A Theory of Quantum Reference Frames and Its Implied Theory of Gravity

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Abstract

This paper considers a quantum reference frame theory, discussing the Ricci flow theory of this spacetime reference frame under quantum second-moment fluctuations and the implied gravitational theory. Due to the non-trivial spacetime second-order quantum fluctuations in the theory, we examine its relationship to problems including cosmic accelerated expansion (the cosmological constant problem), the radial acceleration anomaly at galactic scales, early inflation, as well as the origin of spacetime entropy. Finally, we also present some possible predictions of this theory.

Full Text

Preamble

A Theory of Quantum Reference Frame and its Implications to Gravity
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Contents

I. Early Attempts and Difficulties in Formulating and Applying Quantum Theory to General Spacetime Coordinate Systems

The formulation and interpretation of quantum theory in its current form are built upon the concept of inertial frames. The Schrödinger equation for quantum state evolution, as currently formulated, only holds in inertial frames using

Newtonian time. One important motivation for inventing general relativity was to deprive the inertial frame concept of its special status in physics and to establish physical theories (including quantum theory) on concepts valid for general coordinate systems.

Although quantum theory was initially attempted to be formulated in a form valid for any Lorentz inertial frame, early attempts were unsuccessful, leading Schrödinger to return to Newtonian inertial frames to write his wave equation, which proved successful. Only later was it understood that correctly formulating wave equations for general Lorentz reference frames required proper treatment of particle spin. This partially explains the failure of the Klein-Gordon equation in describing the hydrogen atom and the success of the Dirac equation. However, these early attempts to base quantum theory on general Lorentz inertial frames were not so straightforward. It was later realized that these relativistic wave equations, while valid in any Lorentz frame, encountered problems of negative probability or negative energy—inevitable consequences of the Lorentz spacetime metric allowing indefinite signatures. However, it was soon shown that with appropriate reinterpretation of the theory' s mathematical results, these were no longer theoretical difficulties but correct predictions—namely, the existence of antiparticles. This reinterpretation required that our fundamental physical description be built on "field theory" rather than "mechanics," with continuous "fields" in spacetime as the basic dynamical variables, rather than "mechanical quantities" like coordinates and momenta. The Klein-Gordon equation describes not quantum states but classical field equations for spin-0 scalar fields. The Dirac equation describes not quantum states but classical field equations for spin-1/2 spinor fields. These classical field equations require further quantization treatment. The quantum negative modes produced by the indefinite metric of relativistic spacetime are largely not eliminated but reinterpreted; negative modes are inevitable in relativistic quantum field theory and likely play crucial roles in important physical effects. However, efforts to formulate quantum theory in more general coordinates did not cease. Lorentz coordinate systems are merely a class of inertial frames, and applying quantum field theory to general fixed curved spacetime backgrounds led to quantum field theory in curved spacetime.

Such theories achieved certain successes, being successfully applied to semiclassical considerations of curved spacetime, such as studying early universe inflation and primordial fluctuations, and black hole physics. However, fundamental difficulties were soon discovered. For instance, it was quickly found that such theories are non-unitary: particle number is no longer conserved, and different ground states are not unitarily equivalent. In other words, performing a coordinate transformation to an accelerated frame results in loss of information and unitarity. Currently, this appears to be an inevitable consequence of quantum field theory on fixed curved spacetime backgrounds. Setting aside the so-called ultimate quantum gravity theory of applying quantum principles to general dynamical spacetime backgrounds, quantum theory formulated on fixed curved spacetime already seems to conflict with the principle of general relativity—

namely, if this non-unitarity under general coordinate transformations is real, we could potentially distinguish inertial from accelerated frames through quantum physics experiments, seemingly violating the principle of general coordinate relativity.

Not only does formulating quantum theory in general spacetime coordinate systems face difficulties, but applying quantum principles to general dynamical (pseudo-)Riemannian coordinate systems encounters even greater conceptual and technical challenges. In early attempts at canonical quantization of general spacetime, it was found that for general relativity—a theory independent of coordinate system time choices—the Hamiltonian of spacetime or gravity always equals zero. In other words, there simply exists no Schrödinger evolution equation for gravity or spacetime; instead, there is the time-independent Wheeler-DeWitt constraint, a conceptual problem ubiquitous in quantizing generally covariant theories. The reason for this problem is simple: since general relativity itself is spacetime-coordinate-independent, it lacks the absolute time required for quantum mechanics as an externally measured quantity. Technically, the non-polynomial form of the Wheeler-DeWitt constraint as a fundamental dynamical variable also creates significant technical difficulties in solving this constraint. Even if solutions could be found—for instance, through Hamiltonian constraints reformulated using Ashtekar variables—these "frozen" canonical quantum state wave functions were found to be related to certain knot topological invariants, but their physical meaning remains unclear.

Besides canonical quantization methods, early attempts at covariant quantization also revealed that classical general relativity is non-renormalizable. Quantum corrections to spacetime cannot be absorbed into the coefficients of the original Einstein-Hilbert action. If one attempts to eliminate these divergent quantum corrections using conventional renormalization methods, one must continuously introduce new terms and new coefficients into the original action. If one continues calculating quantum corrections for these new terms and coefficients, even more new terms and coefficients must be introduced to eliminate previously calculated divergences. This process appears to continue endlessly, leading to the realization that obtaining a renormalizable quantum spacetime and gravity theory requires infinitely many input parameters and infinitely many counterterms. It remains unknown whether this theory possesses a bare UV fixed point that makes the theory mathematically well-defined at all energy scales in the quantum regime without encountering mathematical singularities—a property called UV completeness. Non-renormalizability is perhaps the most notorious difficulty, but strictly speaking in terms of UV completeness, quantum theory is incompatible with most field theories, including quantum electrodynamics with its "Landau pole" problem that terminates conventional perturbative renormalization calculations. Only asymptotically free quantum chromodynamics is strictly well-defined in the UV (but only definable in the UV; its infrared behavior remains poorly understood).

In path integral quantization, should we integrate over spacetimes with

Lorentzian metrics or those with Euclidean metrics? Both choices face respective objections. If, following conventional quantum field theory path integral quantization considerations—namely, that real actions offer better integral controllability and convergence than imaginary actions—one performs path integrals in Euclidean time rather than pseudo-Riemannian time, unfortunately, the integration measure for gravitational dynamical variables (the metric) is not mathematically well-defined. Moreover, it was discovered that the Einstein-Hilbert action under Euclidean signature is unbounded from below, making such defined path integrals impossible to converge.

This divergence, independent of the previously mentioned UV divergence problem, constitutes one of the fundamental difficulties in applying quantization methods to gravitational systems. It is worth noting that general relativity' s coordinate-independent (pseudo-)Riemannian geometry describing real spacetime itself (the geometrization of spacetime) relies on another fundamental assumption: the equivalence principle. The classical equivalence principle ensures symmetry in treating material particles and material reference frames, making their relative motion satisfy the principle of relativity. The equivalence principle is the foundation for the universal geometrization of spacetime and gravity (gravity is not simply a conventional "force"), so current mainstream geometrized quantum gravity candidates also build upon the assumption that the equivalence principle remains valid at the quantum level. However, in standard Schrödinger equation treatments, the mass dependence of particle free-fall processes severely conflicts with the mass independence of free-fall claimed by the equivalence principle, lacking symmetry between material particles and material reference frames. This can be summarized by the following simple argument: during free-fall, kinetic energy and gravitational potential energy are proportional to inertial mass and gravitational mass respectively. Since inertial mass equals gravitational mass, this yields mass-independent free-fall velocity and acceleration at the classical level. But at the quantum level discussed by the Schrödinger equation, the uncertainty principle involves uncertainty between particle position and (mass-dependent) momentum. Particle wavefunction quantum uncertainties are directly linked to particle energy, so wavefunction evolution amplitudes and quantum fluctuation broadenings are generally mass-dependent. For instance, coordinate broadening $\Delta x^2 \quad \hbar/m$ differs for particles with large versus small mass. In this sense, although material reference frames of different masses have mass-independent geodesics for classical spacetime measurement, the fluctuations of material reference frames of different masses cannot measure universal spacetime fluctuations. More precisely, the contradiction between the equivalence principle and quantum uncertainty principle mainly appears in second-order moments (such as quantum uncertainties in coordinates or momentum Δx^2 or Δp^2). The semiclassical approximation of quantum mechanics, namely the Ehrenfest theorem, gives first-order moments (averages x or p) consistent with classical Newtonian mechanics and the classical equivalence principle (a widely used example is that gravitational frequency shift $\Delta\omega$ and gravitational time dilation Δt effects for material particles like photons due to gravitational potential changes are equivalent and consistent). The reason is that the Schrödinger equation treats free-fall of waves as dynamical evolution driven by (mass-dependent) Hamiltonian energy on a fixed "spacetime container," whereas in general relativity and the equivalence principle, free-fall of waves is equivalently viewed as the wave not moving while the universal "spacetime container" carrying the wave changes universally. These two perspectives on the relative free-fall between waves and spacetime reference frames in quantum theory and general relativity are asymmetric or inequivalent: first, because in the quantum theoretical picture, waves have second-order moment quantum uncertainties while the classical "spacetime container" lacks symmetric second-order moment quantum uncertainties; second, even if one considers second-order moment quantum fluctuations of material reference frames in the standard quantum theoretical sense, their second-order moment quantum fluctuations are controlled by mass-dependent Hamiltonian energy, so material reference systems of different masses seemingly should have different second-order moment quantum uncertainties and cannot measure universal spacetime second-order fluctuations independent of material reference frame mass. These reasons create a fundamental contradiction between the quantum uncertainty principle and the equivalence principle. In our view, if the equivalence principle is violated at the quantum level—meaning rulers or clocks of different masses as material reference systems (lacking universal second-order uncertainties) can no longer measure a universal (fluctuating) spacetime—this would be a disaster for physics built upon spacetime measurement. We must either start from scratch to build a non-universal spacetime theory dependent on reference frame mass (where gravity reverts to being an ordinary "force" before relativity), or reconsider a more general quantum theory for material reference systems to make universal spacetime measurements (where gravity remains a universal property of spacetime measurement, not simply a "force"). In this paper, we choose the latter. In a more general and fundamental quantum theory, mass independence of material reference systems does not come from the Schrödinger dynamical equation with non-zero Hamiltonian energy, because essentially any quantum system containing non-zero Hamiltonian energy-driven evolution implicitly contains a preferred inertial coordinate system—a concept we must abandon from the start. Instead, it comes from the Ricci flow effect of spacetime coarse-graining under general coordinates (a zero-Hamiltonian constraint system). A quantum version of the equivalence principle serves as the foundation for the geometrization of quantum gravity and quantum spacetime, which we will discuss in (II and V A). A generally coordinate-covariant quantum system with Hamiltonian constraints will provide a more symmetric conceptual framework for uniformly treating material particles/waves and material reference systems. The mass independence of second-order quantum fluctuations of material reference systems required by the quantum equivalence principle, along with the symmetry and equivalence between ordinary material particles/waves and material reference systems, will be realized in this framework.

