clague et al., 2000; Sattar et al., 2022; Nie et al., 2023). For countries where hydropower constitutes a large proportion of national income, GLOF impacts are particularly severe (Carrivick and Tweed, 2016). With expanding glacial lakes and increasing human activities within their influence zones, GLOF disaster impacts tend to intensify, making GLOF susceptibility analysis particularly crucial (Gardelle, J et al., 2011; Carrivick, J. L et al., 2013; Nie et al., 2023). Such analysis helps us better understand dynamic changes in glacial lake threats (Emmer et al., 2018) and provides scientific guidance for disaster prevention and mitigation.

2 Research Status

GLOF prediction differs significantly from traditional hydrological flood frequency analysis, primarily because GLOFs often manifest as one-time events (Steijn, H. V., 1996; Hegglin, E., 2008). Predicting this phenomenon is a complex scientific problem involving high nonlinearity and uncertainty, requiring consideration of complex dynamic relationships between inputs and outputs, which increases prediction difficulty (Zhou, B et al., 2023).

Consequently, the academic community has developed various GLOF susceptibility assessment models. Based on model composition, indicator selection, and the degree of subjectivity in determining indicator importance, these models can be broadly classified as qualitative, semi-quantitative, and quantitative (Clague, J.J et al., 2000; Wang, X et al., 2007). Qualitative models exhibit the highest subjectivity, semi-quantitative models moderate subjectivity, and quantitative models the least subjectivity. Qualitative models overly rely on evaluator experience, generalizing assessment indicators based on subjective judgment, with threshold settings also entirely subjective, resulting in vague outcomes (Carey, M., 2005; Costa, J. E et al., 1988; Huggel, C et al., 2004).

Compared to qualitative models, semi-quantitative models process indicators quantitatively based on subjective experience, reducing reliance on evaluator judgment through computational methods (Reynolds, J. M, 2003; Bolch, T et al., 2011). Although quantitative models appear more objective in indicator weighting and computational processes, they remain influenced by subjective experience in indicator selection and tool choice (Mckillop R J et al., 2007; Wang, W et al., 2011; Mergili, M et al., 2011). Furthermore, existing models have not sufficiently considered coupling effects among different indicators, potentially affecting prediction accuracy (Zhou, B et al., 2023). Model results are often regionally limited, lacking universality, leading to situations where a model is only effective in specific areas (Jiajia Gao et al., 2023). Different evaluators may produce varying classification results for the same area, causing confusion for decision-makers.

Assessment models based on the Analytic Hierarchy Process (Nitesh Khadka et al., 2021; Zhang, D et al., 2023), fuzzy comprehensive evaluation (Wang, W et al., 2011), logistic regression (Mckillop R J et al., 2007), and fuzzy matterelement extension (Liu, J.F et al., 2012) play crucial roles in specific contexts. However, they deviate from GLOF physical processes and cannot finely reflect dynamic changes in GLOF susceptibility indices. The GLOF susceptibility index should vary with environmental changes, yet existing model outputs do not align with these physical characteristics.

3.1 The Process of Glacial Lake Outburst

A deep understanding of GLOF processes is the first step in model construction. GLOFs occur through long-term interaction between lake water and moraine dams, catalyzed by climatic factors such as temperature and precipitation (IPCC, 2013; Harrison et al., 2018). They are triggered by factors including earthquakes, ice/snow avalanches, landslides, upstream floods/debris flows, and internal dam ice melting (Richardson and Reynolds, 2000; Liu Jing-Jing et al., 2014; Worni et al., 2014). Partial spontaneous or non-spontaneous dam failure allows lake water to flow continuously, generating scouring effects and releasing lake water’ s physical processes (Liu, J et al., 2013; Westoby et al., 2014; Robin Neupane et al., 2019).

Typically, GLOFs involve a dynamic cascading process of potential and kinetic energy conversion (Somos-Valenzuela et al., 2016; Cui, P et al., 2019). Notably, the dam failure point under wave action is not necessarily the dam’ s weakest part. Due to varying wave energy levels and different resistances across dam sections, failure must occur where wave energy precisely causes dam failure. To achieve an outburst, this failure point must also have the potential to generate sustained overflow within a short period. In reality, the mechanical energy acting at the dam failure location and the dam’ s resistance at that location truly influence the GLOF. Therefore, defining potential dam failure locations is crucial for enhancing model accuracy. Although we cannot currently predict the specific failure location precisely, we can estimate potential failure areas through empirical analysis, thereby improving GLOF susceptibility index calculation accuracy.

3.2 Analytic Hierarchy Process (AHP)

The Analytic Hierarchy Process (AHP), proposed by American operations researcher Thomas L. Saaty in the early 1970s, is a method specifically designed for complex multi-objective, multi-criteria decision-making problems (Saaty, T. L., 1980). The general process involves first decomposing the decision problem into objective, criterion, and alternative levels to form a hierarchical structure. Next, pairwise comparisons are made among elements at the criterion and alternative levels, scoring based on relative importance. Consistency checks are then conducted on the pairwise comparison matrices to ensure evaluation rationality. Weight vectors are calculated through the pairwise comparison matrices, typically involving eigenvalue and eigenvector computation. Finally, criterion-level weights are combined with alternative-level weights to compute comprehensive alternative scores and rankings (Saaty, T. L., 1990; Saaty, T. L., 1994; Saaty, T. L., 2008; Saaty, T. L. and Vargas, L. G., 2001; Saaty, T. L., 2005).

The AHP's core lies in its hierarchical decision structure, importance assessment and quantification, and weight calculation and synthesis. Even with variations in computational details or technical methods, retaining these core steps preserves the AHP spirit. Mathematical tools such as fuzzy logic (Zadeh, L. A., 1965; Dubois, D. & Prade, H., 1980) and the ideal point method (Tzeng, G. H. & Huang, J. J., 2011) can be integrated to assist in weight determination or consistency checks. Hierarchization allows complex decision problems to be logically divided into different levels, facilitating systematic understanding and handling. Weight synthesis enables decision-makers to consider multiple criteria impacts and determine relative importance, yielding comprehensive evaluation results. This method provides quantitative comparison and selection while helping decision-makers make more rational, scientific choices when facing complex problems.

