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## Research on Pulsar-Time-Steered Atomic Time Algorithm Based on DPLL Postprint

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### Abstract

In today's society, there is a wide demand for high-precision and high-stability time service in the fields of electric power, communication, transportation and finance. At present, the time standard in various countries is mainly based on atomic clocks, but the frequency drift of atomic clocks will affect the long-term stability performance. Compared with atomic clocks, millisecond pulsars have better long-term stability and can complement with the excellent short-term stability of atomic clocks. In order to improve the long-term stability of the atomic timescale, and then improve the timing accuracy, this paper proposes an algorithm for steering the atomic clock ensemble (ACE) by ensemble pulsar time (EPT) based on digital phase locked loop (DPLL). First, the ACE and EPT are generated by the ALGOS algorithm, then the ACE is steered by EPT based on DPLL to calibrate the long-term frequency drift of the atomic clock, so that the generated steered atomic time follows both the short-term stability characteristics of ACE and the long-term stability characteristics of EPT, and finally, the steered atomic time is used to calibrate the local cesium clock. The experimental results show that the long-term stability of atomic time after steering is improved by 2 orders of magnitude compared with that before steering, and the daily drift of a local cesium clock after calibration is less than 9.47 ns in 3 yr, 3 orders of magnitude higher than that before calibration on accuracy.

### Full Text

### Preamble

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ChinaXiv Research on Pulsar Time Steered Atomic Time Algorithm Based on DPLL

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## Abstract

Modern society exhibits widespread demand for high-precision, high-stability time services across electric power, communications, transportation, and finance sectors. Currently, national time standards rely primarily on atomic clocks, yet frequency drift in atomic clocks compromises long-term stability performance. Compared with atomic clocks, millisecond pulsars demonstrate superior long-term stability and can complement the excellent short-term stability characteristics of atomic clocks. To improve the long-term stability of atomic timescales and thereby enhance timing accuracy, this paper proposes an algorithm for steering an atomic clock ensemble (ACE) by ensemble pulsar time (EPT) based on a digital phase-locked loop (DPLL). First, ACE and EPT are generated using the ALGOS algorithm. Then, the ACE is steered by EPT through DPLL to calibrate the long-term frequency drift of atomic clocks, producing a steered atomic time that inherits both the short-term stability characteristics of ACE and the long-term stability characteristics of EPT. Finally, this steered atomic time is used to calibrate a local cesium clock. Experimental results demonstrate that the long-term stability of atomic time after steering improves by two orders of magnitude compared with that before steering. Furthermore, the daily drift of a local cesium clock after calibration remains below 9.47 ns over three years, representing a three-order-of-magnitude improvement in accuracy compared with the uncalibrated state.

Key words: (stars:) pulsars: general -time -methods: data analysis -instabilities

## 1. Introduction

High-precision timing is founded upon establishing an accurate and stable timescale, which finds extensive applications in power systems, communications, transportation, finance, and other fields. High-precision time references are primarily generated through the combined use of atomic clocks, including common types such as rubidium clocks, hydrogen masers, and cesium clocks. These devices exhibit excellent short-term stability with minimal timing error

during short-duration autonomous timekeeping. However, due to long-term frequency drift in atomic clocks, scenarios involving extended autonomous operation produce increasingly severe frequency drift and phase deviation, resulting in progressively larger 1 pulse per second (1PPS) timing errors.

Coordinated Universal Time (UTC) represents a widely adopted atomic time (AT) standard. Local timing laboratories regularly submit atomic clock comparison data for UTC 0h daily to the Bureau International des Poids et Mesures (BIPM). BIPM obtains time differences between local AT data and the international comparison center through remote time comparison links, deriving a weighted average free atomic time. International Atomic Time (TAI) is calibrated using fountain clocks, with UTC obtained through leap seconds [?].