In summary, quantum theory not only rejects the equivalence principle and geometrization of gravity, but also has numerous conflicts and difficulties with general relativity regarding its formulation in general spacetime coordinate systems and its mathematically singularity-free application to handling quantum fluctuations of spacetime gravitational degrees of freedom. To find a theory simultaneously satisfying (a) quantum mechanical principles, (b) general (generalized) coordinate relativity principle, and (c) equivalence principle, at least one of them requires appropriate modification or extension. How our current physical theories can be built upon a unified conceptual foundation is a question that contemporary physicists must spend considerable time contemplating.

If the validity of the equivalence principle can be extended to the quantum level, we can still establish quantum gravity phenomena on a geometric foundation. We see that geometry evolved from the analytic geometry method of global Descartes coordinate systems, to Gauss' s recognition that measuring from local coordinate systems on two-dimensional surfaces suffices to determine the geometry of the surface itself without embedding it in three-dimensional space, to Riemann's general development of Gauss's ideas about intrinsic metric geometry through local quadratic distance forms. From Gauss to Riemann, the spirit or program of coordinate-independent intrinsic geometry reached its physical peak in Einstein' s classical general relativity, bringing a historic synthesis to classical physics. However, this spirit of intrinsic geometry has not been implemented in quantum theory. Relative to intrinsic geometry and the spirit of general relativity, the current framework of quantum theory still requires and depends on descriptions using external absolute inertial coordinate systems—it remains an "extrinsic" theory requiring embedding into external coordinate systems for description. From this perspective, current quantum theory is still a "Newtonian mechanics" built upon Cartesian coordinate systems. This is also an important reason for the conflict between quantum theoretical frameworks and general relativistic principle frameworks. The two need to be built upon a unified conceptual foundation. Moreover, because traditional quantum theory divides the world into an observed quantum system and an external absolute classical instrument or classical "observer," interpreting this theory requires introducing some currently controversial assumptions to be self-consistent, such as the standard Copenhagen interpretation' s requirement that a spatially large and extended quantum wavefunction must "instantaneously collapse superluminally"across distant spatial points (the so-called EPR paradox). Einstein used such paradoxes to express his perceived intrinsic incompatibility between quantum theory and relativity. Current technological means have enabled long-distance quantum mechanics experiments over several kilometers or even hundreds of kilometers (such as entangled photon distribution and measurement experiments). While measurement results of entangled photon pairs align perfectly with quantum mechanical predictions, this is not a problem with the predictions of quantum mechanical mathematical formalism, but rather a problem of physical interpretation of the mathematical formalism—namely, how an observer at one spatial location locally prepares experimental instruments to measure this local quantum state, and how the "relative relationship" (rather than two independent quantum states themselves) with the quantum state at the other spatial location should be properly explained. Currently, there is no better interpretation of quantum mechanical mathematical formalism to provide a self-consistent unified understanding of these intrinsic contradictions. The Copenhagen interpretation remains the closest to "laboratory common sense" and most convenient interpretation of current quantum theoretical mathematical formalism, making it widely accepted by laboratory-scale physicists, although we know that once some interpretations (through thought experiments) are pushed to extreme situations, they may lead to intrinsic problems. Both the current mathematical formalism of quantum theory and its Copenhagen interpretation certainly have much room for improvement, particularly needing to develop into a theory describing "relationships" between quantum states—for instance, a theory of the relationship between the quantum state of a measured system and the quantum state of measuring instruments (this is the basic idea of quantum reference frames in this paper), rather than a theory describing a single quantum state relative to an external absolute classical instrument. This interpretation is particularly urgent for understanding the quantum nature of gravity, as this is already a coordinate-independent relativity theory without absolute external space. We will also see that the description of entangled states provides a suitable foundation for "intrinsifying" or "relationalizing" quantum mechanical interpretation, thereby avoiding many puzzling issues brought by the standard Copenhagen "external" interpretation.

The scientific community, as a robust and conservative system, is usually not easily willing to invent or accept a new image and framework unless the old framework encounters severe and fundamentally irreconcilable crises. Therefore, we must also ask: Have past quantum theories and gravity theories truly encountered serious crises? Indeed, from experimental and observational perspectives, quantum theory at laboratory scales and general relativity at astrophysical scales both make very precise predictions in their respective domains, except for "dark energy" at cosmological scales and "dark matter" problems at galaxy, galaxy cluster, and large scales. While the dark matter problem might be solved by introducing additional matter assumptions into the Standard Model (although this hypothesis is gradually facing another crisis as half a century has passed without detecting any form of dark matter), our current understanding of dark energy with properties very close to a cosmological constant touches upon the fundamental difficulty of how to consistently incorporate quantum fluctuations into gravitational theory. Vacuum quantum fluctuations essentially lead to instability of gravitational systems—the fine-tuning problem of the cosmological constant cannot be solved simply by introducing new constants or new energy forms in current standard theory. In our view, this is a true crisis between current observations and fundamental theory, requiring us to understand what actually happens when quantum theory and gravitational theory coexist, and to revise our understanding of gravity from galactic to cosmological large scales, which we will discuss carefully later.

We more or less face a situation similar to that before the birth of relativity at the turn of the 19th and 20th centuries. The confusion then was whether electromagnetic or optical experiments could distinguish whether we were in an absolutely moving reference frame. The contemporary version of similar questions is: Can we distinguish whether we are in an inertial frame, accelerated frame, or more general coordinate system through quantum experiments? And a series of difficulties derived from contradictions between quantum theory and general relativity principles. We see that Einstein, to maintain the principle of relativity—that electromagnetic experiments cannot reveal whether we are in absolute motion—needed to modify the concept of time, causing traditional Euclidean space to no longer be strictly realizable. In general relativity, Einstein further pointed out that we cannot even distinguish inertial from accelerated frames through classical physics experiments. Classical physics was formulated to be valid in general (non-inertial) coordinate systems, independent of coordinate choices. This not only did not create fundamental contradictions but also automatically gave rise to a new phenomenon: a correct classical gravity was automatically included in such a theory based on (pseudo-)Riemannian geometry, no longer requiring an additional gravitational hypothesis (as Newton did with the inverse-square law), achieving a historic synthesis of classical physics. Gravitational phenomena themselves are intimately connected with general coordinate relativity principles. If the spirit of general (generalized) coordinate relativity is to be maintained (where "general" in this paper refers not only to general classical curved coordinate systems but also attempts to extend it to general quantum coordinate systems with quantum fluctuations)—that is, if we still cannot distinguish inertial, accelerated, or even general (non-inertial) coordinate systems through any quantum experiments, and if generalized quantum theory applies equally to inertial, accelerated, and general coordinate systems—then the current classical equivalence principle must be appropriately extended at the quantum level. We will see that this not only does not create fundamental contradictions (such as the previously mentioned unitary inequivalence between different coordinate systems), but also automatically gives rise to new phenomena: a gravitational theory satisfying general quantum principles (caused by non-unitarity and quantum anomalies between different coordinate systems, see V C and VI D), and (non-unitary) thermodynamic effects produced by quantum fluctuations of spacetime coordinate systems themselves (see VII B), are automatically included in such a quantum spacetime geometry.

This paper can be regarded as an effort and attempt to consistently implement the Gauss-Riemann-Einstein program of coordinate independence or intrinsic geometry at the quantum level, based on the quantum equivalence principle (geometrization of quantum gravity). Applying this new spacetime geometry and gravitational theory to possible domains to achieve understanding of current difficulties in physics is another attempt of this paper: such as late-time cosmic accelerated expansion (see VI C), anomalous acceleration at galactic scales (see VI E), the very early universe (see VI H), stability at the quantum level of gravity (see VI F and VI G), non-equilibrium statistics of spacetime geometry Ricci flow (see V E), thermodynamics and microscopic degrees of freedom statistics of black holes in equilibrium (see VII D), effects of accelerated frames on spectral lines (see VII E), etc. This paper also strives to provide understanding and discussion based on this framework.

II. From Classical to Quantum Equivalence Principle

The equivalence principle originates from Einstein' s desire to extend his special principle of relativity, which only held for inertial frames, to general coordinate systems, such as accelerated frames. In his free-fall thought experiment, he realized that acceleration is also relative, and that in a theory valid for general coordinate systems, gravity would be automatically included. His inspiration can be illustrated through a hypothetical Leaning Tower of Pisa free-fall experiment. Iron balls and wooden balls of different masses, when air resistance is neglected, will free-fall with the same acceleration. Even the Earth should "fall" toward the iron/wooden balls with the same acceleration. Furthermore, light rays used to measure spacetime, despite having very different masses from iron and wooden balls, will also free-fall with the same acceleration. This independence of acceleration from material properties was generalized and no longer interpreted as a property of the material' s own (motion), but rather as describing a universal property of spacetime itself. In other words, in gravity, the bending of light rays measuring spacetime due to acceleration is no longer the light' s own bending, but spacetime' s bending.

This independence of acceleration from material mass has been confirmed in extremely precise experiments. The equivalence principle' s approach of attributing universal physical properties of matter to spacetime geometry is actually the foundation for our measurement of spacetime and the geometrization of gravity. Otherwise, we could not discuss how to measure "spacetime," this hypothetical container where all physical events occur, nor could we connect spacetime properties with gravity, because matter's motion in gravity is now viewed as motion relative to the abstract intermediate equivalent of spacetime, rather than relative to material reference systems. Thus, once you describe spacetime' s curvature itself, you have already described gravity itself.

