Long-term Evolution of Solar Activity and Prediction of the Following Solar Cycles (Postprint)

Authors: Peixin Luo and Baolin Tan

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Abstract

Solar activities have a great impact on modern high-tech systems, such as human aerospace activities, satellite communication and navigation, deep space exploration, and related scientific research. Therefore, studying the long-term evolution trend of solar activity and accurately predicting the future solar cycles are highly anticipated. Based on the wavelet transform and empirical function fitting of the longest recorded data of the annual average relative sunspot number (ASN) series of 323 yr to date, this work decisively verifies the existence of the solar century cycles and confirms that its length is about 104.0 yr, and the magnitude has a slightly increasing trend on the timescale of several hundred years. Based on this long-term evolutionary trend, we predict solar cycles 25 and 26 by using phase similar prediction methods. As for solar cycle 25, its maximum ASN will be about 146.7 ± 33.40, obviously stronger than solar cycle 24. The peak year will occur approximately in 2024, and the period will be about 11 ± 1 yr. As for solar cycle 26, it will start around 2030, and reach its maximum between 2035 and 2036, with maximum ASN of about 133.0 ± 3.200, and period of about 10 yr.

Full Text

Preamble

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Long-term Evolution of Solar Activity and Prediction of the Following Solar Cycles
Pei-Xin Luo[1],[2] and Bao-Lin Tan[1],[2]

[1] National Astronomical Observatories, Chinese Academy of Sciences, Beijing 100101, China; luopeixin23@mails.ucas.ac.cn
[2] School of Astronomy and Space Science, University of Chinese Academy of Sciences, Beijing 100049, China

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Abstract

Solar activity exerts profound impacts on modern high-technology systems, including human spaceflight, satellite communications and navigation, deep space exploration, and related scientific research. Consequently, investigating the long-term evolutionary trends of solar activity and accurately predicting future solar cycles represents a topic of considerable importance.

Based on wavelet transform and empirical function fitting of the longest available record of annual mean relative sunspot numbers (ASN)—spanning 323 years to date—this work decisively confirms the existence of solar century cycles and establishes their length at approximately 104.0 years, with magnitude exhibiting a slight increasing trend on timescales of several hundred years. Leveraging this long-term evolutionary trend, we predict solar cycles 25 and 26 using phase-similarity prediction methods. For solar cycle 25, the maximum ASN will reach approximately 146.7 ± 33.40, significantly stronger than solar cycle 24. The peak year will occur around 2024, with a period of about 11 ± 1 years. For solar cycle 26, it will commence around 2030, reach maximum between 2035 and 2036, with a maximum ASN of approximately 133.0 ± 3.200, and a period of about 10 years.

Key words: Sun: activity -Sun: evolution -(Sun:) sunspots -methods: data analysis

1. Introduction

As the nearest star to Earth and the unique star in our solar system, the Sun plays a crucial role in the formation and evolution of Earth and its near-Earth space environment. Simultaneously, solar activity can cause significant damage to modern high-technology human systems, including aerospace operations, satellite communications and navigation, large power grids, deep-space exploration, national security, and related scientific research. Studying long-term trends in solar activity helps address fundamental questions about how Earth' s future environment will change and humanity' s future trajectory. At minimum, such research enables prediction of the intensity and developmental trends of future solar activity.

Solar activity exhibits cyclical characteristics reflected in various indicators, including sunspot number, sunspot area, and 10.7 cm solar radio flux. The most prominent periodicity is the solar cycle with an average period of approximately

11 years, known as the Schwabe cycle, first reported by Schwabe & Schwabe Herrn (1844).

Over the past two centuries since the discovery of the Schwabe cycle, evidence for other periodic characteristics has emerged, including grand minima, the G-O rule showing differences between even and odd cycles, and the century cycle (Hathaway 2015). While some features are now well understood, others remain inconclusive. For example: Does the century cycle truly exist? What is its period? Does it change, and if so, how?