Pulsars are rapidly rotating neutron stars possessing exceptional long-term rotational stability. Pulsar signals can be employed to steer atomic clocks and generate timescales that incorporate both long-term and short-term stability advantages. High-precision real-time observation of pulsar times of arrival (TOAs) by advanced telescopes such as FAST can circumvent the latency issues associated with precise timing sources like UTC and Terrestrial Time at BIPM (TT(BIPM)), enabling further application as timing signal sources in deep space navigation, power grids, intelligent transportation, and other fields to achieve high long-term stability and high-precision timing [?, ?, ?]. Additionally, pulsar pulse signals are considerably more complex than Global Navigation Satellite System (GNSS) timing systems that can be easily identified through machine learning methods [?], and pulsars can serve as auxiliary signal sources when existing atomic clock timing systems are deliberately compromised, spoofed, or lose lock in weak signal areas [?].

Pulsar time (PT) generation relies on pulsar timing observation data and ephemeris data, obtaining the difference between predicted pulse TOA and observed values at the solar system barycenter—namely, the timing residual. Here, the predicted value represents PT generated by the pulse phase model, while the observed value is based on local AT. If all error terms are corrected, the timing residual equals the difference between PT and AT [?, ?]. By tracing the AT recorded by the station's atomic clock to TAI, the clock error between PT and TAI is obtained. Combining timing residuals from multiple pulsars through appropriate algorithms enables establishment of ensemble pulsar time (EPT).

Based on the complementary stability characteristics of PT and AT, their combination can improve time reference accuracy and stability. In recent research on joint timekeeping using pulsars and atomic clocks, Liu et al. [?] proposed using Vondrak-Cepek filtering to combine PT and AT, generating a timescale with good short-term and long-term stability, though the resulting timescale suffers from latency and insufficient real-time performance. Li et al. [?] employed a dual-steering algorithm to steer atomic clocks using pulsars, where monthly steering forms a closed loop ensuring real-time performance of the post-steering time reference [?]. However, the pulsar timing residuals in that work simulate

future data, with credibility decreasing as prediction time increases.

This paper proposes an algorithm for steering AT based on digital phase-locked loop (DPLL). The atomic clock signal serves as the DPLL' s local oscillator, while the pulsar signal functions as the DPLL' s steering frequency source. The loop filter removes most low-frequency noise from atomic clocks and most high-frequency noise from pulsars, generating a timescale that combines the advantages of both sources to produce high-precision timing signals and provide 1PPS for timing sources such as power grid terminals.

The remainder of this paper is organized as follows: Section 2 introduces the principles of timekeeping algorithms, including the EPT algorithm and atomic clock ensemble (ACE) algorithm. Section 3 presents the pulsar steering algorithm based on DPLL. Section 4 describes experiments and analyzes results, calculating clock errors for EPT(1PPS), ACE(1PPS), and (ACE+EPT)(1PPS) based on DPLL, calibrating a local cesium clock using (ACE+EPT)(1PPS) to achieve high-precision daily drift, and analyzing discrepancies between experimental and actual timing scenarios. Section 5 concludes the paper.

## 2. Principle of Timekeeping Algorithm

This section introduces the fundamental principles of timekeeping algorithms, including the EPT algorithm and ACE algorithm. The ALGOS algorithm serves as the basis for clock error weighting for both pulsars and atomic clocks, with corresponding 1PPS clock error signals generated by the EPT and ACE algorithms respectively. These signals function as the external frequency source and local frequency source for DPLL.

Calculating EPT and ACE requires multiple pulsar clocks and atomic clocks respectively. When using these two clock groups to compute the average timescale, methods such as ALGOS, Kalman filtering, and wavelet analysis can be applied. The ALGOS algorithm, as a typical weighted average method, can improve the stability of EPT and ACE, and is therefore employed to calculate the comprehensive timescale. In this algorithm, Allan variance is used for atomic clocks and the  $\sigma_z(\tau)$  variance (also known as Hadamard variance) for pulsar clocks, expressed as Equations (1) and (2) respectively [?, ?]. In these equations, residuals and errors are divided into subsequences with equal intervals  $\tau$ , with each subsequence fitted by a cubic polynomial where  $c_3$  represents the coefficient of the highest-order term and  $\diamond$  denotes averaging across all subsequences. The weight is taken as inversely proportional to the square of the  $c_3$  error.

Pulsars exhibit substantial, a priori unknown frequency drifts, making Allan variance unsuitable for pulsar data [?]. By defining one additional difference beyond Allan variance,  $\sigma_z(\tau)$  variance ignores fixed frequency drift in pulsar timing residuals.