However, when quantum effects of matter are carefully considered, the previous classical thought experiments need to be re-examined. For example, consider an electron' s free-fall. We know in quantum electrodynamics that an electron can emit a virtual photon and later reabsorb it. Such quantum processes produce the so-called electron self-energy correction. When the electron free-falls, not only does the electron itself free-fall, but the virtual photons or electromagnetic fields around it also free-fall together. Therefore, it appears that the electron' s self-energy due to quantum fluctuations should also contribute to the electron' s inertia. The famous Lamb shift confirms the existence of such contributions to electron inertia from quantum fluctuations. So does this inertial mass from quantum fluctuations, as claimed by the classical equivalence principle, completely equivalently act as gravitational mass, being attracted by Earth' s gravity in the same way?

If the answer is affirmative, we encounter another problem: the cosmological constant problem. If quantum fluctuations indeed contribute to inertia and thus to gravity, the gravitational mass contributed by zero-point quantum fluctuations in vacuum would be enormous, making the universe unstable. This was the initial confusion about the cosmological constant problem: how to cancel out this contribution from vacuum zero-point quantum fluctuations? Later, with observations of cosmic accelerated expansion, people began to realize that we cannot completely cancel all contributions from zero-point fluctuations to gravitational mass; we need to leave a tiny remainder to exactly produce the observed accelerated expansion. This is the later so-called new cosmological constant problem: why is the cosmological constant so small? Solving the cosmological constant problem also requires appropriate quantum-level extension of the classical equivalence principle.

We will return to the cosmological constant problem when discussing effective gravity later. We see that to establish a quantum theory valid in general coordinate systems, we need to extend our equivalence principle. A natural assumption is that we fundamentally cannot distinguish which part of energy or inertia comes from classical physics contributions and which from quantum fluctuation contributions. We similarly need a quantum equivalence principle to extract the universal (quantum) properties from a (quantum) free-falling material system and attribute them to universal properties of (quantum) spacetime, serving as a general intermediate equivalent for comparing all (quantum) motions: the spacetime reference frame. We take this as our fundamental principle for establishing a universal quantum spacetime and quantum reference frame. Under such a principle, particles with different quantum fluctuation self-energy corrections all free-fall with the same acceleration. The wavefunctions of these particles with different energies, due to having the same falling velocity, all undergo the same Doppler frequency shift. Moreover, they all have the same acceleration. It can be proven that local acceleration and local gravity cause Gaussian broadening of wave frequency spectra (see V B 2), so the wavefunctions of these energetic particles also undergo the same Gaussian broadening. Moreover, this universal frequency shift and broadening are no longer properties of these particles themselves but have been extracted and reinterpreted as quantum properties of spacetime. This is precisely what we observe in cosmic accelerated expansion observations. We will see later that local Ricci flow evolution of large-scale spacetime itself also produces local spacetime and spectral broadening completely equivalent to that produced by local acceleration (see VI C), which is the fundamental cause of large-scale cosmic accelerated expansion. Distant spectral lines of different energies in the universe not only redshift in a universal way (Hubble expansion) but also broaden in a universal way (accelerated expansion). In the distance-redshift relation, when expanding distance in terms of redshift to second order, the Gaussian (second-order) broadening of spectral line redshift contributes a universal deceleration parameter. Not only in large-scale cosmology does Ricci flow produce an effect equivalent to accelerated expansion through broadening of local spacetime coordinates.

A quantum-level equivalence principle is the physical foundation for us to establish a universal measurement of spacetime through quantum material fields (frame fields). If those material frame fields possess certain universal quantum properties independent of specific matter—for instance, universal second-order fluctuation broadening and even higher-order fluctuations that we will see later —then the quantum fluctuations of these frame fields will also be extracted and reinterpreted as quantum fluctuations of spacetime geometry itself, rather than being interpreted as quantum fluctuations of the material fields serving as frames. Not only at the classical level is there a classical equivalence principle at the level of averages (first-order moments) between accelerated frames and gravity, but at the level of quantum Gaussian broadening (second-order moments) of spacetime coordinates, besides the two classical dimensions of acceleration and gravity, the dimensions of spacetime Ricci flow renormalization and thermal effects will also be incorporated into a more general quantum equivalence principle. We will see that (1) coordinate broadening in accelerated frames, (2) spacetime curvature and gravitational effects, (3) spacetime coordinate broadening and spacetime geometry curvature produced by local spacetime Ricci flow renormalization, and (4) spacetime thermal effects caused by accelerated frames or spacetime Ricci flow are all completely equivalent and indistinguishable at the quantum level, and all have meaning only relative to an observer' s specific coordinate system, with no absolute meaning.

Current observations provide the following evidence supporting the validity of extending the equivalence principle to the quantum level:

First, in cosmic observations, although spectral line energies can differ greatly, at large cosmological distances when the proper motion of light sources becomes small compared to cosmic expansion comoving motion, these cosmologically comoving spectral lines of different energies almost all have the same universal redshift (i.e., different energy spectral lines "fall" universally with spacetime comoving recession). This can be regarded as a certain test of the quantum equivalence principle. Although we currently have no direct measurement of universal spectral broadening, we actually observe that the distance-redshift relation for spectral lines indeed measures a universal deceleration factor q_0 -0.64 at the square order of redshift, which is almost homogeneous and isotropic in the cosmic background. This is equivalent to a universal second-order moment broadening of spectral line redshift and also serves as an indirect quantum test of the equivalence principle to some extent.

Second, similar to the Leaning Tower of Pisa free-fall thought experiment, consider a heavy ball and a light ball connected by a string. If the heavy and light balls have different falling velocities, there will be a mutual dragging force between them. (This dragging force would slow the heavy ball and speed up the light ball, but this contradicts the fact that when the two balls are combined, their center-of-mass mass becomes larger, so the whole should fall faster than a single ball.) The quantum version of this thought experiment can correspondingly consider a hydrogen atom composite particle composed of a relatively heavy proton and a relatively light electron, following the cosmic background' s accelerated recession. If the heavy proton and light electron have different "falling" (comoving recession) velocities in the cosmic expansion background, then besides the Coulomb force between proton and electron, there would be an additional contribution from mutual dragging force. This would lead to measuring a slight shift in the comoving recession hydrogen atom spectrum compared to the stationary laboratory inertial frame spectrum, or equivalently measuring a deviation of the fine-structure constant from the laboratory value. Therefore, measuring the spectral redshift of the fine-structure constant for hydrogen atoms comoving in the cosmic expansion background can be used to indirectly test the validity of the quantum equivalence principle. First, within error margins, current measurements of the fine-structure constant in the cosmic expansion background show very small deviation from laboratory measurements. This can also be regarded as a quantum test of the equivalence principle to some extent.

Third, current Eötvös factor measurements (describing differences in their falling acceleration) using matter-wave interferometers for cold atoms of different masses (rubidium Rb and potassium K) yield results almost zero at $O(10^{-7})$, indicating that atoms of different masses still undergo universal free-fall at high precision at the quantum level. Free-fall experiments for quantum antiparticles also give almost the same free-fall as ordinary (positive) particles. Additionally, neutron interference experiments in gravitational fields (Colella-Overhauser-Werner (COW) experiments) can be explained not only by adding gravitational potential energy to the Hamiltonian to produce quantum phase shifts, but also self-consistently through universal time dilation of neutrons in different gravitational potentials. Therefore, COW experiments are also considered indirect quantum tests of the equivalence principle.

We know that classical orbits are given by energy variation $\Delta H = \Delta E_{kinetic} + \Delta V = 0$ (more strictly, action variation $\Delta L = \Delta E_{kinetic} - \Delta V = 0$). Since kinetic energy $E_{kinetic}$ is proportional to inertial mass $m_{inertial}$ and gravitational potential V is proportional to gravitational mass $m_{gravitational}$, and because inertial mass equals gravitational mass $m_{inertial} = m_{gravitational} = m$, classical orbit variation $\Delta H = \Delta E_{kinetic} + \Delta V$ is proportional to mass m, so the equation $\Delta H = 0$ becomes independent of mass m, giving mass independence of classical geodesic orbits. However, at the quantum level, quantum fluctuations in inertial frames are generally non-zero ($\Delta H \neq 0$), so even if inertial mass equals gravitational mass, gravitational effects at the quantum level (such as free-fall) appear mass-dependent. More specifically, although the Schrödinger equation can give correct semiclassical approximations at the level of first-order averages (Ehrenfest' s theorem yields Newton' s laws and classical equivalence principle in terms of averages), the mass independence of particle second-order quantum fluctuation broadening (and even higher-order fluctuations) required by quantum equivalence principle is inconsistent with standard quantum uncertainty principle. Standard quantum mechanics gives coordinate uncertainty or broadening inversely proportional to mass, i.e., Δx^2 $\hbar/m$. It is commonly believed that mass independence required by quantum equivalence principle only holds in the large quantum number or large mass limit, and is no longer strictly valid at the quantum level. However, in this paper' s framework, mass independence required by quantum equivalence principle must be strict—only then can quantum fluctuations of material frame fields be interpreted as universal quantum fluctuations of spacetime.

To achieve mass independence or universality at the quantum level, the equality of inertial and gravitational masses is insufficient; a stronger condition than $\Delta H = 0$ given by classical equations of motion is needed. In (V A) we propose that this condition is actually the total energy constraint in any coordinate system: Hamiltonian constraint $H = 0$ (but $L \neq 0$). This is natural for quantum theories valid in general coordinate systems, as a generally coordinate-invariant quantum theory is naturally a constraint system without Hamiltonian, because any quantum system containing non-zero Hamiltonian energy-driven evolution implicitly contains a preferred (at least local) inertial coordinate system, implicitly implying a specific and preferred time splitting of the complete 4-spacetime M^4, such as $M^4 =$ M^3 (where i indexes different local coordinate patches), or implying an external larger space $M^5 = M^4$ s and external global time s—concepts we must abandon from the start. This is the drawback of traditional non-covariant canonical quantization. Moreover, we will later see that inertial frames are also impossible to achieve strictly in the sense of quantum fluctuating clocks. In this paper, we no longer discuss the Hamiltonian of systems (under general coordinates). Instead, the more appropriate fundamental concept is the action under general coordinates. The action method or path/functional integral method (we do not distinguish between path integrals and functional integrals in terminology) is the main method for handling generally coordinate-invariant quantum systems in this paper, because this method does not depend on specific choices of M^4 time splitting, nor does it depend on the existence of an external larger global space $M^5 = M^4$ s assumed by canonical quantum mechanics (Schrödinger method). From (V B) we will see that in general coordinates, it is precisely a Hamiltonian-constrained frame field system whose functional integral quantization yields general quantum coordinate transformations independent of frame field mass.