The solar century cycle refers to a periodicity of approximately one century, first proposed by Gleissberg (1939). By smoothing the amplitude of sunspot numbers between 1750 and 1928, Gleissberg identified a long period of 7–8 solar cycles—approaching a century—also known as the secular or Gleissberg cycle. In 1994, Rozelot (1994) applied Fourier analysis and suggested a century cycle period of 97.2 years. However, Garcia and Mouradian' s analysis (1998) four years later indicated periods of 78 or 81 years. Other researchers (Peristykh & Damon 2003) obtained evidence for an 88-year period through cosmogenic isotope analysis. Ogurtsov et al. (2002) more explicitly described the century cycle as a mixture of biperiodic structures with periods of 50–80 years and 90–140 years. Le & Wang (2003) proposed a period of approximately 101 years. More recently, some researchers (Ma 2009) have extended the century cycle length to between 60 and 150 years. Tan (2011) analyzed average relative sunspot numbers from 1700–2009 AD and suggested three types of solar cycles: the well-known 11-year Schwabe cycle, a 103-year century cycle, and a 51.5-year cycle. The century cycle periodicity also appears in solar proton events and may be linked to grand minima (McCracken et al. 2001).

Various explanations for the century cycle have been proposed. In 1999, Pipin (1999) developed a numerical model based on the dynamo mechanism describing solar magnetic field generation processes, estimating that the century cycle results from the re-establishment of differential rotation following magnetic feedback on angular momentum transport. Some researchers hold different views. In 1999, Hathaway et al. (1999) used the century cycle to predict future cycles and found that the best-fitting value of the century cycle varied continuously rather than remaining stable.

Schwabe (1843) made the first solar cycle prediction (Attia et al. 2020). After nearly two centuries of development, Petrovay (2010) classified solar cycle prediction methods into three categories: physics-based models, empirical precursors, and extrapolation methods. The model-based method primarily refers to solar magnetic field dynamo models. The precursor method relies on previous solar activity indicators or magnetic fields from earlier times to predict solar cycles (Petrovay 2020), mainly divided into two categories: geomagnetic index predictions and polar magnetic field predictions.

Polar magnetic field prediction as a precursor was first proposed by Schatten et al. (1978). Nandy (2021) considers the precursor method relatively superior to other methods in accuracy, particularly approaches using the polar field near solar cycle minimum as the precursor factor. Using geomagnetic indices as precursors dates back to 1966, when Ohl found that the aa index indicating geomagnetic activity near solar cycle minimum correlated well with the amplitude of the subsequent cycle (Hathaway 2015). According to the precursor method definition, not only geomagnetic indices and polar magnetic field magnitude can serve as precursors, but also sunspot numbers in the declining phase of a cycle (Brajša et al. 2022; Nagovitsyn & Ivanov 2023), the number of spotless days in the declining phase (Burud et al. 2021), etc.

Extrapolation methods are based on statistical analysis of various solar activity indicators to model behavior, then apply this model to reasonably extrapolate and predict subsequent solar cycles. Recently, this method was used to predict solar cycle 25 (Kakad & Kakad 2021). While many metrics are available for extrapolation methods, the most widely used is the sunspot number sequence. Sunspots have been observed for over three centuries and can be considered the world' s longest-running time series (Bhowmik & Nandy 2018), which is one of the main reasons the sunspot sequence was chosen as a predictor in this work. Additionally, similarity theory can be used to predict forthcoming cycles, assuming that two solar cycles with similar activity indicators should also have similar features (Du 2023).

Beyond these classifications, interdisciplinary approaches such as machine learning and neural networks for solar cycle prediction represent very popular research directions. Ramos et al. (2023) summarize results on using machine learning to predict solar cycle 25.

This work' s methodology combines extrapolation methods and similarity theory. First, we confirm the existence and long-term evolution trend of the solar century cycle, obtaining an empirical century cycle function through fitting methods. We then propose that an 11-year solar cycle at a similar phase in the century cycle should have similar characteristics (such as length, magnitude, etc.). Based on this, we make predictions for the current solar cycle 25 and the upcoming solar cycle 26. Section 2 describes the data and methods in detail. Section 3 presents the main results on the long-term evolution of solar activity and predictions for forthcoming solar cycles, with comparisons to other results. Finally, Section 4 summarizes the conclusions of this work.

