Centroiding Algorithm for the Dwarf Planet Haumea System (Postprint)

Authors:

Date: 2024-06-03T00:00:00+00:00

Abstract

Atmospheric effects frequently render binary star systems or primary-satellite systems with small angular separations unresolvable when observed with ground-based optical telescopes, potentially introducing systematic offsets between the system’ s photocenter and barycenter positions. Precise determination of barycenter positions for solar system objects is crucial for refining orbital parameters and advancing our understanding of solar system formation and evolution. Using the dwarf planet Haumea and its bright satellite Hi’ iaka as a case study, we simulate the photocenter’ s perturbation about the system barycenter and investigate how seeing variations—characterized by the full width at half maximum (FWHM) of stellar images—affect accurate photocenter measurement. Our simulations demonstrate that photocenter positions derived from a two-dimensional Gaussian centering algorithm are dependent on seeing conditions, whereas results from a modified moment centering algorithm remain unaffected. These findings highlight the necessity of developing centering algorithms that can effectively mitigate seeing-induced errors; furthermore, such algorithms must also account for brightness variations of the primary body. We observed the Haumea system from February 7 to May 25, 2022, using the 2.4;m and 1;m telescopes at Yunnan Observatories and the 0.8;m telescope at Yaoan Station of Purple Mountain Observatory, acquiring 463 CCD images over 29 nights. The photocenter positions determined by our new algorithm exhibit superior fitting quality compared to those obtained with the two-dimensional Gaussian method. Additionally, we identify significant positional discrepancies for Hi’ iaka between the Jet Propulsion Laboratory (JPL) and Institut de M'{e}canique C'{e}leste

Full Text

Research on the Centering Algorithm for the Dwarf Planet Haumea System

HAO Jia-nan[12], **PENG Qing-yu**[12], **GUO Bi-feng**[12], **CAO Jiang-long**[12], **HONG Yao-yao**[12]

[1] Department of Computer Science, Jinan University, Guangzhou 510632

[2] Sino-French Joint Laboratory for Astrometry, Dynamics and Space Science, Jinan University, Guangzhou 510632

Abstract

Ground-based optical telescope observations of binary or primary-satellite systems with small angular separations are often affected by Earth' s atmosphere, resulting in unresolved stellar images. Consequently, deviations may exist between the system' s photocenter position and its center of mass. Accurate measurement of the center of mass for solar system objects can improve their orbital parameters and help reveal the formation and evolution of the solar system. Taking the dwarf planet Haumea and its bright satellite Hi' iaka as examples, we simulate the perturbation of the system photocenter around the center of mass and investigate how variations in seeing (represented by the Full Width at Half Maximum of the stellar image) affect accurate photocenter position measurement. Simulation results demonstrate that photocenter positions determined using the two-dimensional Gaussian centering algorithm vary with seeing conditions, whereas results from the modified moment centering algorithm remain unaffected by seeing. Based on these findings, it is necessary to develop centering algorithms that effectively reduce the impact of seeing variations on accurate photocenter measurement; such algorithms must also account for the influence of primary star luminosity variations. We observed the Haumea system from February 7 to May 25, 2022, using the 2.4 m telescope, 1 m telescope, and the 0.8 m telescope at the Yaoan Station of Purple Mountain Observatory, obtaining 463 CCD images over 29 nights. The photocenter positions determined by our new centering algorithm show better positional fitting compared to those obtained using the two-dimensional Gaussian centering algorithm. Additionally, we discovered significant positional deviations for the bright satellite Hi' iaka between the Jet Propulsion Laboratory (JPL) ephemeris and the Institut de Mécanique Céleste et de Calcul des Éphémérides (IMCCE) ephemeris.