Due to its ability to integrate qualitative and quantitative factors, AHP has been widely applied across various fields. In GLOF susceptibility assessment, Wang et al. (2011) developed a first-order AHP-based method to identify potentially dangerous glacial lakes in southeastern Tibet. Five variables were selected: parent glacier area, lake-glacier terminus distance, lake-glacier slope, moraine dam average slope, and glacier terminus steepness. Weights were assigned using the Fuzzy Consistent Matrix (FCM) method. Each variable was classified using statistical thresholds, successfully identifying 8 extremely high-risk glacial lakes among 78 moraine-dammed lakes, with effectiveness proven by verification against 6 historical outbursts. Nitesh Khadka et al. (2021) applied AHP to assess GLOF susceptibility in the Mahalangur Himalaya, determining weights for six key factors (lake area, expansion rate, glacier distance, dam front slope, ice/snow avalanche potential, upstream GLOF potential) through expert pairwise comparisons. The susceptibility index was then calculated, categorizing lakes into levels from very low to very high. Zhang, D et al. (2023) established an AHP-based assessment method combining digital elevation models (DEM), glacier data, remote sensing imagery, and field surveys to quantify GLOF susceptibility in the Nidu Zangbo Basin, Tibetan Plateau, successfully identifying and validating high-risk glacial lakes.

3.3 Safety Factor

In constructing an analytical model for single glacial lake outburst susceptibility index, relevant concepts from reliability theory regarding safety factors align well with our approach. Reliability theory' s application in assessing existing structure safety has been widely recognized (Diamantidis, D et al., 2024). When exploring existing structure safety, reliability theory provides a powerful analytical framework, particularly through the core safety factor concept. Safety factor (SF) definitions may vary slightly across engineering fields and standards, but the core concept remains similar. SF is typically defined as the ratio of a structure’ s load-carrying capacity considered during design to its actual load requirements under the most unfavorable load combinations (ISO 13822:2017; IBC2018). This factor ensures structures can safely withstand anticipated and unexpected loads throughout their design life. In traditional engineering practice, SF is often defined as the ratio of existing strength parameters to strength parameters required for stability (Song, E.X et al., 2016). Glacial lakes, as natural systems, consist of components including instantaneous triggering bodies, the glacial lake itself, and the dam. During GLOF processes, these components form a natural structural system. The safety factor concept should therefore apply to this analysis.

4.1 Analysis and Construction of the Indicator System

From a practical standpoint, achieving absolute accuracy in GLOF susceptibility index calculation is unrealistic. Our goal is to approximate the true susceptibility index as closely as possible, requiring accurate understanding of random and other influencing factors. However, due to limitations in technical means, information access, theoretical knowledge, and natural environmental conditions, human understanding of all GLOF-influencing factors is limited. Particularly for complex influencing factors, technical understanding limitations and subjective uncertainties are especially evident. This paper aims to improve GLOF susceptibility index understanding under existing conditions.

Therefore, we approach from mechanical energy transfer and conversion perspectives, dividing the three main participants directly involved in the process: instantaneous triggering bodies, the glacial lake itself, and the dam. Instantaneous triggering bodies include ice/snow avalanches, landslides, debris flows, and floods, while the glacial lake itself includes lake water and the lake basin. All GLOF-influencing indicators act directly or indirectly on these three main bodies, thereby participating in GLOFs and influencing the susceptibility index.

For instantaneous triggering bodies, indicators affecting GLOF susceptibility can be divided into two parts: those affecting their occurrence probability, and those affecting mechanical energy calculation entering the lake. Probability-affecting indicators include air temperature, precipitation, earthquakes, and inherent endowments. Mechanical energy calculation indicators include mass, shape, density, volume, distance from lake, height above lake, and path friction coefficient. For the glacial lake itself, susceptibility-related indicators are divided into lake basin indicators (shape, slope, volume) and lake water indicators (volume, density, distribution). Dam resistance is also crucial, with susceptibility-influencing indicators divided into dam shape indicators (height, width, upstream slope) and internal composition indicators (grain size distribution, mineral composition, soil structure, ice content). Instantaneous triggering body and glacial lake indicators are mainly used for energy and probability calculations, while dam-related indicators are used for resistance calculations.

Previous GLOF susceptibility assessments have often not fully considered these indicators. However, as physical properties of main GLOF participants, these indicators cannot be ignored in any assessment model. For example, commonly used indicators such as lake water area, lake water area change rate, and lake depth-to-dam height ratio can all be seen as lake water representations, ultimately reflecting lake water's mechanical energy state at the dam failure location. Similarly, commonly used indicators such as rear-edge glacier area, thickness, lake distance, and fracture development can be seen as instantaneous triggering body representations, ultimately reflecting mechanical energy calculation and occurrence probability. Furthermore, from a mechanical energy transfer perspective, we have identified new indicators including instantaneous triggering body distance relative to dam failure location, entry angle into the lake, wind speed during movement, and block disintegration number upon entry, which help more accurately calculate triggering body mechanical energy.

4.2 Energy Transfer Efficiency Construction

Since instantaneous triggering bodies generally do not directly contact the dam failure location, their mechanical energy acts on the failure location through glacial lake water as a surge. We consider the surge energy acting on the dam failure location as the energy contributing to the GLOF. Therefore, the ratio of lake water mechanical energy at the dam failure location caused by the triggering body to the triggering body' s mechanical energy is defined as energy transfer efficiency.

In the formula, represents energy transfer efficiency, Eb is the lake water mechanical energy at the dam failure location caused by the triggering body, and Eh is the lake water mechanical energy from the triggering body. Since varies with time, is a function of time t, denoted as (t), with t being the moment when the dam failure location receives energy transmitted by the triggering body, in seconds.

4.3 Construction of Dam Resistance

Glacial lakes are categorized by dam type into ice dams, moraine dams, and rock dams (Otto, J.C, 2019). During GLOFs, dams may be subjected to various forces, broadly divided into surface forces and body forces. Surface forces include buoyancy and lake water pressure on the dam, while body forces include seepage forces and gravity. Failure modes also exhibit diversity, potentially including overflow scouring failure, piping seepage failure, tensile fracture failure, and impact shear failure.

Dam failure complexity stems from multiple factors: diverse failure modes, various acting forces, complex material properties, irregular dam geometry, and random surge-dam interactions. These factors collectively determine that the critical failure Newton' s force required for dam failure cannot be precisely calculated, meaning moraine dam resistance is unsolvable. However, despite the inability to obtain exact values, resistance remains expressible.

Numerous factors affect dam resistance unsolvability, mainly summarized as material performance, geometric parameters, and calculation models. Material performance involves strength, elastic modulus, Poisson' s ratio, and other physical properties. Due to composition, structure, and environmental condition differences, dam material performance may vary. Geometric parameters such as height, width, length, and slope may also change under environmental influence. These changes may have minor short-term impacts on dam resistance and are usually treated as constants. Dam resistance calculation is typically based on basic assumptions that may not fully match reality, or formulas may involve approximations, introducing variability. Despite this, we can be certain that a specific critical Newton' s force exists at dam failure.