In this paper, quantum second-order moment fluctuations/quantum uncertainties of material particles/waves and material reference systems generally come from two parts: if there is non-zero Hamiltonian energy in a specific inertial frame, it gives the non-universal dynamical part of Schrödinger dynamics (therefore this broadening cannot be interpreted as universal spacetime broadening, just as various redshifts from proper motions of stars cannot be interpreted as cosmic redshift from spacetime expansion). The dynamical part can be eliminated by coordinate transformation, so there is no contribution from this part in generally coordinate-invariant theories due to Hamiltonian constraints. However, there is always a contribution from the non-dynamical pure geometric coarse-graining (Ricci flow) part (see IV). The geometric broadening part is coordinate-invariant, so unlike the dynamical part, it cannot be eliminated by coordinate transformation. Instead, geometric broadening brings coordinate transformation anomalies (see V C) that can only be eliminated through anomaly cancellation (see VI A). The geometric broadening part is universal and can be extracted and reinterpreted as universal spacetime broadening (just as cosmic redshift is interpreted as redshift from spacetime expansion and recession when stellar proper motions are small). In ordinary laboratories, the former broadening dominates (for instance, this broadening can be interpreted as various particle lifetimes), while the latter is relatively small, so spacetime can be considered classical. When the latter broadening becomes important (such as at cosmological scales, where universal second-order moment broadening of comoving spectral lines contributes second-order corrections to the distance-redshift relation and explains cosmic accelerated expansion), the quantum nature of spacetime cannot be ignored.

Thus we see that to reconcile the mass dependence of free-fall at the level of the Schrödinger equation or quantum uncertainty principle with the mass independence required at the level of quantum equivalence principle, we need to recognize that the particle/wave free-fall described by the Schrödinger equation is not strictly geodesic free-falling. Particles treated this way simultaneously carry both non-universal broadening from their own proper motion relative to inertial frames and universal geometric broadening comoving with the "spacetime container." Strict free-fall should be described by a generally coordinate-invariant Hamiltonian-constrained system, where there is only universal broadening without dynamical broadening from proper motion, such as in nonlinear σ-models. As given by the equivalence principle, its classical equations of motion are mass-independent geodesic equations, where gravity is introduced covariantly through geometric metrics rather than dynamically adding gravitational potentials to the Hamiltonian.

In systems with non-zero Hamiltonian, quantum second-order moment fluctuations driven by Hamiltonian energy dynamics are only part of the complete quantum second-order moment fluctuations, and this part is coordinate-dependent and non-universal. A quantum-level equivalence principle ensures we can always transform (particles without internal degrees of freedom) through coordinate transformation to a relatively "free-falling frame" (such as the aforementioned cosmic comoving expansion coordinate system), turning the original Hamiltonian in a specific inertial frame into a Hamiltonian constraint (i.e., one can always transform to a coordinate system at rest relative to the particle where particle broadening is zero). What holds in all general coordinate systems and is more natural is the Hamiltonian constraint. In Hamiltonian constraint systems, there is no evolution driven by Hamiltonian energy; zero Hamiltonian energy drives neither changes in first-order averages nor second-order quantum fluctuation broadening. Changes in first-order averages and second-order quantum broadening of "free" (without interactions other than gravity) material particles both come from universal relative changes occurring during relative motion between material particles and material reference systems, such as universal changes in coordinate system measures and/or quantum renormalization or coarse-graining of coordinate systems. More specifically, the quantum equivalence principle requires that in a free-falling frame, the universal quantum fluctuation broadening of material reference systems does not come from dynamical broadening due to Hamiltonian energy of material reference systems in inertial frames, but from coarse-graining broadening at the cutoff scale of quantum fluctuations of material particles and/or material reference systems under general coordinate systems (Ricci flow obtained from path integral quantization of zero-Hamiltonian constraint systems). The Ricci flow coarse-graining effect of material reference systems also brings changes in reference system measures due to diffeomorphism anomalies (see V C).

Mass independence (or universality) of particle propagation (all orders of moments) at the quantum level is only one aspect of the quantum equivalence principle. On the other hand, we see that besides being able to transform non-universal dynamical broadening given by non-zero Hamiltonian to zero through coordinate transformation, those broadening and anomalies caused by coarse-graining (see V C) can also be universally subtracted through anomaly counterterms (see VI A), so locally in spacetime one can always return to standard results of a broadening-free, gravity-free, temperature-free relatively stationary inertial frame. Based on this, analogies to other equivalent formulations of classical equivalence principle can also be obtained, such as the indistinguishability at quantum level between local gravity, acceleration, and thermal effects of frames (see VII E), etc. We can view the geodesic motion or free-fall process of particles/waves entirely as a universal general geometric coordinate transformation process, and even all phenomena related to gravity can be viewed as processes of general coordinate transformations. (Quantum) gravity can be universally introduced and eliminated through (quantum) general coordinate transformations (producing quantum anomalies and counterterms) (see V C and VI). The (quantum) equivalence principle leads to (quantum) geometrization of (quantum) gravity.

III. Quantum Reference Systems

Essentially, we have never "truly" measured spacetime itself; all our measurements of spacetime are indirect measurements using some material reference system (such as light or other material fields as reference systems). For example, a contemporary version of Gauss's consideration of large-scale triangulation measurement: to measure whether the interior angles of a triangle sum to 180 degrees in Earth' s gravitational background, one needs to set up three angle-measuring observation points at the triangle' s vertices to measure the angles between the other two points as observed from each point. However, due to the equivalence principle, light inevitably free-falls in the same way as other objects of different masses, and light rays are bent by Earth' s gravity, making it impossible to strictly conclude that the sum of interior angles is 180 degrees in Euclidean geometry within a gravitational background.

A quantum equivalence principle is the physical foundation for us to measure spacetime through material reference systems to establish a theory of quantum reference frames. Based on this, we know how to use material rulers and clocks to measure those universal parts (independent of material reference system mass and other properties) that are reinterpreted as geometric properties of spacetime, rather than merely properties of the rulers and clocks themselves. These universal parts reflected in material reference systems can be not only first-order moments (averages) independent of material reference system mass at the classical level (such as coordinates of material reference systems), but also second-order moment quantum fluctuations of material reference systems at the quantum level (such as coordinate broadening of material reference systems). Attributing universal properties from material reference systems to spacetime may be a human instinct for a priori conceptual simplicity, just as in early human history when transactions were merely "barter," comparing values between different items was very inconvenient. Later, the development of universal currency became an abstract general value measure in all transactions, playing the role of a general equivalent and greatly simplifying transactions. Spacetime reference systems similarly play the role of a general equivalent for comparing different motions.

Let us first examine the simplest example of a reference system—the clock [1, 2] —to see how a reference system enters the description of physical motion in a not-so-obvious way.

A. Classical Clocks

Newton's laws of motion were established based on people's ability to physically measure time through mechanical clocks. Galileo discovered the isochronism of pendulums, which could serve as the working principle of mechanical clocks. Although there was no "standard clock" at the time to tell Galileo why pendulums were "isochronous," he could only rely on his only available "standard" : his pulse. By Newton' s time, he had mathematized the concept of time. Similar to parameterization of complex figures, Newton assumed the existence of mechanical instruments like Galileo' s pendulum that automatically "generate" a parameter: (Newtonian) time. Complex motions in nature, when parameterized by the time parameter, become greatly simplified. To establish his laws of motion, Newton first needed to define a clock through his first law, under which the equations of Newton' s second law take their simplest form. What Newton' s laws of motion aim to determine is the form of an object' s spatial coordinates X, Y, Z parameterized by Newtonian time t during motion in three-dimensional space, called the object' s equations of motion X(t), Y(t), Z(t).

However, after people gradually mastered electromagnetism through experiments in the 19th century, this Newtonian image of parameters generated by pendulums was challenged. Because Maxwell' s equations of electromagnetism do not satisfy Galileo' s principle of relativity—when Maxwell' s equations are transformed into a uniformly moving coordinate system, their form changes—meaning you seem to be able to determine through electromagnetic experiments whether an observer is in some kind of absolute motion, which contradicts the principle of relativity that motion is only relative. So were Maxwell' s equations wrong in moving coordinate systems, or was Galileo' s principle of relativity wrong? It wasn' t until Einstein clarified that the crux of the problem was that to make Maxwell' s equations in moving coordinate systems and the principle of relativity both hold, Newton' s global parameter t as time must be abandoned. In Einstein' s view, time is a value T that an observer reads from a clock, not Newton' s parameter t. This value T read by the observer from the clock differs in coordinate systems moving at different velocities, if the speed of light is the same in all inertial frames. Thus, to describe an object' s motion, besides needing three spatial coordinates $X(\tau)$, $Y(\tau)$, $Z(\tau)$, there is also Einstein' s clock $T(\tau)$, assuming a global parameter τ still exists. This assumed global parameter τ is merely a parameter serving as a common parameter for $X(\tau)$, $Y(\tau)$, $Z(\tau)$, $T(\tau)$, not necessarily related to time, although for historical reasons we often call it "proper time." Einstein abandoned the unobservable Newtonian global parameter t as time and adopted the physical clock reading T in coordinate systems moving at different velocities as the intermediate equivalent for comparing different motions (within the same inertial frame). Thus, the equations of motion for objects in inertial frames moving at different velocities changed from a set of functions X(t), Y(t), Z(t) to a set of functionals $X[T(\tau)]$, $Y[T(\tau)]$, $Z[T(\tau)]$.