Keywords: astrometry, techniques: image processing, methods: data analysis, Kuiper belt objects: individual: Haumea

1 Introduction

The Kuiper Belt, located beyond Neptune' s orbit in the solar system, has a vast structure whose study is crucial for revealing the formation and evolution of the solar system. Many Kuiper Belt objects have companions or satellites (hereinafter referred to as primary-satellite systems). However, when observed with ground-based optical telescopes, they often appear as single blended images on CCD frames. This phenomenon occurs in primary-satellite systems such as the dwarf planets Pluto, Makemake, and Haumea, all certified by the International Astronomical Union (IAU). When resolution is insufficient, what is observed is the system's photocenter position. When the mass ratio and luminosity ratio between the primary and satellite differ significantly, the photocenter and center of mass will not coincide. In 1970, Morel demonstrated that the photocenter orbits the center of mass in a Keplerian ellipse. In 2011, Ortiz et al. discovered periodic perturbations in the photocenter position—known as wobble—during 33 days of ground-based observations of the large Kuiper Belt object Orcus (which has a satellite Vanth). Accurate photocenter motion trajectories obtained through long-term high-precision astrometry are commonly used in binary systems for orbital solution, mass calculation, and derivation of the actual center of mass position; this method is equally applicable to primary-satellite systems.

Accurate measurement of the system photocenter position depends on numerous factors. According to van de Kamp, the photocenter is theoretically the luminosity-weighted center of the primary and satellite. The angular distance from the photocenter to the primary, r_L, depends only on the magnitude difference and angular separation between the primary and satellite. However, ground-based observations may be affected by atmospheric seeing variations. This paper uses computer simulation to model the motion of primary-satellite systems and investigates in detail how photocenter position measurement is influenced by seeing variations and corresponding correction methods. We selected the dwarf planet Haumea and its bright satellite Hi' iaka as our research target. Since Haumea' s other faint satellite Namaka has an apparent magnitude of approximately 21.8 mag, exceeding the imaging threshold of the CCDs used in this study within moderate exposure times, we ignore its positional measurement impact.

The Haumea system is the largest known member of the Kuiper Belt collisional family. Therefore, the primary Haumea and satellite Hi' iaka should have similar physical properties, supported by the fact that their surfaces are covered with extensive water ice. According to the latest photometric results, the average magnitude difference between Haumea and Hi' iaka is 2.76 mag (J and H bands). The Haumea system has an albedo value close to 1 and belongs to a collisional family, indicating similar colors between the primary and satellite (in visible and near-infrared bands). Thus, their magnitude difference remains relatively constant. We adopt an average magnitude difference of 2.76 mag (corresponding to a flux ratio of 12.71) for our experiments. Satellite Hi' iaka orbits the primary with a period of (49.12 ± 0.03) d, with angular separation varying between 0.44" and 1.45" (from JPL ephemeris and IMCCE ephemeris). The mass ratio between the primary and bright satellite Hi' iaka is 223.80 ± 13.75. Additionally, dwarf planet Haumea exhibits a pronounced bimodal rotational light curve; the influence of primary star luminosity variations on the system photocenter must also be considered.

The remainder of this paper is organized as follows. Section 2 describes the motion laws of primary-satellite systems and proposes a new centering algorithm for photocenter correction based on simulation results. Section 3 presents the observational data and reduction process for the Haumea system, yielding corrected photocenter position measurements. Section 4 analyzes and discusses the correction effectiveness and positional differences of bright satellite Hi' iaka between JPL and IMCCE ephemerides. The final section provides conclusions and future prospects.

2.1 Overview of Primary-Satellite Systems

[Figure 1: see original paper] illustrates the motion of a primary-satellite system. Assuming the system' s center of mass (CM) remains fixed, both the primary P and satellite S orbit the center of mass in the same direction along Keplerian ellipses, while the photocenter CL (star symbol) also orbits the center of mass CM (circle) in the same elliptical motion. In the figure, the angular distances from the photocenter and center of mass to the primary are denoted as r_L and r_M, respectively; the angular distance between the photocenter and center of mass is Δ; and the angular separation between primary and satellite is a. According to the literature, the photocenter position is determined by the combined luminosity of the primary and satellite, always lying on the line connecting them and closer to the more luminous primary (consistent with the lever principle). Therefore, as shown in Equation (1), r_L is proportional to a, where L_P and L_S represent the luminosities of the primary and satellite, respectively.