This force can be expressed as the product of stress and area, where stress represents material performance and area represents geometric parameters. Through the critical failure Newton' s force, we can characterize the dam' s overall resistance at the failure location.

The critical failure Newton's force is the combined surface and body force acting on material at critical failure state. A critical failure Newton' s force exists for any dam failure mode, whether overflow scouring, piping seepage, impact shear, or coupled modes.

In the formula, δ represents the ratio of actual dam resistance to calculated resistance; c denotes dam soil shear strength, characterizing material performance; s denotes dam shear area, characterizing geometric parameters; F denotes the dam' s critical failure Newton' s force, representing resistance; F_1 denotes surface force resultant; F_2 denotes body force resultant; denotes the angle between surface and body force resultants.

The formula shows that critical failure Newton's force is influenced by combined surface and body forces at critical failure state. Any change in magnitude or direction of either force will alter the critical failure Newton' s force. In practice, we can only obtain soil samples in front of the instantaneous triggering body for analysis, as once the triggering body enters the lake, mechanical energy transfer begins and dam failure may occur at any time. When the dam failure location receives energy transmitted through lake water, soil properties at that location change, and consequently the critical failure Newton' s force also changes. At this time, the critical failure Newton' s force varies with time, denoted as F(t), representing the critical failure Newton' s force as a function of time t from when the dam failure location receives energy, in seconds.

4.4 Selection, Decoupling, and Valuation of Indicators

This paper has analyzed and constructed important indicator system parameters for GLOFs, which can be summarized into three major indicators: those affecting mechanical energy contributed by instantaneous triggering bodies to the dam failure location, those affecting mechanical energy contributed by lake water to the failure location, and those affecting resistance at the failure location. As mentioned in the research status, existing models do not fully decouple indicator interactions, and assessment indicator selection and weighting remain influenced by subjective experience. This section addresses these issues.

Since GLOF is a dynamic cascading process involving mechanical energy transfer and conversion, energy transfer runs throughout the entire process. Therefore, we incorporated mechanical energy in model construction for two purposes: indicator valuation and decoupling indicator interactions. We found that when calculating mechanical energy contributed by instantaneous triggering bodies and lake water to the dam failure location, they are independent and non-interfering. The model reflects this by making indicator weights equal to their mechanical energy contributions—larger values receive greater weight. This achieves natural valuation of the two major indicators (triggering body energy and lake water energy) while resolving coupling and valuation issues. This paper constructs the model from a mechanical energy transfer perspective, selecting objective physical process indicators that include all influencing factors without subjective experience. Resistance at the dam failure location does not belong to the mechanical energy transfer link, so it naturally does not couple with other indicator types. However, resistance remains an important GLOF-influencing indicator, and its valuation also affects susceptibility index solution and analysis. Here we use critical failure Newton' s force to value dam failure location resistance.

4.5 Determination of Failure Core and Location

Landslides have sliding surfaces, and dam breaches have failure cores. Just as sliding surface position must be determined when calculating landslide stability index, failure core position must be determined when calculating GLOF susceptibility index. Due to dam material anisotropy—where critical failure Newton' s force varies everywhere—and different energy received by dam sections, when a dam section' s critical failure Newton' s force cannot resist water energy, that section will fail. If this section can continue producing overflow after failure, the glacial lake has breached. A dam may have multiple failure points after a surge, but the failure core is the most likely failure location—where the ratio of acting energy to critical failure Newton' s force is largest. Based on experience, the failure core is more likely to appear at lower dam surface positions where piping easily occurs, slightly above lake level, and where soil mechanical properties are poor.

5 Model Proposal

After in-depth analysis of GLOF physical processes and theoretical foundations, we categorize the three main participants into two primary categories: the attacking party and defending party. The attacking party includes instantaneous triggering bodies and the glacial lake itself, contributing mechanical energy at the dam failure location through triggering bodies and lake water. The defending party refers to the dam itself, with defensive capability represented by critical failure Newton's force at the failure location. Consequently, we construct a GLOF susceptibility index solution and analysis model. This model aims to comprehensively consider various GLOF system factors and quantitatively solve and analyze the GLOF susceptibility index.

$$\text{GLOFSI} = \frac{\psi \cdot \eta(t) \cdot E_h + G_b}{F(t)} \div 1\text{m}$$

At the moment when the Glacial Lake Outburst Flood Susceptibility Index (GLOFSI) reaches its maximum value, we have:

$$\text{GLOFSI}_{\max} = \max\left(\frac{\psi \cdot \eta(t) \cdot E_h + G_b}{F(t)} \div 1\text{m}\right)$$

In the formula, GLOFSI (Glacial Lake Outburst Flood Susceptibility Index) is the susceptibility index. We are only interested in the specific numerical value of this ratio, so we divide by 1m to eliminate units and obtain a dimensionless number; represents instantaneous triggering body occurrence probability, with maximum value 1 and minimum value 0; t represents energy transfer efficiency at time t; Eh represents triggering body mechanical energy to lake water; Gb represents lake water potential energy at dam failure location; Eall represents initial triggering body potential energy; Es represents mechanical energy lost by triggering body before entering lake; Ft represents critical failure Newton's force at dam failure location at time t, characterizing overall dam resistance. Here, energy units are joules, force units are newtons, and time units are seconds.

If we directly use critical failure energy at the selected location in the denominator, we could directly obtain a dimensionless number. However, we choose to represent dam resistance with critical failure Newton' s force for two main reasons: First, critical failure Newton' s force is more easily obtainable through technical means compared to critical failure energy; both c (cohesion) and s (shear area) can be measured, leaving only δ as an empirical coefficient. In contrast, estimating critical failure energy involves too many uncertain parameters, leading to significant estimation errors. Second, like critical failure energy, critical failure Newton' s force can strictly represent dam resistance, and both exist under all circumstances. Third, based on our understanding of breach phenomena, due to diverse dam failure modes and current lack of clear breach definition, determining what extent of dam failure constitutes a GLOF remains debated.

Therefore, this model defines a GLOF as the dam' s ability to quickly generate sustained overflow after a certain failure volume, where any failure mode has a significant failure location. During overflow failure, the significant failure location has relatively small failure volume; during wave-induced breach, it has relatively large volume. During piping failure, fine particle migration occurs over a certain timescale, requiring identification of piping failure location volume.

We define the critical Newton' s force causing estimated failure volume at the significant location to fail as the dam's critical failure Newton's force. We define the volume at the significant failure location as the failure core. Based on this understanding, this model assumes the failure location can be infinitely large or small. For these considerations, this paper chooses critical failure Newton' s force to represent dam resistance.