B. Quantum Clocks

Although Einstein reinterpreted time in physical systems more strictly, Newton' s global parameter t still lives in current textbook quantum theory. In quantum mechanics, Heisenberg first abandoned the concept of unobservable electron orbits in atoms and only used observable quantities, such as atomic spectral lines. In Bohr' s early interpretation, since each spectral line is produced by a transition between two electron energy states, in Heisenberg' s picture, the electron' s spatial coordinate is no longer an ordinary number (c-number) but a square matrix (q-number) of electron transitions between different energy states. In other words, the electron' s spatial coordinate changes from a number to a matrix. Matrix mechanics took the first step toward what we recognize today as (textbook) quantum mechanics. Although microscopic particle spatial coordinates were reinterpreted in quantum mechanics, time in quantum mechanics remains Newton' s global parameter. To make quantum theory Lorentz invariant and valid in general Lorentz inertial frames, quantum field theory merely replaced Newton' s single global parameter t with four global parameters x, y, z, t, interpreting them as Minkowski spacetime coordinates. Quantum mechanics presupposes a classical external observer holding the generator of Newton' s parameter t, while quantum field theory' s classical external observer generates four global parameters. This existence of an external observer leads to some essential difficulties in textbook quantum theory, such as its inability to apply to the entire universe, because by definition, the universe has no exterior. Moreover, quantum theory' s division of the world into an observed quantum system and an external classical observer/instrument requires additional assumptions to handle the measurement process between the external classical observer and the observed quantum system, such as wavefunction collapse in the Copenhagen interpretation.

Einstein' s relativity tells us that time is no longer Newton' s global external parameter t, but the reading $T(\tau)$ of a clock in each inertial frame. A clock is a physical instrument that "generates" an idealized, standard motion $T(\tau)$, such as the uniform motion of objects in Newton' s first law, like the uniform circular motion of clock hands. Quantum theory tells us that all motions in the world are not idealized because they always undergo quantum fluctuations. In other words, there may be no absolutely ideal quantum clock without fluctuations in the world. However, the fact that an object used as a clock always undergoes quantum fluctuations may not be that important, because a clock only serves as an intermediate equivalent or reference object for comparing other motions, simplifying complex motions. Just as when you barter items directly, their values are complex and not easily discernible, but when you use a common intermediate equivalent "money" to exchange items, their values become clear at a glance. The value fluctuations of "money" itself are unimportant; what matters is only the relationships between different items and different motions.

To mathematically define a quantum clock as a reference system between different motions, the first question to ask is how to quantize Einstein' s clock $T(\tau)$ so that the clock satisfies quantum mechanics while constantly undergoing quantum fluctuations, and also satisfies the principle of relativity. The second question is what the classical-level motion equation functionals $X[T(\tau)]$, $Y[T(\tau)]$, $Z[T(\tau)]$ become at the quantum level.

For simplicity, we consider a one-dimensional moving object with only spatial coordinate $X(\tau)$, whose Hamiltonian for motion with parameter τ is H_X. We also consider a clock whose hand coordinate $T(\tau)$ evolves with parameter τ according to Hamiltonian H_T. Here X and T share a global parameter τ that can be interpreted as a global Newtonian parameter. We also assume no interaction between object $X(\tau)$ and clock $T(\tau)$; they evolve independently with parameter τ, so the total system Hamiltonian is H = H_X + H_T. The total system quantum state space is the direct product space H_X   H_T of the quantum state space H_X spanned by object coordinate $X(\tau)$ and the quantum state space H_T spanned by clock hand coordinate $T(\tau)$. However, the total system quantum state, as an eigenstate of the total Hamiltonian, is not necessarily simply a direct product state $|X(\tau)\quad|T(\tau)$, but generally an entangled state:

$$|X;\, T\ =\ \tau\ C\tau\ |X(\tau)\quad|T(\tau)$$

All measurement processes are comparisons between the measured system state and the measuring instrument state. Before measuring this system, we must first initialize it, called instrument calibration or scaling, which adjusts the states of object coordinate X and clock T so that the instrument (clock T) and measured system (object position X) establish a one-to-one correspondence, allowing measurement of the measured system' s state by reading the instrument state. The superposition coefficients C_τ of entangled state (1) are given by the calibrated state preparation. For example, adjusting at $\tau = 0$ so that the clock hand is in state $T(0) = 0$ while the object' s coordinate is at position $X(0) = X_0$. This process is equivalent to letting them undergo an instantaneous interaction, after which they no longer interact but begin to evolve independently with τ according to their respective Hamiltonians. In subsequent evolution, when the clock is in some evolving state |T(τ) , the object must also be in some evolving state |X(τ) , so the total system state is |X(τ) |T(τ) with amplitude C_τ and probability $|C\_\tau|^2$. This amplitude or probability of the object position being in the corresponding state when the clock is in some state is prepared during instrument calibration. Moreover, quantum principles also allow the existence of superpositions of these one-to-one states, superimposing all possible one-to-one states of clock and object position with amplitude C_τ—this is the entangled state (1). Mathematically, this entangled state establishes a one-to-one (probabilistic) mapping |X; T : |T(τ) → |X(τ) between all possible clock states |T(τ) and object position states |X(τ) . This mathematical mapping is essentially no different from the functional mapping X[T(τ)]: T(τ) → X(τ). It can be regarded as a quantum version of the motion equation functional X[T(τ)].

Unlike deterministic events described by motion equation functionals, this entangled state describes a probabilistic event: when the clock is in state |T(τ) , the object position is in state |X(τ) , with normalized joint probability $|C\_\tau|^2$ for these two events occurring simultaneously. Unlike simple direct product states |T |X , the entangled state |X; T is an inseparable state, meaning the joint probability $|C\_\tau|^2$ does not equal the simple product of the normalized probability $|A\_\tau|^2$ of measuring the clock alone in state |T(τ) and the normalized probability $|B\_\tau|^2$ of measuring the object position alone in state |X(τ) :

$$|C\_\tau|^2 \neq |A\_\tau|^2|B\_\tau|^2$$

where |T = τ $B\tau$|X(τ) is some quantum state of each subsystem (called the conditional state or reference state of the subsystem), expanded in terms of basis vectors in the entangled state. Therefore, the probability of object position being in state |X(τ) given that the clock is definitely in state |T , is a conditional probability (or relational probability). From the conditional probability formula we obtain:

$$P(X(\tau)|T) = P(X(\tau) \quad T(\tau)) \ / \ P(T(\tau)) = |C\_\tau|^2 \ / \ |A\_\tau|^2$$

where $P(T(\tau)) = P(T(\tau)|T) = |A\_\tau|^2$ is the probability of reference state |T being in entangled subsystem state |T(τ) , and $P(X(\tau) \quad T(\tau)) = |C\_\tau|^2$ is the joint probability of the entangled state being simultaneously in |X(τ) and |T(τ) .

We call the quantum state of the quantum clock or measuring instrument |T the conditional state (or reference state), which is arbitrarily input by the observer and depends on under what conditions the observer wants the entangled state |X; T to output relative probabilities—this is the "relative probability" interpretation of entangled states. This is an interpretation different from standard quantum mechanics's "absolute probability" interpretation, adding consideration of the state of the instrument subsystem on the basis of absolute probability interpretation of entangled states. In the current framework, to extract physically meaningful measurement results from entangled states describing the relationship between measured quantum systems and quantum instruments, one must use relative probability interpretation.

We see that the entangled state |X; T cannot be expressed as a direct product state |X; T $\neq$ |X |T . The relative probability of this object position being in state |X(τ) given that the clock is in state |T is not equal to the probability appearing when measuring object position alone, i.e., $|C\_\tau|^2 \neq |B\_\tau|^2$. Therefore, the inseparable entangled state |X; T can only be interpreted probabilistically based on the relationship between two states. The state |X(τ) of object position only has physical meaning relative to the state |T(τ) of the clock; discussing the object position state alone or the clock state alone has no absolute meaning, and their quantum probabilistic behaviors are completely different.

The inseparability of entangled state (1), or the inability to write it as a global direct product state, means mathematically that entangled state |X; T cannot be covered by a global clock state for all possible position states. It can only locally map out a clock state |T(τ) on local basis |X(τ) , then superpose (glue) all local basis states to obtain the overall quantum state. In other words, the entangled state |X; T = τ $C\tau$ |X(τ) |T(τ) is a non-trivial fiber bundle that grows fibers |X(τ) on local base space |T(τ) , rather than a trivial fiber bundle of global direct product state |X |T . |T(τ) is the local base space, |X(τ) is a section on the fiber growing from the local base space, and the entangled state |X; T is the fiber bundle. The projection from fiber bundle to fiber section |X(τ) gives the conditional amplitude of |X(τ) occurring under the condition of |T .

Only when the probability P(X(τ)|T) of object position state appearing equals the probability of measuring object position and clock separately—i.e., when separable $|C\_\tau|^2 = |A\_\tau|^2|B\_\tau|^2$—can we use a global time to cover all possible position states. Then the entangled state |X; T returns to a separable direct product state |X |T or trivial bundle, allowing the entire background state of the clock to be separated out, becoming the standard textbook quantum state |X describing the probability $|B\_\tau|^2$ of measuring object position X alone at global time τ. Therefore, we need a relative quantum mechanical interpretation based on "relative relationship between states |X(τ) and |T(τ) " described by local entangled states, to extend the textbook "absolute quantum state |X(τ) " Copenhagen interpretation of quantum mechanics. This is analogous to extending global Cartesian geometry to curved Riemannian geometry based on tangent bundles over local base spaces. Establishing a relational quantum theory and intrinsic interpretation that measures one subspace (quantum measured system) through another subspace (quantum measuring instrument) in a large quantum state space is particularly urgent for interpreting quantum gravity, as this is already a coordinate-independent relativity theory without absolute external space.

An important difference between non-trivial fiber bundles described by entangled states and global trivial bundles described by ordinary quantum mechanics is that, due to the lack of a global base space in non-trivial bundles, local basis vectors on base space T are not necessarily orthogonal, forming a non-orthogonal basis, but give the metric on base space T: T(τ)|T(τ ') = s_{$\tau\tau$' }, which is not necessarily gauge-equivalent to Kronecker' s delta, i.e.,

T(τ)|T(τ') $\neq$ δ_{$\tau\tau$' }

This means the base space of non-trivial fiber bundles can be completely "curved," unlike the "flat" basis vectors of global trivial bundles in ordinary quantum mechanics. Orthonormality or invariant inner product must now be written in the form of basis vector |T and dual basis vector T|.