The center of mass follows the same principle, and the expression for Δ is given below, where M_P and M_S represent the masses of the primary and satellite, respectively:

$$\Delta = r_L - r_M = \left(\frac{1}{1+L_P} - \frac{1}{1+M_P}\right) a = ka$$

According to the literature, we use coefficient k to represent $\frac{1}{1+L_P} - \frac{1}{1+M_P}$. When coefficient k is constant, the photocenter trajectory (smaller ellipse) has the same shape as the satellite' s orbital trajectory around the primary (larger ellipse), with proportional sizes. From ground-based observations, only the system photocenter CL is visually detectable and practically measurable; thus, [Figure 1: see original paper] also explains why observed photocenter positions exhibit periodic fluctuations during long-term observations of certain targets.

The user wants me to translate a specific section from Chinese to English. The section is "### 2.2 Investigation of Photocenter-Seeing Relationship" . Let me analyze the requirements:

1. **Critical Requirements:**
   - Preserve ALL math exactly as given
   - Preserve ALL citations exactly

- Preserve ALL FIGURE/TABLE markers exactly
- Maintain academic tone, precision, and technical accuracy

2. **Translation Guidelines:**
   - Produce flowing, readable prose
   - Merge fragmented text into complete sentences
   - Skip garbage text
   - Organize by logical sections
   - Each paragraph should express a complete thought (3-8 sentences)
   - Use clear topic sentences and logical transitions
3. **Latex Formatting Rules:**
   - LaTeX commands must be inside math mode
   - Use ... for inline math,

     ...

     for display math
   - Use proper brace grouping
   - Use full Greek commands
   - Use markdown lists, not LaTeX
4. **Specific Issues Detected:**
   - Chinese characters: 律, 规
   - These appear to be mixed technical content and proper nouns
5. **Translation Rules for This Section:**
   - Author names: Romanize using pinyin
   - Journal names: Use English title if available, otherwise translate meaning
   - Institution names: Use official English names when known
   - Technical terms: Use standard English equivalents
   - DO NOT leave any Chinese characters
6. **Output Format:**
   - Return ONLY the translated section with the same structure
   - Use ## for major headings, ### for subsections
   - Separate paragraphs with blank lines
   - Preserve math inline or in display blocks

Now let me translate the section:

2.2 Investigation of Photocenter-Seeing Relationship

This study follows the simulation method of Ortiz et al., using the photutils library to simulate the motion of the Haumea system and investigate the variation 规律 of its photocenter trajectory.

Let me break this down: - "This study follows the simulation method of Ortiz et al." - This is straightforward - "using the photutils library" - Keep as is, it' s a library name - "to simulate the motion of the Haumea system" - Keep Haumea as is (proper noun) - "and investigate the variation 规律 of its photocenter trajectory" - Here "规律" means "pattern" or "regularity" or "behavior"

So: "This study follows the simulation method of Ortiz et al., using the photutils library to simulate the motion of the Haumea system and investigate the variation pattern of its photocenter trajectory."

Next sentence: "The simulation function requires parameters including the central positions of the primary and satellite, atmospheric seeing, and peak flux values to generate two-dimensional Gaussian point source images."

This is good as is.

Next: "To control for a single variable, this experiment temporarily excludes Poisson noise."

Good.

Next: "The primary is placed at the image center, and the relative position deviations of bright satellite Hi' iaka with respect to the primary are obtained from the IMCCE ephemeris at 1-hour intervals from February 7 to March 30, 2022 (approximately 52 days, about one orbital period)."

Good. Hi' iaka is a proper name, keep it. IMCCE is an institution, keep as is.

Next: "These are converted to pixel coordinates using the CCD scale of $0.348''$ pixel^{-1} from the 0.8 m telescope at Yaoan Station."

Good. Yaoan Station should be romanized.

Next: "According to the literature, when the stellar image FWHM exceeds 2 pixels, the image is not undersampled."

Good.

Next: "Therefore, in this simulation, undersampling does not occur when seeing is greater than $0.7''$."

Good.

Next paragraph: "Under seeing conditions of $1.0''$, $1.5''$, $2.0''$, $2.5''$, and $3.0''$, the fluxes of the primary and satellite are set according to their average magnitude difference of 2.76 mag to simulate the motion of bright satellite Hi' iaka around the primary."

Good.

Next: "The photocenter positions are obtained by fitting the blended images with a two-dimensional Gaussian function (GS)."

Good.

Next: "The results are shown in Figure 2: see original paper."