However, if other forms (including critical failure energy) are used to represent dam resistance, a similar E/F calculation mode constructed here would remain valid.

6 Model Analysis

When quantitatively analyzing single glacial lake outburst susceptibility index, we find all influencing factors can be directly or indirectly reflected in corresponding susceptibility expressions of participating entities, collectively determining GLOF susceptibility index changes. Importantly, susceptibility index changes do not always correspond to dramatic single-parameter changes; instead, comprehensive system perspective assessment is required. For instance, if dangerous ice avalanche body cracks increase, raising occurrence probability, but lake water simultaneously decreases significantly due to evaporation and seepage, we cannot conclude susceptibility index increased solely due to higher avalanche probability, nor can we conclude it decreased solely due to lower lake water. It is necessary to consider changes in remaining expression parameters. Meanwhile, changes in factors we subjectively believe influential may not necessarily affect the susceptibility index, as these changes must result in expression parameter changes to influence the index. This systemic perspective provides more comprehensive understanding of single-lake susceptibility index changes.

Each glacial lake has unique natural characteristics and conditions, so each lake' s susceptibility index expression is established based on its specific situation. The primary step in analyzing single glacial lake outburst susceptibility index is identifying and understanding the lake' s natural endowment, including geographic location, morphology, and hydrological conditions, and determining relevant parameters and values as accurately as possible. Based on these parameters, we can construct a lake-specific susceptibility index expression and perform analysis.

By analyzing the susceptibility index expression in division form, we observe that within a certain period, if the numerator (representing attacking party energy) decreases while the denominator (representing defending party resistance) increases, the GLOF susceptibility index decreases; conversely, if the numerator increases while the denominator decreases, the index rises. When numerator and denominator increase or decrease simultaneously, the trend may not be apparent. In such cases, if we can obtain specific amplitude changes in numerator and denominator through monitoring or other means, we can more accurately determine susceptibility index changes. This is analyzed using ratio concepts.

The GLOF susceptibility index expression also contains addition, where the numerator is divided into two parts, each divided by the denominator and summed, representing different concepts. The first part represents susceptibility contributed by instantaneous triggering bodies, and the second part represents susceptibility contributed by lake water potential energy at the dam failure location. Thus, the GLOF susceptibility index consists of two components: one from triggering bodies and one from lake water. When analyzing single glacial lake outburst susceptibility index, if both components increase, the index increases; if both decrease, the index decreases. When one component increases and the other decreases, we need to solve for the corresponding component, though current technical means cannot achieve precise valuation. At this time, we can conduct quantitative analysis to determine which component dominates the susceptibility index, as dominant component changes often dictate overall index changes, especially when dominant component changes exceed non-dominant component changes.

In some cases, although errors may exist in judging susceptibility index changes, when dominant and non-dominant component changes are not easily distinguishable—i.e., when dominant component change is small and non-dominant component change is large—the combined susceptibility index change will not be significant.

Specifically, for moraine lakes without instantaneous triggering bodies, we can analyze their susceptibility index expression and changes under specific environmental conditions to determine susceptibility index variations. When the ratio of new to original susceptibility index equals 1, the GLOF susceptibility index remains unchanged; when the ratio exceeds 1, the index increases; when the ratio is less than 1, the index decreases. For instance, under rising temperatures, glacier meltwater increases, but precipitation and evaporation-seepage changes may be uncertain. Simultaneously, critical failure Newton' s force at the dam failure location may decrease due to buried ice and ice lens melting. Under such circumstances, we typically expect GLOF susceptibility index to increase. However, according to our model, more precise assessment of glacial lake water volume changes is required, achievable by monitoring water levels corresponding to assumed dam breach ranges. We also need to evaluate whether resistance at the dam failure location has decreased or remained unchanged. If water level at the dam failure location rises, we can essentially determine that the GLOF susceptibility index has increased. If evaporation and seepage are too strong, causing water level at the dam failure location to drop, then specific susceptibility index changes require further analysis. However, it can be preliminarily judged that in this case, susceptibility index increase may not be significant, and may even decrease.

For glacial lakes with only a single dangerous ice avalanche body, establishing an exclusive susceptibility index expression is crucial. With continuous global warming impact, instantaneous triggering body occurrence probability may increase. However, as the ice body continues melting, its volume and potential energy continuously decrease, with high potential energy gradually transforming into lake water’ s low potential energy. During this mechanical energy transformation, parameters involved in susceptibility index calculation also constantly adjust and change. Notably, during this process, a phenomenon may occur where the GLOF susceptibility index actually decreases. For example, in the past, ice avalanche bodies entering lakes could trigger outbursts, but when the dangerous ice body melts to the point where waves caused by its fall are insufficient to cause outburst, this empirically observed phenomenon contradicts intuitive expectations.

Simultaneously, as glaciers gradually melt, the distance between glacier rear edge and glacial lake increases, leading to greater mechanical energy consumption of dangerous ice avalanche bodies entering the lake and decreased mass of dangerous ice avalanche bodies. Meanwhile, glacial meltwater does not increase glacial lake water volume due to infiltration and evaporation. At this point, numerator decrease exceeds denominator decrease, providing evidence of GLOF susceptibility index decrease under global warming for some glacial lakes.

Previous studies have considered minimum glacial lake area thresholds (Nie et al., 2018; Veh et al., 2019). Glacial lakes below certain area thresholds are inevitably not considered by researchers and are usually ignored in remote sensing interpretation. However, small-area glacial lakes can potentially cause “small breaches leading to large disasters” under right conditions (Liu, M et al., 2020; Zhang, T et al., 2022). By establishing GLOF susceptibility index expressions, it is not difficult to discover that overlooked small-area glacial lakes may have higher susceptibility indices than large-area lakes, a phenomenon not captured in assessment models overly reliant on area and scale indicators. This provides clear signals to GLOF disaster prevention personnel: although small-area breaches may not be as severe as large-area breaches, when small-area glacial lakes have higher susceptibility indices compared to large-area lakes, small-area lakes should also receive due attention. Simultaneously, we find that regardless of glacial lake area size, each has its own susceptibility index expression. That is, regional GLOF susceptibility assessment based on this model does not need to consider minimum glacial lake area thresholds, further strengthening model universality.