2. Data and Methodology

The data used in this work are ASN and the 13-month smoothed monthly total sunspot number, compiled and released by the World Data Center (WDC)–Sunspot Index and Long-term Solar Observations (SILSO), Royal Observatory of Belgium, Brussels. These can be downloaded from www.sidc.be/SILSO/datafiles and include 323 years of ASN data recorded from 1700 AD to the present. This represents the longest continuous solar activity time series available to date and constitutes one of the most important fundamental datasets for studies of solar activity' s long-term evolution.

To better reflect the long-term trend of cycle maxima in the ASN series, we first assume a sinusoidal empirical function expressed as:

$$R(x) = A + B \sin\left(\frac{2\pi}{C}x + D\right) + Ex \tag{1}$$

Here, x represents years after 1700 AD, and R denotes the ASN value for that year. A, B, C, D, and E are five unknown parameters to be determined. The assumed empirical function (Equation (1)) is a nonlinear fit function, with parameters determined using mathematical analysis software. The constant A represents the background level, B is the magnitude of the century cycle, C is the period, D signifies the initial phase, and E reflects a long-term trend of the century cycles. Obviously, the century cycle is quasi-periodic. For convenience, we first determine parameters C and D before fitting the function. The wavelet analysis method is used to analyze the ASN sequence, combining actual data with consideration of solar cycles' quasi-periodicity to determine whether the century cycle ends, and finally to determine parameter values C and D.

Tan (2011) found $E = 0.06$, indicating a slowly increasing trend in solar activity on century timescales. Javaraiah (2017) also found that solar cycle maxima have a linear tendency to increase, though it has been argued that no such tendency exists since the Maunder Minimum (Svalgaard 2011). Therefore, the empirical function assumed in this work also incorporates linear variation, with its coefficient denoted by E. The function is fitted to determine whether there is a trend for the cycle maximum to increase ($E > 0$) or decrease ($E < 0$).

This work assumes that a Schwabe cycle located at the same or similar phases in the century cycle will have similar cycle characteristics, such as cycle maximum, rise time, and cycle length. We may then adopt the phase similarity method to predict the basic features of solar cycles 25 and 26. The specific steps are: (1) determine the empirical function corresponding to the century cycle; (2) extend the fitting function forward to determine the phases of solar cycles 25 and 26 in the century cycle; (3) extend the fitting function backward to identify previous cycles with the same or similar phase; and (4) compare solar cycles 25 and 26 with their corresponding similar cycles to determine their characteristic parameters.

3.1. Result

3.1.1. Determination of the Century Cycle

We employ wavelet transform analysis for the ASN time series. To ensure analytical accuracy, we subtract the mean from the original ASN series and perform symmetric extension on both sides to reduce boundary effects.

[Figure 1: see original paper] (a) shows the variance plot of wavelet coefficients after wavelet transform of the ASN sequence, while (b) depicts the contour plot of the real part of the wavelet coefficients. Figure 1(a) reveals that variance at approximately 104.0-year period reaches the maximum across all periods, indicating that the 104.0-year period constitutes the most significant periodic component of solar activity. Furthermore, in terms of peak sharpness, solar cycle peaks appear sharper, while the 104.0-year period peak is broader, indicating stronger quasi-periodic characteristics for the century cycle.