Good.

Next: "Larger seeing values (represented by FWHM) produce photocenter trajectories that more closely approximate an ellipse, but their size remains smaller than the true photocenter trajectory (pink)."

Good.

Next: "Moreover, when seeing is less than 2.0″, the trajectory shows noticeable central symmetric concavity, which contradicts the photocenter trajectory 规律 described in Section 2.1."

Here "规律" again means "pattern" or "behavior". So: "which contradicts the photocenter trajectory pattern described in Section 2.1."

Next: "This phenomenon may be related to the centering algorithm used."

Good.

Next: "When the satellite is at its maximum separation from the primary, the blended image becomes irregularly shaped, with more irregularity at smaller seeing values."

Good.

Next: "Consequently, the two-dimensional Gaussian centering algorithm is subject to certain limitations."

Good.

Next paragraph: "To verify this hypothesis, we also employed the modified moment algorithm (MM) to determine the photocenter."

Good.

Next: "The modified moment algorithm determines the object' s centroid by calculating first-order moments, and the object' s luminosity centroid corresponds to the photocenter definition."

Good.

Next: "However, it is most sensitive to positional errors caused by stellar images near the fitting region boundary."

Good.

Next: "Therefore, we implemented two iterations of the modified moment algorithm, using the initial centering result as input for the second iteration to ensure the stellar image remains centered in the fitting region."

Good.

Next: "Repeating the above process with the modified moment algorithm yields the photocenter trajectory simulation results shown in Figure 2: see original paper."

Good.

Next paragraph: "Clearly, the photocenter trajectories under different seeing conditions are identical and coincide with the true photocenter trajectory (pink)."

Good.

Next: "This indicates that modified moment centering results are unaffected by seeing variations and accurately measure the photocenter position."

Good.

Next: "In the modified moment algorithm expression, the centroid position is proportional to the first power of the distance from pixels in the fitting region to the centroid."

Good.

Next: "This linear relationship ensures that the centroid position remains unchanged when the stellar image' s FWHM varies (i.e., when seeing changes)."

Good.

Next paragraph: "We now conduct quantitative analysis of the photocenter positions determined by these two methods."

Good.

Next: "Each centering method simulated five different seeing values across one satellite orbital period (1248 photocenter positions per period)."

Good.

Next: "The angular distance from the photocenter to the primary, r_L, is calculated as the vertical coordinate, and the angular separation between primary and satellite, a, as the horizontal coordinate, as shown in [Figure 3: see original paper]."

Good.

Next: "Under modified moment centering (MM), the relationship between r_L and a is a consistent straight line across all seeing values, with measured r_L values (MM) overlapping the true values (real) calculated from Equation (1) in Section 2.1."

Good.

Next: "Under Gaussian centering (GS), the relationship between r_L and a exhibits curves with different slopes across various seeing values, indicating that the centering results are significantly affected by atmospheric seeing variations."

Good.

Next: "The discrepancy between measured and true r_L values increases with greater satellite-primary separation and smaller seeing values."

Good.

Next: "In this simulation, the maximum difference between measured and true r_L values under Gaussian centering (GS) is approximately 0.284 pixel (0.099″), with seeing variations causing a maximum impact of approximately 0.214 pixel (0.074″)."

Good.

Next: "Based on both qualitative and quantitative analysis, the modified moment method is more suitable than the two-dimensional Gaussian method for measuring photocenter positions in primary-satellite systems under noise-free conditions."

Good.

Now I need to check for any Chinese characters I might have missed. Let me scan the original text again:

"### 2.2 Investigation of Photocenter-Seeing Relationship

This study follows the simulation method of Ortiz et al., using the photutils library to simulate the motion of the Haumea system and investigate the variation 规律 of its photocenter trajectory. The simulation function requires parameters including the central positions of the primary and satellite, atmospheric seeing, and peak flux values to generate two-dimensional Gaussian point source images. To control for a single variable, this experiment temporarily excludes Poisson noise. The primary is placed at the image center, and the relative position deviations of bright satellite Hi' iaka with respect to the primary are obtained from the IMCCE ephemeris at 1-hour intervals from February 7 to March 30, 2022 (approximately 52 days, about one orbital period). These are converted to pixel coordinates using the CCD scale of 0.348″ pixel^{-1} from the 0.8 m telescope at Yaoan Station. According to the literature, when the stellar image FWHM exceeds 2 pixels, the image is not undersampled. Therefore, in this simulation, undersampling does not occur when seeing is greater than 0.7″.