Glacial lakes at lower water levels generally have lower susceptibility indices compared to those at higher water levels, aligning with subjective understanding and reflected in our constructed GLOF susceptibility index model. At lower water levels, without considering triggering body changes, mechanical energy contributed by lake water is relatively small, and breach location often shifts downward accordingly. Generally, lower dam sections are thicker, requiring greater Newton’ s force for failure. According to established susceptibility index expression analysis, numerator decreases and denominator increases, resulting in decreased susceptibility index. Simultaneously, wave-induced breach possibility is smaller because waves need to breach thicker sections to produce sustained overflow. Therefore, in this case, GLOF breach mode is mainly dominated by seepage/piping.

Similarly, without considering triggering body changes, as water level rises, GLOF susceptibility index tends to increase. When wave energy can easily breach the dam, breach mode should be wave-induced breach, seepage/piping dominated. As water level continues rising, susceptibility index continues increasing, and overflow scouring becomes a new dominant breach mode, at which point GLOF breach modes should be overflow scouring, wave-induced breach, seepage/piping dominated.

Through previous analysis, we understand that water level changes can cause GLOF susceptibility index changes and even determine breach mode, indicating water levels play a controlling role in GLOFs. When water level approaches zero, instantaneous triggering body entry (except floods/debris flows) will not cause GLOFs; when water level reaches maximum, glacial lakes can easily experience outbursts. Between these extremes, there must be a critical water level at which triggering body entry will not trigger outburst, referred to as the safe water level for glacial lakes. Similarly, when glacial lake water level remains stable within a certain range for many years, there is also a suitable instantaneous triggering body that will not trigger outburst when entering the lake. This triggering body is referred to as the safe instantaneous triggering body. These analyses provide new insights for engineering management: in addition to improving dam resistance, it is also possible to effectively regulate individual glacial lake outburst susceptibility indices by lowering water level to safe level, reinforcing triggering bodies, reducing triggering body mass, or simultaneously controlling water level, triggering bodies, and dam. This control essentially adjusts influencing factor parameter values to reduce GLOF susceptibility indices, thereby achieving ideal engineering management effects for glacial lakes, aligning with subjective understanding.

This model is entirely constructed based on objective physical processes. Although some parameters may be difficult to obtain precisely due to objective reasons, in certain cases, the model can accurately determine trends in individual glacial lake outburst susceptibility index changes. If this model is applied to multiple glacial lakes within a region and results are ranked, regional GLOF susceptibility assessment can be conducted, thereby constructing a GLOF susceptibility assessment model. For conclusions drawn from other evaluation methods, this model can serve as a verification tool under certain conditions.

7 Model Explanation

Since this model is constructed based on physical processes, its architecture coincides with safety factor concepts, potentially causing confusion with stability analysis. Therefore, it is necessary to clarify differences between this model and stability analysis models, as well as similarities with AHP models, to demonstrate that this is a susceptibility analysis model rather than a stability analysis model.

In the previous section, we divided the three main entities into attacking and defending parties, both existing within the same GLOF system. Therefore, attacking and defending parties hold equal importance but offset each other. In basic arithmetic operations, adding or multiplying quantities representing the two parties is clearly inappropriate, leaving subtraction and division as remaining options. When subtracting the attacking party quantity from the defending party quantity, we find large-scale glacial lakes generally have higher susceptibility indices than small-scale lakes, clearly contrary to common sense. This leaves division as the only viable option. Thus, we construct the model by dividing quantities representing the two parties, finding it meets initial expectations. This architectural form of dividing quantities is similar to safety factor form but fundamentally different. Safety factor is usually dimensionless, while the E/F calculation mode in this architecture produces a quantity with units. Since we are only interested in numerical values, we divide by 1m to eliminate units. If this architecture used an E/E calculation mode, it would directly produce a dimensionless number. In summary, this model does not require numerator and denominator quantities to have the same dimension, fundamentally differing from safety factor. Moreover, numerical values obtained from this model are much larger than typical safety factor ranges, and these values alone do not necessarily indicate problems; they only become meaningful when compared with values calculated for other glacial lakes (including those that have already burst) or with the model’ s own historical values. The reciprocal calculation mode of safety factor is a special case of susceptibility index calculation mode, where numerator and denominator must have the same dimension (E/E mode, F/F mode) and numerator must not include probability terms (such as instantaneous triggering body occurrence probability). Simultaneously, safety factor is generally defined as the ratio of load-carrying capacity considered during structure design to actual load requirements under most unfavorable load combinations. The most significant difference between safety factor and GLOF susceptibility index expression is that the former does not include probability terms while the latter is the ratio of load side to resistance side. This sufficiently explains that this model does not perform safety factor analysis (stability analysis).

As mentioned earlier, AHP’ s core lies in its hierarchical decision structure, importance assessment and quantification, and weight calculation and synthesis. Even with computational detail variations, retaining these core steps preserves the AHP spirit. The entire GLOF susceptibility index solution analysis model is also very similar in composition to AHP. When applying AHP to this model, the objective layer is GLOF susceptibility, the criterion layer consists of values for three major indicators (mechanical energy contributed by triggering body to dam failure location, mechanical energy contributed by lake water to failure location, and resistance at failure location), and the alternative layer consists of susceptibility indices for each glacial lake awaiting assessment. This satisfies the hierarchical decision structure. We find that mechanical energy contributed by triggering body and lake water belong to the same category, both representing attacking party quantities with consistent units, so they can be directly added after separate calculation. Thus, there are only two indicator types: attacking party representative quantity (potential energy) and defending party representative quantity (critical failure Newton’ s force). After classifying various GLOF susceptibility index influencing factors based on physical processes and quantifying the three major indicators, each indicator’ s valuation is completed, meaning this model does not need consistency tests. When constructing pairwise comparison matrices for criterion and alternative layers, especially since the two indicator types belong to different categories, attacking and defending party representative quantities should be separated when constructing criterion layer matrices, with judgment matrices constructed separately. More specially, after solving each assessed glacial lake indicator’ s valuation, they all exist objectively and independently without subjective judgment. Therefore, each assessed glacial lake can have its own judgment matrix (the purpose of criterion layer judgment matrix is to measure each indicator’ s relative importance and assign corresponding weights), eliminating the need to construct a single judgment matrix followed by all GLOF susceptibility index calculations through subjective experience. Ultimately, the criterion layer pairwise comparison matrix consists of two 1$×$1 matrices. Alternative layer judgment matrix construction is similar to ordinary AHP. In this way, criterion and alternative layer weights can be determined. The two indicator types are classified based on physical processes and follow physical relationships (E/F calculation mode), which serves the objective layer problem, so this calculation mode must be followed when calculating and synthesizing weights.

Simultaneously, each indicator’ s impact on GLOF susceptibility index can be comprehensively reflected in the calculation expression. Surprisingly, calculation results obtained by directly substituting each indicator’ s valuation into the calculation mode and results obtained through normal AHP mode maintain a multiple relationship, with order unchanged, realized through hypothetical data.