Since expansion coefficients A_τ are normalized, we have τ *Aτ^** A_τ = τ *s*{$\tau\tau$} = 1. Therefore:

T(τ)|T(τ ') = A_τ^* A_{τ' } = T(τ)|T(τ ') = A_τ A_{τ' }^* = δ_{$\tau\tau$' }

Thus, the second method for calculating relative amplitudes is that the relative amplitude is the "conditional projection" of entangled state vector |X; T in the double quantum state space H_X H_T onto the reference state |T of the clock, where the reference state |T can be expanded in terms of the clock part basis |T(τ) in the entangled state:

T| = *{τ,τ '} A_τ^* T(τ)/* = {τ,τ' } δ{τ' τ} *A_τ^* T(τ')/* = τ s_{$\tau\tau$ '} T(τ')| = A_τ^* A_{τ '}^* T(τ')| = *{τ,τ '} A_{τ' } T(τ')/ Aτ^**

The conditional projection yields:

T|X; T = τ *Cτ* |X(τ) T|T(τ) = *{τ,τ '} C_τ A_τ^* T(τ')/T(τ) /X(τ)* = {τ,τ '} C_τ A_τ^* s_{$\tau\tau$' }|X(τ) = τ *Cτ* A_τ |X(τ)

The coefficient C_τ A_τ now reveals the relative phase between subsystems of the entangled state, giving relative probability |C_τ|^2 that no longer has absolute meaning (relative to absolute normalization coefficient |A_τ|^2), but only relative meaning relative to components A_τ (length |A_τ|^2) of subspace H_T. More precisely, what we call "relative relationship" between two entangled systems in this paper essentially describes the "relative component" of entangled state vector components on one subspace relative to another subspace. This "relative component" is the relative amplitude or conditional amplitude.

The third method for calculating relative amplitude or relative probability notes that this relative probability is also equivalent to taking the partial trace of the entangled state' s density matrix _{X,T} = |X; T X; T| with respect to the reference state |T :

Tr_T(_{X,T}) = T|X; T X; T|T = *{τ,τ '} C_τ A_τ |X(τ) X(τ')|* = $C\tau^{*}$ A_τ' * s_{ττ '}|X(τ) X(τ')|

Here, taking the partial trace of the entangled state is not simply summing diagonal elements in the subspace. This is similar to taking the trace of a matrix in curved Riemannian space no longer being simply adding all diagonal elements, but requiring contraction with the curved metric (now s_{ττ' }), i.e., Tr_T(O) = O_{ }s^{ } ≠ O_{ }δ^{ }. This is an important difference between entangled-state-based quantum mechanics and ordinary (global) quantum mechanics.

Since entangled state amplitude C_τ is determined by instrument calibration preparation, not by fundamental equations of the theory, and does not change with any hypothetical external parameters (remaining constant during experiments), the entangled state is merely a "comparison table between instrument readings and measured system states (or relative probabilities)" prepared during calibration between the measured system and quantum instrument. The relative amplitude C_τ A_τ obtained through metric projection onto conditional states, its evolution, is completely determined by the metric s_{ττ' } of entangled state subsystems (formula (3)) and its evolution relative to another subsystem of the entangled state. The relative probability given by relative amplitude is physically measurable.

The metric s_{ττ '} of T subspace is given by the Ricci-flat Kähler-Einstein equation (see Appendix). This equation relates the intrinsic curvature of the non-flat subspace H_T embedded in the flat quantum state space H_X H_T to the extrinsic curvature of relative X subspace evolution, serving as the equation of motion for the metric s_{ττ' } of T subspace H_T relative to state vectors of X subspace. Similarly, we have the Ricci-flat Kähler-Einstein equation for the metric of X subspace H_X describing its relative evolution with respect to state vectors of T subspace, giving measured systems and measuring instruments completely equal status. More discussion on Ricci-flat Kähler-Einstein equations and their relationship to the Schrödinger equation will be placed in the Appendix, which is a formulation based on quantum states. Below we will approach this problem from the perspective of operator-based relative evolution. The advantage of the operator method is that operators, as q-numbers (quantum numbers), can be treated as ordinary c-numbers (classical numbers) when making mean-field approximations, making it easier to return to past standard quantum theory and facilitating comparison with past standard quantum theory to see how this theory transcends standard theory.

As mentioned earlier, what has physical meaning is the relative state of object position relative to clock hand position. Since the action form does not involve clock T as an operator but can be treated as an ordinary number, to see how the object' s position X operator changes with clock hand position T operator, we consider the action form of this theory. Assuming the actions of object and clock are:

S_X = ∫ dτ [½ m_X (dX/dτ)^2 - V(X)],
S_T = ∫ dτ ½ m_T (dT/dτ)^2

where the global parameter τ can be interpreted as laboratory proper time. The object' s action is kinetic energy minus potential energy V(X) integrated over laboratory time, while we assume the clock hand moves uniformly with laboratory time τ, so the clock' s action only has kinetic energy. Since there is no interaction between them, the total system action is the direct sum:

S[X, T] = S_X + S_T = ∫ dτ [½ m_X (dX/dτ)^2 - V(X) + ½ m_T (dT/dτ)^2]

We observe that since the total action S[X, T] contains no first-order term in T, the system' s total average energy ⟨E⟩ = 0. Thus the total Hamiltonian H_X + H_T of the system equals 0, and the quantum state |X; T⟩ of the system satisfies the Schrödinger equation that is strictly speaking a Wheeler-DeWitt equation:

(H_X + H_T) |X; T⟩ = iℏ ∂|X; T⟩/∂T = 0

The object' s position X evolves relative to laboratory proper time τ. To make it evolve relative to clock T, we only need to rewrite the action as:

S[X, T] = ∫ dτ [½ m_X (dX/dτ)^2 - V(X) + ½ m_T (dT/dτ)^2] = ∫ dT [½ m_X (dX/dT)^2 (dT/dτ) - V(X) (dτ/dT) + ½ m_T (dT/dτ)]

where dτ/dT is the lapse function, i.e., the Jacobian matrix of time integration measure, transforming the original integral over τ to an integral over T. Quantizing this theory is equivalent to calculating the partition function:

Z = ∫ DX DT e^{iS[X,T]}

If we adopt the semiclassical approximation, considering only the average value ⟨T⟩ of the clock and ignoring the fluctuation variance ⟨(ΔT)^2⟩, then the contribution from the classical path ⟨T⟩ dominates the path integral over T. Taking its average value in the action, the partition function becomes:

Z ≈ ∫ DX e^{iS[X(⟨T⟩)]}

Up to a constant, we obtain an action very similar to S_X, only replacing the laboratory proper time parameter τ with clock time T, and replacing the derivative of X with respect to τ in the kinetic term with the functional derivative of X with respect to T.

S[X(⟨T⟩)] = ∫ dT [½ m_X (dX/dT)^2 - V(X)] + const

where the effective mass of the object relative to clock time T becomes:

M_X = m_X (d⟨T⟩/dτ)$^{-1}$

Only when the clock runs at the same rate as laboratory proper time, i.e., when the lapse function d⟨T⟩/dτ = 1, does M_X equal m_X. Thus we see that mass is not an invariant under general coordinate transformations, but only under inertial coordinate transformations.

When quantum fluctuations of the clock $(\Delta T)^2$ cannot be ignored, i.e., when the clock eigenstate τ' $|T(\tau)$ can no longer be semiclassically approximated by a delta function but must consider its second-order moment width, such as becoming a finite-width coherent state, this will lead to many important consequences, the main topics discussed in this paper. First, although the system' s average energy $E = 0$, clock fluctuations $(\Delta T)^2$ will lead to vacuum energy fluctuations $(\Delta E)^2 \neq 0$, giving the correct contribution of vacuum energy to the cosmological constant. Second, the Schrödinger equation no longer holds strictly, leading to strict violation of temporal unitarity and spacetime thermal effects, which we will discuss more generally later in the context of anomalies in general spacetime coordinate transformations. To not limit ourselves to considering only quantum effects of clocks, but to consider quantum effects when measuring spacetime more generally, we need to extend the treatment of quantum clocks to more general quantum spacetime reference frames.

C. Quantum Spacetime Reference Systems and Nonlinear σ-Models

In (II) we saw that non-universal (mass-dependent) dynamical fluctuations given by general test particle systems cannot measure the intrinsic fluctuations of spacetime itself. Quantum dynamical broadening controlled by system Hamiltonian can be eliminated by coordinate transformation, because the Hamiltonian energy of a particle without internal degrees of freedom can always be transformed to zero by coordinate transformation to the particle' s rest frame —a Hamiltonian constraint. To find a coordinate-independent, covariant material reference system describing universal quantum fluctuations of spacetime (independent of material reference system mass) that generalizes the quantum clock from the previous section, we have a material reference system frame field theory described by a nonlinear σ-model (NLSM). The Hamiltonian of this theory is automatically zero, i.e., valid in general coordinate systems (no need for coordinate transformation to the rest frame to make Hamiltonian zero). Its classical equations of motion are mass-independent geodesic equations, and gravitational action is introduced geometrically (free-fall is mass-independent) rather than dynamically adding gravitational potentials to the Hamiltonian (free-fall is mass-dependent). Based on this, the quantum equivalence principle ensures we can measure universal quantum properties of spacetime through this quantum material reference system.

For simplicity, we consider a scalar field (x) on laboratory spacetime coordinates $x = (x^0, x^1, x^2, x^3)$, as a generalization of the one-dimensional moving object position $X(\tau)$ from the previous section. As a generalization of the quantum clock $T(\tau)$ from the previous section, we consider that measuring an event where the scalar field occurs requires at least 4 spacetime coordinates. For example, the usual Global Positioning System (GPS) requires measuring distances to at least 3 satellites to locate a person' s position on Earth, because at least three spheres intersect at a point in space. If we also want to measure the time when the person is at that position, we need to measure the distance to at least one additional satellite. Therefore, to measure when and where an event occurs in spacetime for a scalar field, we need at least 4 scalar fields X^ = (X^0, X^1, X^2, X^3) as 4 satellites for spacetime positioning. Without loss of generality, we can assume scalar fields $X^1(x)$, $X^2(x)$, $X^3(x)$ measure the three spatial coordinates of the event, while scalar field $X^0(x)$ measures the time of the event. We call these 4 spinless scalar fields frame fields, as a generalization of quantum spacetime reference systems from the previous section's quantum clocks. The 4 scalar fields X^ (x) share the same set of laboratory global parameters as the event scalar field (x) to be measured, i.e., laboratory spacetime coordinates x = (x^0, x^1, x^2, x^3), as a generalization of laboratory proper time parameter τ. We also similarly assume no interaction between the scalar field event (x) and the 4 frame fields X^ (x); they are distributed and evolve independently in the laboratory frame. Therefore, the total system action is the sum of their actions S = S[ ] + S[X].