Under seeing conditions of 1.0″, 1.5″, 2.0″, 2.5″, and 3.0″, the fluxes of the primary and satellite are set according to their average magnitude difference of 2.76 mag to simulate the motion of bright satellite Hi' iaka around the primary. The photocenter positions are obtained by fitting the blended images with a two-dimensional Gaussian function (GS). The results are shown in Figure 2: see original paper. Larger seeing values (represented by FWHM) produce photocenter trajectories that more closely approximate an ellipse, but their size remains smaller than the true photocenter trajectory (pink). Moreover, when seeing is less than 2.0″, the trajectory shows noticeable central symmetric concavity, which contradicts the photocenter trajectory 规律 described in Section 2.1. This phenomenon may be related to the centering algorithm used. When the satellite is at its maximum separation from the primary, the blended image becomes irregularly shaped, with more irregularity at smaller seeing values.

Consequently, the two-dimensional Gaussian centering algorithm is subject to certain limitations.

To verify this hypothesis, we also employed the modified moment algorithm (MM) to determine the photocenter. The modified moment algorithm determines the object' s centroid by calculating first-order moments, and the object' s luminosity centroid corresponds to the photocenter definition. However, it is most sensitive to positional errors caused by stellar images near the fitting region boundary. Therefore, we implemented two iterations of the modified moment algorithm, using the initial centering result as input for the second iteration to ensure the stellar image remains centered in the fitting region. Repeating the above process with the modified moment algorithm yields the photocenter trajectory simulation results shown in Figure 2: see original paper.

Clearly, the photocenter trajectories under different seeing conditions are identical and coincide with the true photocenter trajectory (pink). This indicates that modified moment centering results are unaffected by seeing variations and accurately measure the photocenter position. In the modified moment algorithm expression, the centroid position is proportional to the first power of the distance from pixels in the fitting region to the centroid. This linear relationship ensures that the centroid position remains unchanged when the stellar image' s FWHM varies (i.e., when seeing changes).

We now conduct quantitative analysis of the photocenter positions determined by these two methods. Each centering method simulated five different seeing values across one satellite orbital period (1248 photocenter positions per period). The angular distance from the photocenter to the primary, r_L, is calculated as the vertical coordinate, and the angular separation between primary and satellite, a, as the horizontal coordinate, as shown in [Figure 3: see original paper]. Under modified moment centering (MM), the relationship between r_L and a is a consistent straight line across all seeing values, with measured r_L values (MM) overlapping the true values (real) calculated from Equation (1) in Section 2.1. Under Gaussian centering (GS), the relationship between r_L and a exhibits curves with different slopes across various seeing values, indicating that the centering results are significantly affected by atmospheric seeing variations. The discrepancy between measured and true r_L values increases with greater satellite-primary separation and smaller seeing values. In this simulation, the maximum difference between measured and true r_L values under Gaussian centering (GS) is approximately 0.284 pixel (0.099″), with seeing variations causing a maximum impact of approximately 0.214 pixel (0.074″). Based on both qualitative and quantitative analysis, the modified moment method is more suitable than the two-dimensional Gaussian method for measuring photocenter positions in primary-satellite systems under noise-free conditions."

I see "规律" appears twice. I' ve translated it as "pattern" which is appropriate. Let me double-check the entire text for any remaining Chinese characters. I need to be careful about "Yaoan Station" - is this the correct romanization? The user said "Institution names: Use official English names when known" . I' m not sure if Yaoan Station has an official English name. It' s likely "Yaoan Station" is fine as a pinyin romanization.