Hypothetical Data: Glacial Lake 1 has mechanical energy of 8 joules (J) and resistance of 2 newtons (N); Glacial Lake 2 has mechanical energy of 12 joules (J) and resistance of 6 newtons (N); Glacial Lake 3 has mechanical energy of 15 joules (J) and resistance of 3 newtons (N).

1. **Direct solution by substituting into calculation model:**
    - Glacial Lake 1: GLOFSI = $(8/2) \div 1 = 4$
    - Glacial Lake 2: GLOFSI = $(12/6) \div 1 = 2$
    - Glacial Lake 3: GLOFSI = $(15/3) \div 1 = 5$
2. **AHP solution process:**
    - (1) Construct criterion-level mechanical energy pairwise comparison matrix
    - (2) Construct criterion-level resistance force pairwise comparison matrix
    - (3) Construct alternative-level mechanical energy pairwise comparison matrix
    - (4) Construct alternative-level resistance force pairwise comparison

matrix
- (5) Calculate and synthesize weights

The results show that direct indicator value substitution and normal AHP pairwise comparison matrix construction produce results in a multiple relationship. This multiple always equals the sum of each glacial lake' s mechanical energy divided by the sum of dam resistance forces. For calculation simplicity, when constructing criterion-level pairwise comparison matrices, usually only a single glacial lake' s matrix needs construction.

From this we conclude that the model can directly omit criterion and scheme layer comparison matrix construction steps, without needing consistency checks. Finally, calculation results for burst and assessed glacial lakes can be compared and sorted, and can also be compared with each lake' s own historical susceptibility indices. After comparison with other simultaneously assessed glacial lakes, intervals can be divided for susceptibility assessment, and GLOF susceptibility index changes can be analyzed by comparing with previous indices. Theoretically, all GLOF samples can serve as a historical database, and the GLOF susceptibility assessment model will no longer be regionally restricted, further strengthening model universality. This sufficiently illustrates that this is a model that naturally embeds physical processes into AHP without considering subjective experience, and is also a susceptibility assessment model.

8 Construction Method of the Analytic Hierarchy Process Based on Physical Processes

The GLOF susceptibility assessment model construction, from inception to results, appears very natural and completely uninvolved with subjective experience. For now, let' s call this method for constructing GLOF susceptibility assessment models based on physical processes the Analytic Hierarchy Process Based on Physical Processes (AHPBPP). The GLOF susceptibility assessment model can be considered a typical AHPBPP application, so it is necessary to summarize AHPBPP commonalities from the application model. Therefore, I summarize and refine the GLOF susceptibility assessment model construction process to further introduce AHPBPP.

The first AHPBPP construction step is clearly defining our objective layer, then deeply understanding physical processes involved in the objective layer problem (GLOF susceptibility). Next, classify factors affecting the objective layer problem to establish the criterion layer (values of mechanical energy contributed by triggering body to dam failure location, values of mechanical energy contributed by lake water to failure location, and values affecting resistance at failure location), clarify physical relationships among categorized classes serving the objective layer problem (attacker and defender), and establish physical relationships (the E/F calculation mode in the GLOF susceptibility model). It is essential to find the flow quantity (quantify each indicator with flow quantity; in the GLOF susceptibility model, energy is the flow quantity). Then establish the alternative layer (susceptibility indices of each glacial lake).

Next, construct criterion and alternative layer pairwise comparison matrices, noting that numerator and denominator criteria matrices need separate construction. Each entity awaiting assessment will have its own pairwise comparison matrix. Since numerator items have consistent units, they can generally be combined into one item, as can denominator items, so the criterion layer pairwise comparison matrix consists of two 1\$×1*matrices.This*1×\$1 matrix is essentially self-comparison. Alternative layer pairwise comparison matrix construction is consistent with standard AHP.

Finally, synthesize and calculate weights based on physical relationships. According to objective layer problem needs, rank, zone, or compare these results with previous susceptibility indices of assessed entities. It is found that when calculation results obtained by directly substituting each indicator’ s valuation into calculation mode have a multiple relationship with results obtained by normally constructing pairwise comparison matrices (realized by assuming multiple data sets), the model is essentially constructed. Thus, AHPBPP is constructed.

AHPBPP is a method for solving complex scientific problems, requiring that the problem at the objective layer can be fully summarized by indicators, and these indicators can only be divided into two categories—those written in numerator and denominator. It does not require that numerator and denominator dimensions can cancel each other, but requires that when all indicators are collected in the fraction, the fraction’ s numerical value can exactly describe the problem to be solved, such as GLOF susceptibility index.

8.1 Landslide Susceptibility Analysis Model Based on the Analytic Hierarchy Process of Physical Processes

The GLOF susceptibility model based on the Analytic Hierarchy Process of Physical Processes is a typical application of this method. Through previous model construction and analysis, as well as method refinement, it is not difficult to discover that the Analytic Hierarchy Process of Physical Processes, as a method, should have a certain degree of generalization capability. Therefore, this paper attempts to establish a landslide susceptibility assessment model based on the Analytic Hierarchy Process of Physical Processes.

In this context, the objective layer is landslide susceptibility; the criterion layer consists of slope body potential energy at the foot, additional instantaneous mechanical energy, and slope anti-sliding force; the alternative layer is each slope body’ s susceptibility index. When clarifying physical relationships among criterion layer indicators, they still follow the E/F calculation mode, as shown in equation (10). Then construct criterion and alternative layer pairwise comparison matrices. After merging numerator parts, the criterion layer pairwise comparison matrix remains a 1\$×\$1 matrix. The alternative layer pairwise comparison matrix is constructed according to standard AHP. Results obtained by directly substituting indicator values into calculation mode and those obtained through normal AHP still have a multiple relationship, realized by assuming multiple data sets. Obtain each landslide' s susceptibility index (including those that have occurred), then compare, rank, and divide these results into high and low susceptibility intervals; or compare these results with their previous indices for analysis.

$$\mathrm{LSI} = \frac{G + \psi \cdot E_f}{F} \div 1\mathrm{m}$$

In the formula, LSI refers to landslide susceptibility index. We are only concerned with the specific numerical value of this ratio, so it is divided by 1 meter to eliminate units and obtain a dimensionless number; G represents slope body potential energy at the foot; Ψ is the probability coefficient of instantaneous additional mechanical energy taking effect, with maximum value 1 and minimum value 0; Ef is instantaneous additional mechanical energy; F is slope body anti-sliding force; energy units are joules, and force units are newtons.