### 2.1. Ensemble Pulsar Time Algorithm Flow

This section first uses the timing residuals of selected pulsars to generate EPT, which is traced back to TAI through TT(BIPM19) and converted to UTC, then interpolated into an EPT(1PPS) clock error signal serving as the steering frequency source for DPLL. The detailed flow chart is shown in Figure 1 [Figure 1: see original paper].

To improve the accuracy of the final EPT(1PPS) timekeeping, following the criteria of “more observation points” and “longer observation time” proposed by Shaifullah et al. [?], M superior pulsars are selected as PT sources due to their strict linear relationship between frequency and timing residual. We can use Tempo2 software [?, ?, ?] to fit observation data and calculate timing residuals traced back to TT(BIPM19).

Due to large timing noise in individual pulsars, timing stability can be improved through weighting. It is necessary to set a reference pulsar to Modified Julian Date (MJD) and align the timing residuals and errors of other pulsars to this MJD reference through down-sampling. To calculate the clock error of EPT(1PPS) from EPT, timing residuals and errors must be linearly interpolated at 1PPS intervals using MJD, ensuring subsequent interpolation meets equal-interval conditions and reducing errors.

Since this paper focuses on timing, it is necessary to align the past 1PPS signals of EPT and ACE and input them into the DPLL steering algorithm to fix their time series length. When adding the latest value, the earliest value is discarded. To calculate long-term stability, the  $\sigma_z(\tau)$  method should be used to evaluate stability before truncation for weighting, thereby eliminating pulsar frequency drift.

As AT has been traced back to UTC and pulsar signals are traced back to TT(BIPM19), it is only necessary to trace PT back to UTC to unify the time reference for steering operations. The traceability data interval between TT(BIPM19) and UTC provided by BIPM is 10 days [?], which is relatively large, necessitating linear interpolation of timing residual points to calibrate deviation from UTC. Data are then intercepted according to the preset time series length.

To facilitate interpolation, an MJD index table is created with the horizontal axis as a 1PPS time series, the minimum vertical axis value as 0, and the maximum as the number N of timing residuals within the set MJD range. The abscissa of the 1PPS signal (1, 2, K) is evenly distributed across gaps in timing residuals. If the abscissa of the original residual is replaced by  $x_1, x_2, K, x_N$  and the ordinate corresponds to  $y_1, y_2, K, y_N$  respectively, the interpolation formula can be obtained.

After obtaining the PT(1PPS) clock error for each pulsar through interpolation, normalized weights  $w_i$  are calculated based on the ALGOS algorithm, enabling weighted calculation of EPT(1PPS) clock error from M pulsars and mitigating

instability from individual pulsars [?, ?].

Finally, the EPT(1PPS) clock error must be differentiated to obtain frequency difference, completing the phase steering of ACE(1PPS) in the subsequent DPLL steering algorithm. Since the pulsar signal serves as the steering frequency source, the steering physical quantity required is frequency rather than phase. As the EPT(1PPS) resulting from this step is a differential frequency signal,  $\sigma_z(\tau)$  is no longer required for subsequent stability evaluation—only Allan deviation is needed. While the generated (ACE+EPT)(1PPS) exhibits even smaller frequency drift, Allan deviation can also be used to calculate stability.

Following these steps, real-time availability of the EPT signal is realized while preserving most of the original timing residuals from each pulsar, which serve as the steering frequency source for the subsequent DPLL algorithm.

## 2.2. Atomic Clock Ensemble Algorithm Flow

This section first fits and interpolates the measured clock error of preprocessed atomic clocks using a quadratic model, adds noise at interpolation points, and performs weighting based on Allan deviation to serve as the local oscillator signal for DPLL. The detailed flow chart is shown in Figure 2 [Figure 2: see original paper].

First, the actual clock errors of atomic clocks should be analyzed in detail, eliminating data points with obvious anomalies and performing linear interpolation for very few missing clock error points. If the original data are normal, this step can be skipped. This ensures normal progress of subsequent data processing while the small number of anomalous data points renders their impact on the original data negligible.