The total system quantum state space is the direct product space H_ H_X of the quantum state space H_ spanned by the field (x) to be studied and the quantum state space H_X spanned by frame fields. Similarly, the total system quantum state is not necessarily a simple direct product state, but generally an entangled state:

| ; X = d^4x C(x) | (x) |X(x)

where we generalize the sum over τ to a continuous integral over 4 laboratory global parameters x. The quantum state of frame fields is:

|X(x) |X^0, X^1, X^2, X^3 = *{permutations of* σ*}* { σ} |X^ |X^ |X^ |X^σ

where the sum runs over all index permutations to ensure the 4 frame fields are oriented like laboratory spacetime x and maintain invariance under parity inversion, time reversal, and combined inversion. Therefore, the quantum state of frame fields is essentially also an entangled state. The preparation process of frame field entangled states is to make them interact as little as possible and measure them independently to form as orthogonal frames as possible. It is worth noting that since each frame field particle is labeled, they are not identical. This frame field quantum state can also be viewed as a many-body quantum state of non-identical independent particles, which we will see in (VII B) satisfy Boltzmann statistical distribution rather than Bose statistics at thermal equilibrium.

The reason why | ; X is an entangled state is completely similar. Because the instrument calibration process before measuring the event is the process of preparing the entangled state. For example, still using GPS satellites as an example, we must first precisely calibrate the positions of 4 independent satellites and the clocks on them through a known position on the ground. The entire calibration process is equivalent to letting a preset event on the ground and the states of 4 satellites undergo an interaction, after which they no longer interact but are independently distributed and evolved. This calibration process establishes a one-to-one mapping | ; X : |X(x) $\rightarrow$ | (x) between all possible events | and all possible frame field/satellite configuration states |X . This enables future measurements: once we measure that the frame field is in quantum configuration state |X , we can infer that the event is in the corresponding quantum state | , achieving the purpose of measuring the event through frame field instrument readings. This process is completely similar to the previous section: after calibrating the clock and object position, reading the clock state allows inference of the object position state.

Similarly, according to the standard probability interpretation of quantum states, the relative probability of measuring the event being in state | (x) under the condition that the frame field is in quantum state |X can also be calculated through the joint probability $|C(x)|^2$ of | and |X occurring together and the probability $|A(x)|^2$ of the frame field alone being in quantum state |X(x) (where |X = d^4x A(x)|X(x)):

P((x)|X(x)) = $|C(x)|^2$ / $|A(x)|^2$

This is the relative probability.

The spacetime part |X in entangled state (9) plays the role of a smearing function that blurs any physical quantity generated on spacetime (such as the local field operator (x) here). When you integrate out the spacetime part in the density matrix of the entangled state of this physical quantity and spacetime (partial trace), it is a process of smearing the physical quantity in spacetime, which improves the convergence of physical quantities (such as correlation functions of local field operators). As we will see later, this is the fundamental reason why Ricci flow through spacetime smearing can alleviate singularities in calculations of local operator correlation functions.

Mathematically, | ; X : |X(x) $\rightarrow$ | (x) being an entangled state also shows that this mapping can only be expanded locally on local frame |X(x) to express | ; X , and cannot cover all possible event states through a global frame as basis, i.e., cannot write | ; X as a direct product state. This shows that if there is no global external spacetime embedding all events to write the overall quantum state, then this local mapping of entangled states is particularly suitable for locally describing events on "curved" frame configurations, then superposing (gluing) all possible local frame configurations to obtain the overall quantum state, making the final overall quantum state no longer dependent on the choice of local frames. This is completely analogous to: if there is no flat external 3-dimensional space to view a curved 2-dimensional space, then an ant living on this 2-dimensional space can only first locally establish local flat coordinate patches to describe events on local coordinate systems, then glue all local flat coordinate patches together in a certain way to form an overall curved 2-dimensional space, making the glued entire curved 2-dimensional space no longer dependent on the choice of coordinates on specific local patches. This 2-dimensional space possesses its own intrinsic geometry without needing to study it through extrinsic geometry embedded in an external 3-dimensional space. Current formulations of quantum theory, whether quantum mechanics or quantum field theory, actually depend on the existence of a global external inertial frame: quantum mechanics is formulated on Galilean inertial frames, while quantum field theory is formulated on Minkowski spacetime Lorentz inertial frames. If we view the (absolute) quantum theory with global basis vectors in quantum theory textbooks as analogous to Cartesian geometry, then we can view this (relative) quantum theory without global basis vectors based on entangled states as an intrinsic geometric manifold that does not depend on any external coordinates, independent of the existence of global basis vectors. In this paper, we still assume the existence of global basis vectors x of entangled state (9) and interpret them as laboratory (inertial) frames, because this is more compatible with traditional quantum theory quantization methods based on inertial frames. But we must remember that what has physical meaning is only the relative relationship between the studied system | and reference frame |X in entangled state (9).

Removing the concept of global coordinate systems from quantum theory and making the theory independent of local coordinate system choices can be regarded as one of the main goals of this paper.

Now let us examine how the action form of frame fields in the experimental spacetime generalizes the action of quantum clocks (6) from the previous section. In principle, if the quantum equivalence principle holds, any 4 scalar fields that are as independent as possible can in principle serve as "instruments" for measuring events occurring in spacetime, as long as you can appropriately extract the universal properties of those frame fields. Then these universal properties describe not only the physical properties of the frame fields themselves but also the properties of spacetime. We can imagine that in the laboratory frame, we first use laboratory walls and clocks as initial references to prepare 4 free scalar fields, similar to the actions of 4 quantum clocks. For example, emit scalar field particle beams X^1 along the direction of laboratory wall x^1, etc., and set scalar field X^0 according to the laboratory clock x^0. In this way, the 4 scalar fields are prepared as particle rulers and clocks with the same orientation and arrangement as laboratory spacetime, making the classical solutions of frame fields easier to write:

X^ (x) = e^ _a x^a, X / x e^ _

called the vierbein. According to the corresponding placement of x^a and X^ , and stipulating that frame field quantities are isometric with laboratory coordinates, this vierbein is a Kronecker delta, e^ _a = δ^ _a.

A concrete example can be imagined as a multi-wire proportional chamber placed around accelerator collision points to measure particle spatial trajectories. A multi-wire proportional chamber is a series of very thin metal wires arranged neatly, placed along certain directions with orthogonal laboratory walls as basic references. When a particle event triggers discharge in gas around a certain wire, this discharge electrical signal is transmitted along the wire direction to the detector. This discharge, as an electronic pulse signal, acts like the frame field used, inferring the spatial position and occurrence time of the event by reading these frame field signals such as time-of-flight.

Because the 4 spacetime coordinates of an event can be reduced to measuring distances or time differences to 4 independent detectors (such as independent multi-wire chamber electronic pulse detectors or independent GPS satellites), we write the action of 4 frame fields as the sum of 4 free scalar fields on laboratory flat background spacetime parameters:

S_X = (1/2λ) d^{4}x ^{ab} _a X^ b X^ g{ }

where ^{ab} = diag(-1, 1, 1, 1) is the laboratory flat metric, and we adopt Einstein summation convention to sum over repeated indices. Since the 4 free scalar fields X^ are also mutually independent, non-interacting, and mutually entangled, their actions are simply added together.

The mutual independence and orthogonality of the 4 frame fields X^ only holds approximately in the finite laboratory frame when placing these frame fields with reference to laboratory walls and clocks. When extrapolated beyond the laboratory frame, these frame particle beams can no longer be placed with reference to laboratory walls and clocks. As frame fields gradually broaden in wave packets over long distances, due to the quantum equivalence principle, this broadening of frame fields cannot be distinguished from particle acceleration or experienced gravity. These particle beams appear to universally begin "free-falling" in this effective gravity. When beyond the laboratory frame at relatively large scales, this broadening or "free-falling" becomes important, making these orthogonal frame fields no longer strictly orthogonal in the gravitational background beyond laboratory frame scales. For example, broadening has an effective gravity along the x^3 direction. Although in the laboratory frame the X^1 particle beam is initially placed with reference to x^1 and is orthogonal to x^3, as the particle beam flies increasingly farther, the X^1 particle beam gradually develops a motion component in the gravity direction x^3. Since X^3 is still in the x^3 and gravity direction, the motion directions of X^1 and X^3 gradually become non-orthogonal. Or equivalently, one can view it as: originally at laboratory scales X^1 and X^3 had no wavefunction overlap, but due to X^1 gradually broadening, at large scales the wavefunctions of X^1 and X^3 gradually overlap and become non-orthogonal.

But we can always locally prepare another set of dual local frame fields {X' }^ = ({X' }^0, {X' }^1, {X' }^2, {X' }^3) in the gravitational background through "free-fall" experiments, making them orthogonal to each original frame field component respectively. For example, if you follow the motion of frame particle X^1, X^1 gradually bends due to gravity, then you can locally prepare several other frame fields ({X' }^2, {X' }^3) that are orthogonal to the direction of X^1. From a mathematical perspective, if we view the instantaneous motion direction components of those frame particles as "tangent" frame fields |X H_X, then near the coordinate system following the free-fall of frame fields, we can always establish a local "cotangent" frame field |X̄(x) |{X' }^ = {permutations} { σ} |{X' }^ |{X' }^ |{X' }^ |{X' }^σ where we write indices upstairs to indicate dual cotangent frame fields.