The solar cycle timescale obtained through wavelet analysis is slightly less than the well-known 11-year period. Due to the quasi-periodic nature of solar cycles, we consider this value within the reasonable range of solar cycle timescales. If we assume that the durations of 29 complete solar cycles observed since 1700 AD follow a normal distribution and perform Gaussian function fitting, the mean value is approximately 10.78, close to our obtained value. The third prominent crest shows a 56.20-year period, numerically close to half the century cycle. To more intuitively describe the periodicity of the ASN time series, Figure 1(b) presents the contour plot of the real part of wavelet coefficients. This contour plot can reflect periodic changes in the ASN time series across different periods and their distribution in the time domain, enabling judgment of future ASN trends at different periods. As shown in Figure 1(b), multiple cycle components exist in the ASN time series from 1700 AD to 2022 AD. The 10.74-year and 104.0-year cycle components persist consistently, whereas the 56.20-year period is only partially significant along the time axis, suggesting that the 56.20-year cycle may not be a stable periodic feature of solar activity.

Figure 2 plots the wavelet coefficients at the 104.0-year period. We find three complete cycles, with absolute amplitude gradually increasing over time, also indicating a gradual increasing trend in solar activity since 1700 AD.

Different studies have yielded different results for the century cycle timescale (Gleissberg 1939; Garcia & Mouradian 1998; Peristykh & Damon 2003). However, based on our analysis using the longest data series to date, we believe the more accurate century cycle timescale should be 104.0 years, close to results of 101 years (Le & Wang 2003), 103 years (Tan 2011), and 100 years (Feynman & Ruzmaikin 2014), but far from the 90-year result (Le Mouël et al. 2017).

After determining the century cycle timescale, we require more precise confirmation of the initial phase of the century cycle in the ASN series to fit the empirical trend equation. In an earlier study, Tan (2011) suggested that solar cycle 24 is located in the valley between the 3rd and 4th century cycles. We compare the 13-month smoothed monthly total sunspot numbers of corresponding months during the rising phases of solar cycles 24 and 25. The start of both cycles is defined by the month of minimum 13-month smoothed monthly total sunspot numbers: December 2008 and December 2019, respectively. Results are shown in Figure 3 [Figure 3: see original paper]. We find that the 13-month smoothed monthly total sunspot numbers for solar cycle 25, as of February 2023, are overall greater than corresponding values for solar cycle 24, indicating that solar cycle 25 is significantly stronger than solar cycle 24.

We must then determine the end time of the last century cycle (G3) and the start time of the forthcoming century cycle (G4). Our analytical results show that solar cycle 24 is the weakest cycle in the past half-century, while the current solar cycle 25 is obviously stronger than solar cycle 24. This fact strongly indicates that solar cycle 24 is precisely located at the lowest point between G3 and G4. Therefore, we conclude that the end of G3 and beginning of G4 occur in the middle of solar cycle 24, around April 2014. Moreover, solar cycle 25 is already in the rising phase of the forthcoming century cycle G4. Looking further back, 1702 AD marks the starting year of the first complete century cycle appearing in the ASN series.

The ASN sequence we used was recorded from 1700 AD, though sunspot observations confirm existence much earlier than 1700 AD. This paper takes 1700 AD as the starting point, with 58 data sets in terms of years, ending at the conclusion of solar cycle 24. The processed image appears as the black curve in Figure 4 [Figure 4: see original paper].

3.1.2. Fitting of Hypothesized Empirical Function

To minimize error, this paper focuses on the long-term trend of solar activity known as the century cycle and utilizes it to predict parameters of forthcoming solar cycles. To prevent our study from being affected by outliers in the ASN series, we smooth the ASN series to better emphasize the long-term trend: the maximum of each solar cycle is averaged with its minimum at the beginning and its minimum at the end. These averages serve as elements of the smoothed ASN series, with corresponding times being the closest years obtained by indexing real ASN values.

We assume the empirical function has the form of Equation (1), where parameter C is the length of the corresponding century cycle at 104.0 years, and parameter D, calculated from the fact that 1702 is the starting year of century cycle G1, has a value of -0.5385.

Using the least squares fitting method, we obtain values for parameters A, B, and E in Equation (1), yielding:

$$R(x) = 23.86 \sin\left(\frac{2\pi}{104.0}x - 0.5385\right) + 0.06028x + 52.84 \tag{2}$$

The coefficient of the linear term is 0.06028, indicating that solar activity intensity tends to increase linearly on century timescales. Figure 4 [Figure 4: see original paper] clearly shows the presence of the century cycle. The fitted function is extended outward by 150 years, shown by the red curve in Figure 4. Parameters of future solar cycles are inferred through extrapolation based on similarity of solar cycle parameters in the same phase of the century cycle.