Landslide susceptibility assessment model construction demonstrates that the Analytic Hierarchy Process Based on Physical Processes possesses generalization capability. It also builds a bridge between stability analysis and susceptibility analysis. We often believe that more unstable landslides in a region indicate higher regional landslide susceptibility. This issue is partially explained here. When probability term coefficient Ψ is determined to be 0 or 1 and E/E or F/F calculation modes are employed, more unstable landslides yield greater susceptibility indices from the calculation formula. If a region has more unstable landslides, then more landslides in the region have increased susceptibility indices, implying higher regional landslide susceptibility. Models using E/E, F/F, M/M, and similar patterns still belong to this category. The GLOF susceptibility assessment model is similar; this model will not be restricted by regional limitations.

9 The Law of Relative Quantities with Uncancelled Dimensions

From summarizing the Analytic Hierarchy Process Based on Physical Processes and constructing GLOF and landslide susceptibility models, we discovered that numerical results obtained from direct comparisons between different-dimension data and values obtained through constructing pairwise comparison matrices and synthesizing weights are in a multiple relationship, with this multiple equal to the sum of denominator values divided by the sum of numerator values.

The process of constructing pairwise comparison matrices may seem meaningless but is an extremely important dimensionless process. Later, in the weight determination phase, dimensionless weights are synthesized to ultimately obtain a dimensionless number. This dimensionless number is in a constant multiple relationship with numerical results obtained by directly comparing different-dimension data. This seems to be a mathematical equation but does not satisfy the principle of dimensional consistency. We have discovered this pattern, which can greatly simplify future calculations. It is possible to directly take the ratio of original different-dimension data to obtain results, or divide by the multiple relationship to get desired values. However, the ratio of different-dimension data results in a dimensionally consistent outcome. This can be proven under the principle of dimensional consistency, hence it is named the Law of Relative Quantities with Uncancelled Dimensions.

The Law of Relative Quantities with Uncancelled Dimensions states: When there are two or more data sets that only have two types of dimensions, and each data set’ s dimension form is consistent, and the ratio change of the two data’ s numerical values can reflect some target value’ s change, then the ratio of these data sets’ target values can be solved without considering dimensions, and the ratio of each data set’ s numerical results can be directly used as the target values’ ratio.

It may seem that directly using each data set’ s numerical result ratio as target value ratio without considering dimensions violates the principle of dimensional consistency. However, this law is deduced under the principle of dimensional consistency. Data sets with the same dimensions can not only compare internally to get dimensionless numbers but also compare with other sets to get dimensionless numbers. Data sets with uncancelled dimensions can only compare with other data sets of the same form to get dimensionless numbers. Compared with dimensionless numbers obtained by comparing same-dimension data sets, dimensionless numbers obtained by uncancelled-dimension data sets carry less information. For example, in landslide prediction analysis, if the F/F model is used and a probability term is added to the numerator, a landslide susceptibility index can be obtained. This index can be compared with unit 1; if the landslide susceptibility index exceeds 1, the landslide is dangerous and may slide. Simultaneously, this index can also be compared with other indices, with larger ones being more dangerous. If the E/F model is used in landslide prediction analysis, the dimensionless number’ s size itself is meaningless, only becoming meaningful when compared with other groups’ dimensionless numbers. If a new uncancelled-dimension data set is added, to allow the new set to participate in comparison, all previous comparisons must be redone, requiring reconstruction of pairwise comparison matrices and re-synthesis of weights, making calculations somewhat complex. If the Law of Relative Quantities with Uncancelled Dimensions is used, results can be directly obtained, eliminating complex calculation processes.

9.1 Proof of the Law of Relative Quantities with Uncancelled Dimensions

Assume there are multiple data sets E_1/F_1, E_2/F_2, E_3/F_3, E_4/F_4, ⋯, where E units are consistent and F units are consistent. It is stipulated that E and F can both represent a pair of opposing quantities, such as destructive force and resistance in landslides. For instance, E represents energy and F represents

Newtonian force. Landslide susceptibility index is jointly controlled by destructive force and resistance; actual destructive force and resistance values govern susceptibility index magnitude and variation. When all data sets are in this form, susceptibility index relative size is independent of units. Susceptibility is a relative concept, indicating the relative likelihood of an individual unit exhibiting certain signs within a group. When using E to represent destructive force and F to represent resistance, one must clarify each individual' s susceptibility index relative magnitude within a group. At this point, each E is compared with others to obtain E' s relative value within the group—a dimensionless number; each F is compared with others to obtain F' s relative value within the group. By comparing an individual' s E relative value with its F relative value, we obtain the individual' s relative susceptibility index within the group. The ratio of each individual' s relative susceptibility index is the final susceptibility, which can then be ranked. An individual' s relative susceptibility index will change with group size increase or decrease. This change can cause computational troubles in solving practical problems, and errors are prone when data amount increases. The Law of Relative Quantities with Uncancelled Dimensions effectively addresses this issue. Here, since there is only one numerator and one denominator, numerator and denominator weights when compared to themselves are both 1. This can also be disregarded in the future.

1. **Construct numerator E pairwise comparison matrix:**

$$\begin{bmatrix} 1 & E_2/E_1 & E_3/E_1 & E_4/E_1 \\ E_1/E_2 & 1 & E_3/E_2 & E_4/E_2 \\ E_1/E_3 & E_2/E_3 & 1 & E_4/E_3 \\ E_1/E_4 & E_2/E_4 & E_3/E_4 & 1 \end{bmatrix}$$

2. **Construct denominator F pairwise comparison matrix:**

$$\begin{bmatrix} 1 & F_2/F_1 & F_3/F_1 & F_4/F_1 \\ F_1/F_2 & 1 & F_3/F_2 & F_4/F_2 \\ F_1/F_3 & F_2/F_3 & 1 & F_4/F_3 \\ F_1/F_4 & F_2/F_4 & F_3/F_4 & 1 \end{bmatrix}$$

3. **Calculation and synthesis of weights**

The relative weight of E_1: $\frac{E_1}{E_1+E_2+E_3+E_4}$

The relative weight of F_1: $\frac{F_1}{F_1+F_2+F_3+F_4}$

The relative susceptibility index of E_1/F_1: $\frac{E_1/(E_1+E_2+E_3+E_4)}{F_1/(F_1+F_2+F_3+F_4)} = \frac{E_1}{F_1} \times \frac{F_1+F_2+F_3+F_4}{E_1+E_2+E_3+E_4}$

It is not difficult to discover that, when considering only numerical values without regard to units, the value of E/F is $\frac{\sum F}{\sum E}$ times the relative susceptibility index of E/F.