If the time interval of the original data is too long, it must be interpolated into 1PPS clock error. Generally, atomic clocks exhibit clock drift that deteriorates over time, so the quadratic clock error model [?] is often employed for modeling, completing approximation of clock error, frequency error, and linear frequency drift parameters. The expression is as follows:

Here  $x_0$ ,  $y_0$  and  $d$  represent initial time error, frequency error, and linear frequency drift respectively, which are deterministic components, with the remainder being noise.

To steer atomic clocks using pulsars, it is necessary to align the MJD of the selected atomic clock with the MJD of EPT(1PPS) clock error through interception, and interpolate the atomic clock error into 1PPS clock error using fitted parameters. Similar to pulsar data processing methods, Allan deviation [?] can be calculated from historical observation data before intercepting atomic clock error to compute more accurate weights for each atomic clock. If the 1PPS signal can be obtained directly from original observations, this step can be skipped.

Based on the ALGOS algorithm, weights for each clock can be obtained through

Allan deviation after generating AT(1PPS) clock error, and ACE(1PPS) clock error can then be generated through weighting. After these steps, 1PPS matching between the ACE signal and EPT signal is realized, serving as the local oscillator frequency source for the subsequent DPLL algorithm.

### 3. Principle of Pulsar-steering Algorithm Based on DPLL

This section introduces the algorithmic principles from timekeeping to timing. The timekeeping algorithm completes steering of EPT(1PPS) to ACE(1PPS) based on DPLL, where the steering frequency source and local oscillator frequency source of DPLL are static. The timing algorithm operates based on the (ACE+EPT)(1PPS) steering output, periodically calibrating the clock error of a local cesium clock and achieving dynamic output of actual physical signals.

#### 3.1. Principle of Loop Establishment and Steering of DPLL

DPLL is a negative feedback system that can steer the phase of a local oscillator through the phase of an input signal source. Its basic structure is illustrated in Figure 3 [Figure 3: see original paper] [?]. In this diagram, PD, LPF, and VCO represent phase detector, loop filter, and voltage-controlled oscillator respectively. The product of the gains of these three components is recorded as  $G(z)$ , representing the transfer function of the DPLL open-loop system in the Z domain. EPT(1PPS) denotes the DPLL reference input, also known as the steering frequency source. ACE(1PPS) represents the local oscillator frequency source of DPLL, input to the VCO. (ACE+EPT)(1PPS) is the output after input steering, fed back to the input terminal through the frequency divider. By analyzing the I/O characteristics of the DPLL system in the Z domain, the DPLL output can be expressed as:

Here  $H(z)$  is the transfer function of the closed-loop system, which acts as a low-pass filter that can remove high-frequency noise from the steering frequency source EPT(1PPS) clock error.  $H_e(z)$  is the closed-loop error transfer function of DPLL, which acts as a high-pass filter that can remove low-frequency noise from the local oscillator frequency source ACE(1PPS) clock error. Therefore, the steered (ACE+EPT)(1PPS) clock error simultaneously exhibits small low-frequency noise and small high-frequency noise.

Considering that third-order phase-locked loops can stably track frequency ramp signals [?] and perform better than second-order phase-locked loops, ensuring the steering stability of ACE(1PPS) with EPT(1PPS) clock error in this work, the system transfer function of DPLL is selected from Feng et al. [?]:

Here  $T$  is the sampling period,  $K_d$  is the phase detector gain,  $K_0$  is the voltage-controlled oscillator gain, and the intersection bandwidth of the system transfer function is:

The optimal loop parameters [?, ?] are obtained when and only when the single sideband phase noise intersection of ACE(1PPS) and EPT(1PPS) equals  $K$ . At

this point, (ACE+EPT)(1PPS) simultaneously retains the near-end phase noise of EPT(1PPS) and the far-end phase noise of ACE(1PPS), achieving optimal steering performance.

In actual timing scenarios, according to dynamic changes in the input sequence, the value of  $K$  is automatically adjusted based on the loop parameter calculation formula, maintaining loop stability. When the output signal is in the first steering cycle, the 1PPS clock error of the steered source ACE is output. In subsequent steering cycles, the frequency of ACE(1PPS) is feedback-steered by calculating the difference between the frequency of EPT(1PPS) and that of ACE(1PPS), thereby preventing uncontrollable phase drift of ACE(1PPS). Simultaneously, leveraging the excellent short-term stability of ACE(1PPS) clock error, substantial high-frequency noise from EPT(1PPS) clock error is eliminated, making the second-order difference of the output signal similar to that of ACE(1PPS) clock error.