3.1.3. Predictions for Solar Cycles 25 and 26

Our method employs phase-similarity prediction based on the evolutionary trend of the century cycle. Before prediction, we collate parameters of all solar cycles to date.

International solar cycle numbering began in 1755, with 24 complete solar cycles recorded so far. From 1700 to 1755, ASN data also encompassed five complete solar cycles, which for analysis purposes we have named alphabetically from latest to earliest as A, B, C, D, and E (Tan 2011).

Characteristic parameters describing solar cycles include: (a) t_0, the start year of the solar cycle; (b) R_{min}, the ASN minimum at cycle start; (c) T_{up}, the rising phase length (rise time); (d) t_{max}, the year of cycle maximum (peak time); (e) R_{max}, the ASN maximum of the cycle; (f) T_{de}, the decay phase length (fall time); (g) L, the cycle period; and (h) R_{at}, the asymmetric parameter—the ratio between rise time and fall time, representing whether the cycle shape is left-right symmetrical.

Table 1 lists parametric characteristics of 29 solar Schwabe cycles since 1700 AD. The period L ranges between 9 and 14 years, with 82.76% of all recorded solar cycles having periods between 10 and 12 years, confirming the quasi-cyclical nature of solar cycles. Second, substantial differences exist among maxima of different solar cycles. The smallest observed maximum occurred during solar cycle 6 at 76.30, while the highest maximum occurred during solar cycle 19 at 269.3.

Regarding the asymmetric parameter R_{at}, 75.86% of solar cycles have $R_{\mathrm{at}} < 1$, 13.79% have $R_{\mathrm{at}} > 1$, and 10.34% have $R_{\mathrm{at}} = 1$. This indicates that most solar cycles rise rapidly and fall slowly, belonging to the left-biased asymmetric type.

After clarifying these properties, we predict characteristics of forthcoming solar cycles 25 and 26. Figure 5 [Figure 5: see original paper] plots temporal profiles of 29 Schwabe cycles numbered 1–24 and A–E, overlaid on century cycles G1–G4 since 1700. Solar cycle 24 is located at the valley between century cycles G3 and G4, while cycles 25 and 26 are the first and second cycles in the ascending phase of century cycle G4. Considering solar cycles in the early part of century cycle G1 (A, B, C, D, and E) have substantial missing data (Clette et al. 2015; Clette & Lefèvre 2016), we exclude them when searching for corresponding past cycles.

We find that the phase of solar cycle 25 closely resembles that of solar cycle 6 in century cycle G2 and solar cycle 15 in century cycle G3, while solar cycle 26' s phase closely matches solar cycle 7 in G2 and solar cycle 16 in G3. We therefore apply averaged parameter values from cycles 6 and 15 to predict solar cycle 25, and averaged values from cycles 7 and 16 to predict solar cycle 26, while also considering the effect of the linear increasing term in Equation (2).

Solar cycle 25 predictions:

1. $R_{\max}$ of solar cycle 6 is 76.30, and $R_{\max}$ of solar cycle 15 is 173.6. Based on similarity principles, we speculate that $R_{\max}$ of solar cycle 25 should fall between 76.3 and 173.6. Additionally, since solar cycle 25' s corresponding value in the century cycle phase is higher than solar cycle 24' s trough value ($R_{\max} = 113.3$), and considering the overall centennial trend, we finally predict $R_{\max}$ of solar cycle 25 will be between 113.3 and 180.1, i.e., 146.7 ± 33.40.
2. L (period) for solar cycle 6 is 13 years and for solar cycle 15 is 10 years. In previous analyses, long cycles of 13 years accounted for only 6.9% of all cycles, including cycles 6 and 9. Solar cycle 6, which began in 1810, falls within the historically famous Dalton Minimum (1790–1830 AD). Based on Table 1 and century cycle phase analysis, we do not predict solar cycle 25' s intensity to be lower than solar cycle 24. Therefore, we tentatively conclude that solar cycle 25 will not enter a new minimum similar to the Maunder or Dalton minima. Regarding the relationship between solar cycle period and grand minima, one study (Karak & Choudhuri 2013) noted that periods may be prolonged in 1–2 solar cycles before entering a grand minimum. Combining these analyses, we believe the probability of a 13-year length for solar cycle 25 is very low, and therefore predict its period L will be about 11 ± 1 years.
3. R_{at} for both solar cycles 6 and 15 is less than 1, so we predict R_{at} of solar cycle 25 will also be less than 1, meaning rise time is less than fall time.
4. With predictions for period and asymmetric parameter, we predict a rise time of 4–5 years. Since solar cycle 25 actually started in December 2019, we predict $R_{\max}$ will occur in 2024.

Solar cycle 26 predictions:

1. $R_{\max}$ of solar cycle 7 is 117.4, and $R_{\max}$ of solar cycle 16 is 129.7. We apply the averaged value of cycles 7 and 16 to predict solar cycle 26. Considering the increasing trend of solar cycles (the last term of Equation (2)), we finally predict $R_{\max}$ of solar cycle 26 should be 133.0 ± 3.200.
2. L: both solar cycles 7 and 16 have periods of 10 years, so we predict solar cycle 26' s period will also be 10 years.
3. R_{at} of solar cycle 7 exceeds 1, while R_{at} of solar cycle 16 equals 1. Therefore, we predict R_{at} of solar cycle 26 will be greater than or equal to 1, meaning rise time will not be shorter than fall time.
4. T_{up}: the rise time of solar cycle 7 is 7 years and of solar cycle 16 is 5 years. Since $R_{\mathrm{at}}(= 2.333)$ of solar cycle 7 is the highest of all cycles, and considering period L and asymmetric parameter R_{at}, we predict a rise time of 5–6 years for solar cycle 26. In the previous section, our prediction for L of solar cycle 25 is 11 ± 1 years. In predicting solar cycle 26, we take the average of predicted L values for solar cycle 25. Thus, we consider solar

cycle 25 to end in 2030 and solar cycle 26 to begin in 2030. Therefore, we predict $R_{\max}$ of solar cycle 26 will occur between 2035 and 2036.

5. For $R_{\min}$, according to the fitted empirical function, solar activity shows an overall increasing tendency, so we predict $R_{\min}$ of solar cycle 26 will be greater than or equal to 3.6 (the $R_{\min}$ of solar cycle 25).

We plot prediction curves for solar cycles 25 and 26 as the blue dotted curve in Figure 5. Values are obtained by averaging the previous two similar cycles plus the appropriate solar activity trend. Our predictions above do not consider the influence of the 56.02-year period. Although this period ranks as the third strongest in Figure 1(a), its presence is not stable (Figure 1(b)), leading us to suspect that solar cycle 25 is not affected by this period. However, considering the 56.02-year period' s influence, Figure 5 shows that the first three century cycles all fluctuate at their peaks. Troughs are located in solar cycle A within century cycle G1, cycle 10 in G2, and cycle 20 in G3. Since solar cycles 25 and 26 are in the ascending phase of century cycle G4, this does not affect our predictions.

The good fit of the actual ASN sequence to the fitted century cycle curve in Figure 5 further validates the correctness of our identified century cycle timescale and the G3 ending time.

3.1.4. Comparison with Other Predictions

In addition to predicting solar cycle 25, this paper provides a new prediction for solar cycle 26. We compare our results with existing predictions for both cycles.

Many solar cycle 25 predictions have been published. Nandy (2021) compiled most predictions prior to cycle 25' s start. Since our predictions are made during cycle 25' s ascending phase, we selected more recent predictions made after cycle 25' s beginning (post-2019). We summarize these recent predictions, sorted chronologically by method category (Table 2). Note that different prediction methods yield varying accuracy, reflecting method characteristics, so we do not impose uniform accuracy requirements. According to the classification in the previous section, our method belongs to the extrapolation category, listed separately in the table.