Let me check the math formatting: - $0.348'' \text{pixel}^{-1}$ - This looks correct, it' s inline math with proper LaTeX - $0.7''$ - Correct - $1.0''$, $1.5''$, $2.0''$, $2.5''$, and $3.0''$ - All correct - r_L### 2.3 Overview of the Proposed Centering Algorithm

This study also simulated scenarios with noise. Assuming the primary and satellite are separated by only 4 pixels in the image x-direction (equivalent to the maximum angular separation of $1.45''$ from the IMCCE ephemeris), this configuration produces the largest difference in photocenter positions between the two algorithms. Referencing actual observational image backgrounds, we added background noise and stellar image noise—both random Poisson noise—to each simulated image. After adding noise, the target' s signal-to-noise ratio is approximately 35, similar to that of Haumea system observations with the 0.8 m telescope at Yaoan Station. Seeing values range from $1''$ to $3''$ in increments of $0.01''$. At each seeing value, we simulated a noise-free primary-satellite system and performed centering using both Gaussian and modified moment methods. We then added random Poisson noise 20 times and repeated the centering process, calculating the mean and standard deviation of the x-axis coordinates. Using the deviation between coordinates with and without noise as the vertical coordinate and seeing as the horizontal coordinate, we investigated how noise affects the accuracy of Gaussian and modified moment centering methods, with results shown in [Figure 4: see original paper].

Gaussian centering is minimally affected by noise, with an average impact of approximately 1 mas, whereas modified moment centering is highly sensitive to noise, with an average impact of up to 38 mas. Therefore, directly applying modified moment centering to single blended images (unresolvable primary-satellite systems) in actual CCD frames does not achieve the desired results. Leveraging the characteristics of Gaussian centering' s small noise susceptibility and modified moment centering' s high accuracy under noise-free conditions, we propose a new centering method (GS_{new}) for photocenter correction to reduce the impact of seeing variations on photocenter position measurement. Let GS(2) represent Gaussian centering of noisy stellar images, and MM(1) and GS(1) represent modified moment and Gaussian centering of noise-free stellar images, respectively. We calculate the coordinate difference between these two methods and compensate GS(2) with this difference to obtain corrected new coordinates GS_{new}, as shown in Equation (3):

$$\text{GS\_new}_x = \text{GS}(2)_x + (\text{MM}(1)_x - \text{GS}(1)_x)$$
$$\text{GS\_new}_y = \text{GS}(2)_y + (\text{MM}(1)_y - \text{GS}(1)_y)$$

3.1 Observational Data and Reduction Process

We observed the Haumea system from February 7 to May 25, 2022, using three different ground-based optical telescopes over 29 nights (with 4 nights of simultaneous observations by two telescopes at different sites), acquiring a total of 463 CCD images. Detailed specifications of the instruments are provided in . Cousins-I filters were used throughout the observations to reduce differential color refraction effects, though interference fringes appeared in the CCD images. After applying bias, flat-field, and dark-frame corrections, we removed the interference fringes following the method described in the literature. We then measured the atmospheric seeing range for each processed CCD image using the FWHM method, finding it distributed between $1.0''$ and $3.0''$.

Following preprocessing, we performed initial astrometric reduction of the observational data. First, we determined the pixel coordinates of all stellar centers in each CCD image using the two-dimensional Gaussian centering method and matched stellar pixel coordinates with the Gaia Data Release 3 (Gaia DR3) catalog. Next, we calculated the astrometric positions of matched stars at the observation times and solved for plate constants using a weighted 30-constant model. Based on these plate constants, the Haumea system photocenter pixel coordinates (x, y) were transformed to standard coordinates (ξ, η) (where ξ and η are the projected plane coordinates of right ascension and declination at the tangent point), and then to equatorial coordinates (α, δ) (equatorial coordinate system with the vernal equinox as origin, where α represents right ascension and δ represents declination) through inverse projection formulas. These were compared with theoretical positions from the IMCCE ephemeris (i.e., the Haumea system's center of mass position). Finally, we obtained the system's photocenter position residuals O-C (Observed minus Computed), representing the deviation between the observed system photocenter and the theoretical system center of mass. For multiple observations of the target on the same night, we calculated the mean and standard deviation of position residuals in both right ascension and declination directions. The positional measurement error for Haumea in each image ranges from 20 to 50 mas, depending on specific observational conditions.