If we let the value considering only numerical value without unit be A/B, and the corresponding relative susceptibility index be a/b, with a total of n data sets, then we have:

$$\frac{A/B}{a/b} = \frac{\sum F}{\sum E}$$

The Law of Relative Quantities with Uncancelled Dimensions is proven. Of course, relative quantities with cancelled dimensions also satisfy this theorem, but generally, when dimensions are cancelled, a dimensionless number can be directly obtained without constructing pairwise comparison matrices. Relative quantities with cancelled dimensions are more common in practical applications, while relative quantities with uncancelled dimensions are rarely encountered.

In summary, the Law of Relative Quantities with Uncancelled Dimensions brings three benefits: First, it provides theoretical basis for models such as the E/F model in susceptibility issues, making seemingly impossible models possible and inspiring similar problems in other fields. Second, it greatly simplifies calculations. For example, without using this law, calculating 100 landslide susceptibility indices involves significant computation, including calculating numerator and denominator relative weights separately, then synthesizing them to obtain 100 landslide susceptibility index ratios. If the law is adopted, calculations can directly ignore dimensions and consider only numerical values, then obtain and sort ratios. There is also a worse-case scenario: if 101 landslides exist total and one is missed, susceptibility indices for the original 100 landslides calculated without the law would require recalculation due to the omission, increasing computational load. If the law was adopted from the beginning, results could be directly compared with previous 100 landslides without considering dimensions, only numerical values. Third, numerical values obtained by directly ignoring dimensions have the same functionality as those obtained by considering dimensions, and are even more convenient. The Law of Relative Quantities with Uncancelled Dimensions broadens data comparison scope in applicable problems.

9.2 Some Inferences of Equivalent Quantity Substitution

Based on landslide susceptibility model construction, GLOF susceptibility model construction, and dimensional consistency theorem proof, we find that susceptibility can be represented by F/F, E/F, or F/E models because both energy (E) and Newtonian force (F) can serve as destructive force and resistance indicators. Therefore, E and F are equivalent quantities.

Both E and F can quantify destructive force and resistance, and their values measure destructive force and resistance. In susceptibility calculation, any E-F combination can be used.

The deeper logic behind this phenomenon leads to three properties: (1) **Invariance of Functional Attributes of the Target Quantity:** Under dimensional consistency principle satisfaction, if any parameter in target quantity expression is replaced by an equivalent quantity, the target quantity's functional attributes in the problem remain unchanged. (2) **Equivalence Quantity Discrimination Based on Physical Quantity Attributes:** If physical quantities are equivalent, in addition to having target quantity-required common attributes, each physical quantity' s remaining attributes must not interfere with target quantity solution. (3) **Reduction of Equivalent Quantity Options:** The more quantities associated with the replaced quantity through multiplication or division, the fewer equivalent quantity options for that replaced quantity in the target problem. If multiple quantities are associated with the replaced quantity through addition or subtraction, once the replaced quantity is substituted by an equivalent quantity, quantities associated through addition or subtraction must also be replaced. Moreover, as addition/subtraction-associated quantity numbers increase, quantities needing equivalent quantity replacement also increase.

9.3 Explanation of E and F as Equivalent Quantities in Susceptibility Problems

In mathematics, if E and F are equivalent quantities, then E = kF, meaning E and F have proportional relationship. However, in specific problems, even if E and F lack proportional relationship, they can still be equivalent quantities. In linear algebra, if two quantities are equivalent, they are linearly related. In specific problems, not all attributes of two quantities are involved in solution, only some attributes are involved. If several quantities in the target problem all have attributes required to represent a certain quantity, then these quantities are equivalent representation quantities.

Although they all have represented quantity attributes, they need to meet certain relationships to replace each other—that is, to meet equivalence quantity discrimination rules.

Susceptibility index = destructive force / resistance. Both E and F can serve as destructive force and resistance indicators. The commonly used model is the model with same dimension for destructive force and resistance, such as F/F model. Now, to switch to E/F model, it is necessary to prove that E and F are equivalent quantities in susceptibility index solution problems.

The susceptibility index itself does not mean much; it only makes sense when compared with other susceptibility indices and finds its position in the value group. E has numerical measurement and destructive force representation attributes, and F in F/F model also plays numerical measurement and destructive force representation roles. When switching to E, E can only play these two roles, and remaining attributes will not affect target quantity solution. When sorting susceptibility indices, E only compares with other E to get destructive force relative value. It can also be considered that when E units are the same, only E numerical value plays a role, and unit is irrelevant. Whether using E or F, the assessed landslide is still that landslide at that moment, and obtained susceptibility index is different with different dimensions. However, they can both represent the landslide’ s position in this dimension and obtain corresponding ranking. Thus, the assessed individual’ s danger level can be judged.

For a landslide among a group of landslides, susceptibility can be described by architectures such as E/E, E/F, F/E, and F/F. Let R represent landslide susceptibility ranking in this landslide group. Then we have R(E/E) = R(E/F) = R(F/E) = R(F/F), meaning regardless of susceptibility expression form used, landslide susceptibility ranking remains unchanged.

10 Conclusion

This paper proposes a theoretical model for glacial outburst susceptibility assessment, introduces the Analytic Hierarchy Process Based on Physical Processes, applies this method in landslide susceptibility assessment model construction, and further proposes the Law of Relative Quantities with Uncancelled Dimensions.

While the model is novel, it inherently possesses complexity, especially in accurately obtaining numerous parameters. Pursuing precise parameter estimation is not just a scientific endeavor but a crucial step in enhancing model predictive power and practical application value, representing an important future development area for this model. Model construction is similar in architecture to safety factor assessment, aiming to reflect comparison between attacking forces (triggering events and lake water mechanical energy) and defensive forces (dam resistance). This comparison is not just theoretical construct but the key to systematic assessment and precise approximation of susceptibility index. The model clarifies the relationship between susceptibility and stability.

However, model development faces challenges. Difficulty in accurately determining certain parameters, such as exact dam failure core location, demonstrates necessity for continuous improvement. Nonetheless, model application is not regionally limited and involves no subjective experience from beginning to end, emphasizing robustness and universal applicability. The Analytic Hierarchy Process Based on Physical Processes is expected to be applied in other fields in the future. The Law of Relative Quantities with Uncancelled Dimensions broadens data comparison scope and greatly simplifies calculations in solving practical problems.

Note: The model is in its initial stage and requires rigorous validation and iterative enhancement. We extend an open invitation to scholars and practitioners to contribute insights and suggestions, with the shared goal of improving model accuracy and broadening its practical relevance.

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Note: Figure translations are in progress. See original paper for figures.