Comparing $R_{\max}$ predictions in Table 2 reveals: (a) Guo et al. (2021) in the dynamo model category; Diego & Laurenza (2021) and Bisoi et al. (2020) in the precursor method; McIntosh et al. (2020) and Sarp et al. (2018) in the extrapolation method; and Su et al. (2023) and Prasad et al. (2022) in the interdisciplinary category all agree with our result that solar cycle 25 will have larger $R_{\max}$ than cycle 24. Among these, Diego & Laurenza (2021), who use the relationship between geomagnetic activity recurrence index in the previous cycle' s declining phase and sunspot number in the next cycle, reach the same conclusion that the century cycle ends at solar cycle 24. (b) Nagovitsyn & Ivanov (2023), Lu et al. (2022), Xiong et al. (2021), and Kumar et al. (2021)

in the precursor method also conclude that cycle 25 is stronger than cycle 24 based on their $R_{\max}$ predictions. (c) Javaraiah (2023) in both precursor and extrapolation methods, and the NOAA/NASA co-chaired international panel result (https://www.swpc.noaa.gov/news/solar-cycle-25-forecast-update) predict $R_{\max}$ values similar to cycle 24. (d) Labonville et al. (2019) in the dynamo model category; Brajša et al. (2022), Burud et al. (2021), and Chowdhury et al. (2021) in the precursor method; Kakad & Kakad (2021) in the extrapolation method; Courtillot et al. (2021) in the other category; and Bizzarri et al. (2022) and Attia et al. (2020) in the interdisciplinary category conclude that solar cycle 25 will have smaller $R_{\max}$ than cycle 24, or that cycle 25 lies in the trough between century cycles, or that cycle 25 will enter a new grand minimum. These conclusions are inconsistent with our century cycle-based prediction.

Table 3 compares predictions for cycle 26. We find that $R_{\max}$ predictions for solar cycle 26 vary considerably across studies. Kalkan et al. (2023) using nonlinear autoregressive exogenous neural networks predict cycle 26 to be similar to or weaker than cycle 24, believing cycles 25 and 26 are at the century cycle minimum and solar activity will enter a new grand minimum. Liu et al. (2023) using long short-term memory methods show cycles 25 and 26 as similar, differing from our result that cycle 26 is slightly weaker than cycle 25. Becheker et al. (2023) using function fitting methods consider cycle 26 numerically weaker than cycle 24. Current projections generally agree that solar cycle 26 will peak between 2035 and 2036.

3.2. Discussion

The ASN sequence represents the longest continuously observed time series data for the same celestial body to date, yet compared to solar activity history, these data remain too limited and brief. Therefore, it is practically impossible to determine whether the quasi-periodic properties of solar and century cycles arise from incomplete observed cycles showing quasi-periodicity due to short observation times, or whether they are inherent properties of the cycles themselves.

Solving this problem requires extending the sunspot number sequence further back in time. This issue impacts our prediction errors: our forecasts depend on maxima of solar cycles at similar century cycle phases, but only two similar-phase cycles are available for prediction, resulting in relatively large error ranges for cycles 25 and 26. Our method analyzes long-term trends (centennial scales) and is therefore dependent on data length, contrasting with most prediction methods such as the precursor method considering polar magnetic fields, which utilizes only the previous cycle' s polar magnetic field on approximately decade-long scales.

Prediction errors vary considerably across methods. For example, the method of McIntosh et al. (2020), which depends on terminators, yields large error ranges when terminators are undetermined. However, predictions using comprehensive precursors and multiple regression techniques can significantly improve accuracy, adaptability, and stability, with regression coefficients reaching 0.95 (Xiong et al. 2021; Lu et al. 2022). For interdisciplinary (specifically machine learning) predictions, result quality depends on model selection and construction, so error ranges are generally small after model complexity is determined. However, the quasi-periodicity and complexity of solar activity make predictions challenging, and overly precise predictions risk overfitting.