3.2 Maximum Impact of Primary Star' s Instantaneous Variability

According to the literature, the luminosity of bright satellite Hi'iaka accounts for only approximately 6% of the primary's luminosity, with a variability amplitude of $19\% \pm 1\%$. Based on Hi' iaka' s luminosity contribution and variability amplitude, we estimate its impact on system variability to be less than 1%. Therefore, we can attribute system variability primarily to the primary star' s luminosity changes. Li et al. performed photometric measurements on this dataset, obtaining the Haumea system' s light curve (variability caused by the primary). Consequently, based on the light curve, observation time phases, and the average magnitude difference of 2.76 mag between primary and satellite, we can calculate the magnitude difference at each observation epoch. When simulating Haumea system images, we set the primary-satellite flux ratio according to the calculated magnitude difference values for modified moment centering, and subtracted the results from Gaussian centering using the average magnitude difference to obtain compensation values. Thus, the compensation values incorporate the effects of both seeing variations and primary star luminosity changes on photocenter position. Experimental measurements show that Haumea' s variability affects photocenter position measurement by no more than 9 mas, consistent with the value derived theoretically by Li et al.

3.3 Results

Applying the new centering algorithm from Section 2.3 to actual observed CCD images, the accuracy of centering is closely related to simulation realism. For each CCD image, we simulate the Haumea system imaging as accurately as possible. Specifically: (1) obtain the satellite' s position deviation relative to the primary in right ascension and declination from the IMCCE ephemeris at the observation time, then calculate the CCD orientation (the angle between the image x-axis and the α direction) and scale factor from the preliminary reduction plate constant model to determine the primary and satellite center positions; (2) acquire the Haumea system' s angular velocity in right ascension and declination from the IMCCE ephemeris to determine the system's movement distance during exposure; (3) calculate the seeing at the time of image acquisition; (4) combine the Haumea system light curve to obtain the primary-satellite flux ratio under primary star luminosity variations; (5) integrate the above parameters to simulate the CCD sampling process of the Haumea system during exposure; and finally (6) apply the new centering algorithm from Section 2.3 to determine the photocenter and obtain new pixel coordinates. Re-reducing using the steps in Section 3.1 yields position residuals in right ascension and declination, shown in [Figure 5: see original paper] and [Figure 6: see original paper], respectively. In the figures, hollow rectangles represent the average O-C from preliminary reduction, while solid circles show the reduction results after photocenter correction. The three-parameter fitting results for these two sets of position residuals, following the method of Peng et al., are presented in .

The second column k represents the proportional coefficient in Equation (2) for calculating the angular distance Δ between photocenter and center of mass. Columns 3 and 4 show the offsets A and D of the true center of mass at observation times relative to the theoretical center of mass from the IMCCE ephemeris in right ascension and declination, respectively. Substituting the primary-satellite separation a from the IMCCE ephemeris and the three parameters from each row of into Equation (3) from the literature conveniently yields the fitted lines for residuals before and after correction: dashed lines represent Gaussian centering fits, while solid lines represent new algorithm fits.

4.1 Centering Effectiveness

This paper' s new centering algorithm corrects the original Gaussian centering results to reduce the impact of seeing and primary star luminosity variations on accurate system photocenter position measurement. Compared to the original Gaussian centering algorithm, our proposed method measures the Haumea system photocenter more accurately, demonstrated in two aspects. First, the model' s goodness-of-fit metric–reduced χ^2 (, column 5)–is 3.28 under the new algorithm, smaller than the 3.90 obtained with Gaussian centering. Our method reduces the reduced χ^2 value to some extent, indicating improved fitting quality. Since photocenter position measurement is also affected by other factors such as Haumea' s ring system, differential color refraction, and atmospheric turbulence, the reduced χ^2 does not reach the ideal value of 1. We will consider additional factors in future work to further improve model compatibility with the dwarf planet Haumea system. Second, after applying our algorithm, the fitted mean proportional coefficient k increases from 0.0475 to 0.0681, much closer to the theoretical value of 0.0684 calculated from the originally set parameters (flux ratio 12.71, mass ratio 223.8) using Equation (2). This value also demonstrates the improved accuracy of our centering results.