These two sets of mutually dual frame fields are complete and sufficient to represent all possible quantum states. Quantum state |Ψ can be expanded locally both through "tangent" frame fields:

| ; X = d^{4}x C(x) | (x) |X(x)

and through "cotangent" frame fields:

| ; X̄ = d^{4}x C̄(x) | (x) |X̄(x)

If tangent and cotangent frame fields coincide, this means there exists a global flat frame to expand |Ψ , because the frame fields become global frame fields independent of x and can be taken outside the integral sign of the entangled state. Then | ; X can be written as a direct product state rather than an entangled state.

With the aid of the "invariant" inner product formed by tangent and cotangent frame field states (the "invariant" in quotes only means invariant under classical coordinate transformations and parallel transport; we will see later that quantum fluctuations will change the inner product), we can generalize the sum over same lower indices in the frame field action (11) to an inner product sum over same upper and lower indices of tangent and cotangent frame fields. This makes the action invariant under frame broadening or equivalent acceleration or equivalent gravity:

S_X = (1/2λ) d^{4}x ^{ab} g_{ } _a X^ _b X^

This frame field action is called the nonlinear σ-model (NLSM), as a generalization of the quantum clock action (6) from the previous section. Since both X^ and x^a have length dimension [L], λ is a constant with dimension [L^{-4}]. We will later see that to return to standard gravity theory, this constant needs to take the value of cosmic critical density $\lambda = 3H_0{}^2/(8\pi G)$ $(10^{-3}$ eV$)^4$, where H_0 is the Hubble constant and G is Newton' s constant.

The laboratory global spacetime parameters x^a are called the base space, while the spacetime reference system frame fields X^ are called the target space. NLSM is a mapping model of field theory, mapping base space coordinates to target space coordinates, X(x): x → X. Mathematically, NLSM generates a differentiable manifold X from a flat base space coordinate mapping. This differentiable manifold is generally curved; frame fields X are local linear coordinates (tangent or cotangent space coordinates), and wavefunction overlaps between different locals constitute the quadratic metric g_{ }. Therefore, this differentiable manifold is Riemannian, with g_{ } being the Riemannian metric.

NLSM mapping can also be viewed from the paradigm of double quantum state space direct product of quantum reference systems. If the quantum system to be studied is spacetime itself |X , and the quantum reference system is the laboratory frame |x , then their relationship is described by the entangled state between them |X |x . As we discussed earlier, this entangled state is a superposition of local mappings x → X. Only now we assume the laboratory frame is a flat global coordinate system, so we no longer need to first map local coordinate patches and then glue (superpose) them, but degenerate into a direct product state mapping. In physical terms, frame field X is a function or field on flat coordinate system. If all possible mappings are to be mathematically singularity-free, we will discuss in the next section the mathematical conditions for mapping singularity-free.

Obviously, the NLSM action form is invariant under pointwise general coordinate transformations {X' }^ (X) = {X' } / X X^ + b^ , i.e., e^ _ X^ + b^ . Therefore, this curved Riemannian manifold does not depend on the choice of local coordinate patches. This property is called classical diffeomorphism invariance, or general coordinate transformation invariance. However, this may be broken at the quantum level, which we will discuss in detail in (V C) regarding quantum anomalies of general coordinate invariance and their cancellation.

Since we consider here a metric with negative-definite time component, the quadratic distance form in this target space may be negative-definite, so the target space is actually a (pseudo-)Riemannian spacetime. In many classical physical situations, we do not need to worry about the indefinite metric of pseudo-Riemannian spacetime, as most theorems of Riemannian geometry can be extended to pseudo-Riemannian spacetime. The indefinite metric may cause some problems at the quantum level, which we will discuss later.

Furthermore, considering that the scalar field (x) to be studied will no longer refer to laboratory spacetime coordinates x in the future, but will refer to the quantum reference frame coordinates X constituted by frame fields, the entire theory will give a theory of the relationship between the scalar field state | to be studied and the frame field reference system state |X . Therefore, physically we expect this theory will no longer depend on laboratory coordinates x and their metric ^{ab}. Indeed, we observe that if you perform a base space coordinate transformation, turning Minkowski base space into Euclidean base space, $ix^0 \to x^4$, then det _{ab} → i det δ*{ab}, where* δ{ab} = diag(1, 1, 1, 1). So:

$d^4x(M)$ = det _{ab} $dx^0dx^1dx^2dx^3 = d^4x(E)$ det δ_{ab} $dx^1dx^2dx^3dx^4$

Thus the volume element is formally invariant. Moreover, after Wick rotation of base time from Minkowski to Euclidean, the base spacetime metric also changes from Minkowski to Euclidean _{ab} → δ_{ab}, so the entire action remains formally invariant:

S = (1/2λ) $d^4x(M)$ ^{ab} g_{ } _a X^ *b X^ = (1/2λ) $d^4x(E)$ δ^{ab}* *g*{ } _a X^ _b X^

In short, because the Lagrangian can be regarded as a topological invariant of dimension on base spacetime, we can without loss of generality treat the base spacetime as simple Euclidean flat spacetime, which brings some convenience when functionally integrating to quantize this theory.

D. Semiclassical Approximation of Quantum Spacetime Reference Frames

We discussed spacetime frame field reference systems earlier; now we examine the scalar field (x) to be studied. Without loss of generality, we consider its action to be a general scalar field action:

S_ = d^{4}x [½ ( )^2 - V( )]

as a generalization of the object action (6) from the previous section, where we have also assumed the laboratory frame x is Euclidean flat space. Since there is no interaction between scalar field and frame fields X, the total system action is:

S[, X] = S_ + S_X = d^{4}x [½ ( )^2 - V( )] + (1/2λ) d^{4}x g_{ } _a X^ ^a X^

Similar to the previous section, when quantum fluctuations (ΔX)^2 of the spacetime reference frame can be ignored, we consider only the semiclassical approximation of frame field averages X . As discussed earlier, we formally perform the functional integral partition function in Euclidean base space, but the results are actually the same in Minkowski and Euclidean spaces. We obtain the partition function:

Z = D DX e^{-S[,X]} D e^{-S[(X)]}

where the effective action under semiclassical approximation is:

S[(X)] = dV(X) [½ g^{ } _ _ - V( )]

In the semiclassical approximation, ignoring quantum fluctuations of frame fields, frame fields can simply be treated as numbers. Therefore, semiclassical approximation can be viewed merely as performing a coordinate transformation from base space coordinates x to target space coordinates X, while the Jacobian determinant is the classical change of integration measure from coordinate transformation. This coordinate transformation, at the semiclassical level, has transformed a theory originally formulated on flat Minkowski or Euclidean spacetime inertial frames for scalar field into a theory on general non-flat classical Riemannian spacetime background X. Equation (18) reproduces quantum field theory in a fixed curved spacetime.

From this semiclassical approximation result, we can also see that the dimensions of frame field spacetime reference system and base space are both equal to D = d = 4 under semiclassical approximation, because the Jacobian matrix is a square matrix, so the dimension d of base space must equal the dimension D = 4 of target space. Therefore, semiclassically, we describe frame field spacetime reference systems with a d = 4 NLSM. However, is a d = 4 NLSM well-defined at the quantum level? With current understanding, d = 4 NLSM may be non-renormalizable at the quantum level. Under certain extreme conditions, the mapping x → X may encounter mathematical singularities, causing the theory to be no longer well-defined in those extreme cases. So we only say d = 4 is for semiclassical approximation; at the semiclassical level, we can simply treat d as 4 without serious problems. But at the quantum level beyond semiclassical approximation, this cannot be done strictly. Fortunately, the dimension of base space is not actually an observable of the theory, but only a parameter of the theory, and this theory essentially describes only the relationship between and X, which can exist independently of base space. Base space in the formulation of this theory only serves as an initial laboratory reference to prepare a frame field reference system X and a field system to be studied. Currently, to prepare and describe these two field systems, we can only use the language of well-developed quantum field theory on flat backgrounds of inertial frames. We actually know that at the quantum level, the dimension d of base space suffers quantum corrections of anomalous dimensions, making d effectively deviate from the semiclassical value 4 in quantum fluctuations, even becoming fractional dimensions d < 4. We know that when d = 2 or d = 3, NLSM becomes completely renormalizable, and d = 2 is renormalizable in the sense of perturbative power counting, while d = 3, although non-renormalizable in perturbative power counting, is also non-perturbatively renormalizable as numerical calculations tell us, and fractional dimensions d < 4 are also renormalizable in the sense of analytic continuation. This phenomenon is more obvious from a topological perspective. Since NLSM is a quantum field theory model of mapping x → X, whether this mapping process encounters mathematical singularities determines whether field mapping X(x) encounters mathematical singularities. For simplicity, consider the target space spacetime configuration as a spherical compact spacetime configuration S^{D=4} after Wick rotation of time. Now we want to make a continuous differentiable mapping from a d-dimensional flat base space x to X S^4. All possible continuous differentiable mappings constitute a homotopy group $\pi d(S^4)$. *We observe that this homotopy group is trivial* $\pi\{d<4\}(S^4) = 0$ when d < 4, meaning all possible continuous differentiable mappings X(x) will not encounter essential topological obstacles, so all possible continuous differentiable mappings are mathematically singularity-free and can be well-defined. However, π_{d=4}(S^4) = is not a trivial group, indicating that some mappings will encounter topological obstacles. This topological obstacle cannot be avoided when path integral quantization sums over all possible paths, so mathematically singular and ill-defined mappings may appear, making this mathematical theory ill-defined at the quantum level. In other words, when this theory undergoes renormalization at the quantum level, it may encounter non-convergent mathematical singularities, the theory is non-renormalizable, and the theory fails at that point. So if you want all mappings at the quantum level, i.e., quantum field X(x), to be mathematically singularity-free, we need the homotopy group to be trivial, which can be achieved as long as d < 4, as proven by past perturbative proofs and non-perturbative numerical calculations. Therefore, to make the theory well-defined at the quantum level, we assume that frame fields are an NLSM defined on a d = 4 - dimensional base space, where 0 < 1 acts like a small parameter of dimensional regularization to avoid possible quantum-level singularities at d = 4. At the semiclassical level, as our common sense of laboratory life, we can simply treat d as 4 without serious problems. But at the quantum level beyond semiclassical approximation, this cannot be done strictly.

Note: Figure translations are in progress. See original paper for figures.

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