Our predictions also include the effect of the 56.20-year period component in $R_{\max}$ values. We assume cycles at similar century cycle phases have similar parameter characteristics, while the 56.20-year period component is only implied in relatively small Schwabe cycles near century cycle peaks. Figure 5 shows that cycle A in G1, cycle 10 in G2, and cycle 20 in G3 are located near century cycle peaks but are relatively weak. Based on their statistical trend, we may also predict future cycles near G4' s peak. Obviously, this component has minimal impact on predictions for cycles 25 and 26, which are in the early ascending phase of the century cycle.

Regarding result comparisons, Section 3.1.4 shows that published predictions differ significantly from each other, presenting a chaotic state. We believe this clearly manifests the complex and quasi-periodic nature of solar cycles. Despite large differences, comparing results from different prediction methods provides deeper understanding of solar cycles. We also observe that among all cycle 25 predictions, later predictions show greater probability of concluding that cycle 25 will be stronger than cycle 24, consistent with our prediction. One reason is that later predictions show more pronounced evolutionary trends in cycle 25' s relative sunspot numbers, but this also illustrates that no current prediction method type has clear advantage for long-term forecasting, with the precursor method being most constrained. Unlike these methods, our approach is based on the principle that solar cycles at similar century cycle phases have similar parameters. Therefore, after confirming the century cycle's existence and shape (fitted empirical function) and timescale (parameter C), our prediction is not fundamentally limited by early or late prediction timing. The closer our prediction time is to $t_{\max}$, the closer our prediction will be to true values, without affecting our forecast of long-term solar cycle trends. Our method has data length requirements, but there are two sides to everything: regarding earlier predictions, our work is superior.

Among many cycle 25 predictions, many thought cycle 25 would be weaker than cycle 24, contrary to our prediction, and believed a new grand minimum similar to the Maunder or Dalton minima would occur in cycle 25. However, current cycle 25 data dramatically reduce the probability of it being weaker than cycle 24. In other words, the trough between century cycles G3 and G4 does not exhibit a very pronounced, long-lasting grand minimum, and we can probably assume that the correlation between century cycles and grand minima is weakened. How exactly this will play out, and whether a new grand minimum will emerge, still depends on future sunspot number data development.

4. Conclusion

This work confirms the existence of solar century cycles, determines their main parametric characteristics, and obtains an empirical function (Equation (2)). Solar activity shows a gradually enhancing trend on century cycle timescales. Solar Schwabe cycle 24 is located in the valley between century cycles G3 and G4. Subsequently, cycles 25 and 26 are in the ascending phase of century cycle G4; therefore, a new grand minimum will not occur, or rather, the grand minimum has just occurred around cycle 24.

The existence of solar century cycles indicates that a global physical mechanism on the Sun larger in scale than the solar Schwabe cycle must influence solar activity patterns. However, this global physical mechanism is not yet understood. The century cycle pattern can help predict basic characteristics of forthcoming Schwabe cycles. Based on empirical functions of the solar century cycle and similarity extrapolation assumptions, we predict the main parametric characteristics of solar cycles 25 and 26:

1. Solar cycle 25 will reach maximum in 2024, with peak ASN of 146.7 ± 33.40, stronger than cycle 24, and a period of about 11 ± 1 years.
2. Solar cycle 26 will start from 2030, reach maximum between 2035 and 2036 with peak ASN of 133.0 ± 3.200, and have a period of about 10 years.

This paper ignores some anomalous sunspot data when examining solar activity' s century cycle. This may result in correct long-term trend predictions but with some uncertainties in details. Future studies may interpret anomalous sunspot data and combine multiple solar cycle features for improved prediction accuracy. Additionally, when studying long-term evolutionary characteristics of solar activity, long-term isotope data (especially carbon-14 isotopes) on Earth could be analyzed alongside relative sunspot number observations. This could extend time series to thousands of years and provide a more accurate picture of solar activity' s long-term evolutionary pattern.

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