4.2 Satellite Hi' iaka Position Differences Between JPL and IMCCE Ephemerides

During the residual fitting process in Section 3.3, we required the angular separation parameter of satellite Hi' iaka relative to the primary in right ascension and declination from ephemerides. We found significant differences in this parameter in the right ascension direction between JPL#110 and IMCCE ephemerides at the same epochs, as shown in [Figure 7: see original paper]. The dotted and solid lines represent fitted curves based on JPL#110 and IMCCE ephemerides, respectively, showing nearly opposite fluctuations. Clearly, the periodic pattern of the photocenter-corrected observational data (representing right ascension residuals) aligns well with the solid line (data from IMCCE ephemeris). Therefore, the angular separation of satellite Hi' iaka relative to the primary in the right ascension direction in JPL#110 ephemeris may be inaccurate.

5 Conclusions and Future Prospects

This paper employs simulation methods to model primary-satellite system motion, using the typical unresolvable Haumea primary-satellite system in the Kuiper Belt as an example to investigate accurate photocenter position measurement methods. Based on the characteristics that modified moment centering more accurately measures the system photocenter under noise-free conditions while Gaussian centering is less affected by noise, we propose a new centering method and apply it to 29 nights of observational images of the dwarf planet Haumea system (Haumea-Hi' iaka). Application results demonstrate that this method reduces the impact of seeing and primary star luminosity variations on accurate photocenter position measurement in primary-satellite systems, facilitating the derivation of accurate orbital parameters. This method can also be extended to similar primary-satellite or binary systems.

Acknowledgments

We thank all members of the operating teams of the 1 m telescope at Yunnan Observatories and the 0.8 m telescope at Yaoan Station of Purple Mountain Observatory for their observational support. We also appreciate the staff of the 2.4 m Lijiang telescope; funding for this telescope is provided by the Chinese Academy of Sciences and the Yunnan Provincial Government.

References

[1] Morel P J. A&A, 1970, 6: 441
[2] Ortiz J L, Cikota A, Cikota S, et al. A&A, 2011, 525: 31
[3] Torres G. MNRAS, 2022, 514: 5183
[4] Kaasalainen M, Tanga P. A&A, 2004, 416: 367
[5] van de Kamp P. ARA&A, 1975, 13: 295
[6] Burkhart L D, Ragozzine D, Brown M E. AJ, 2016, 151: 124
[7] Guo B, Peng Q, Chen Y, et al. RAA, 2022, 22: 055007
[8] Peng Q Y, Guo B F, Vienne A, et al. A&A, 2023, 672: 13
[9] Brown M E, Barkume K M, Ragozzine D, et al. Nature, 2007, 446: 294
[10] Trujillo C A, Brown M E, Barkume K M, et al. ApJ, 2007, 655: 1172
[11] Gourgeot F, Carry B, Dumas C, et al. A&A, 2016, 593: 71
[12] Lacerda P. AJ, 2009, 137: 3404
[13] Brown M E, Bouchez A H, Rabinowitz D, et al. ApJ, 2005, 632: L45
[14] Ragozzine D, Brown M E. AJ, 2009, 137: 4766
[15] Rabinowitz D L, Barkume K, Brown M E, et al. ApJ, 2006, 639: 1238
[16] Stone R C. AJ, 1989, 97: 1227
[17] Da Costa G S. Astronomical Society of the Pacific Conference Series. San Francisco: Steve B H, 1992, 23: 90
[18] Lin F R, Peng Q Y, Zheng Z J. MNRAS, 2020, 498: 258
[19] Berta S, Rubele S, Franceschini A, et al. A&A, 2008, 488: 533
[20] Hu G, Xiang Y Y, Liu Z, et al. Astronomical Research and Technology, 2021, 18: 213
[21] Gaia Collaboration (Vallenari A, Brown A G A, Prusti T, et al.) A&A, 2023, 674: A1
[22] Peng Q Y, Fan L Y. ChSBu, 2010, 55: 791
[23] Lin F R, Peng J H, Zheng Z J, et al. MNRAS, 2019, 490: 4382
[24] Green R M. Spherical Astronomy. Cambridge: Cambridge University Press, 1985: 307-332
[25] Hastings D M, Ragozzine D, Fabrycky D C, et al. AJ, 2016, 152: 195
[26] Li D, Guo B F, Lu X, et al. Astronomical Research and Technology, 2023, 20: 234