### 3.2. Daily Drift Calibration of Local Cesium Clock Error

The previous section obtained the (ACE+EPT)(1PPS) clock error signal for the past year through DPLL. However, only the 1PPS clock error from the last second of this output signal can be used for timing, because only the 1PPS clock error from the last second of the two input signals is real-time. Although atomic clock error can be better predicted according to models, pulsars are affected by various complex factors, making clock error prediction uncertainty too strong [?] and making accurate generation of future 1PPS clock error signals difficult. Therefore, the clock error of (ACE+EPT)(1PPS) from the last second is selected, and among  $L$  atomic clocks, the one with the smallest daily drift is chosen. The (ACE+EPT)(1PPS) clock error is then used as its calibration source to obtain Cssteered(1PPS).

When DPLL runs once daily, first, the ACE(1PPS) and EPT(1PPS) clock errors for that day should be introduced in the same manner, while deleting the clock error from the earliest day. Then, the initial phase of the current one-year sequence is calibrated to UTC released with a one-year lag. Finally, the 1PPS clock error for the last second is calculated using the same method. At this time, the local master clock is calibrated using the phase microstepping method [?] to avoid lock loss and stability deterioration caused by phase steps. Therefore, the final daily clock error equals the sum of the daily phase offset and calibration error of the local atomic clock.

### 3.3. Algorithm Flow of Pulsar Steering Atomic Clock Based on DPLL

The flow chart of the clock error steering algorithm based on DPLL is depicted in Figure 4 [Figure 4: see original paper]. This includes the EPT algorithm, ACE algorithm, and clock error steering algorithm based on DPLL. The final generated Cssteered(1PPS) represents the output signal of this algorithm, used for high-precision timing. The clock error steering algorithm based on DPLL

proceeds as follows:

- Step 1. Calculation of ACE(1PPS) and EPT(1PPS);
- Step 2. Generation of the timekeeping signal (ACE+EPT)(1PPS) through DPLL clock error steering;
- Step 3. Periodic calibration of the local cesium clock with (ACE+EPT)(1PPS) to complete high-precision time service.

In summary, the algorithm in this paper can be summarized as: the EPT time-keeping algorithm (represented by the red dotted box), the ACE timekeeping algorithm (represented by the green dotted box), and the DPLL-based clock error control algorithm (represented by the blue dotted box). Although the algorithm is relatively complex, it can realize automatic processing in actual time service operations, generating a real-time time service scale with better accuracy and long-term stability than atomic clocks alone.

## 4. Experiments and Results Analysis

This section first introduces the source and selection of data used, then presents calculation and analysis of EPT(1PPS) frequency error and ACE(1PPS) clock error respectively. Finally, the EPT(1PPS) frequency error is used to steer the ACE(1PPS) clock error through DPLL, the local cesium clock is calibrated daily using the generated signal, and the robustness of loop parameters is analyzed. In each data processing stage, errors compared with the actual timing system are analyzed.

### 4.1. Experimental Data

Pulsar timing data are selected from the pulsar dataset published by IPTA in 2019 [?, ?]. According to Shaifullah et al. [?], nine pulsars are selected from 65 available pulsars: J0437-4715, J0613-0200, J1012+5307, J1643-1224, J1713+0747, J1744-1134, J1909-3744, J1918-0642, and J2145-0750. Key parameters of the selected pulsars are shown in Table 1. Among these, the observation range and TOA data points of the selected pulsars are superior within the pulsar dataset, and the MJD range can cover the selected atomic clock data.

AT data were obtained from the BIPM official website (<https://webtai.bipm.org/ftp/pub/tai/data/>), selecting three atomic clocks (Cs1350104, Cs1351463, and H1400711) from the United States Naval Observatory [?]. The MJD range was 53739-55584, with a clock error sampling period of 5 days. Among these, the cesium clock Cs1350104 data are normal, while the other two clocks have less than 3% missing data. Linear interpolation is performed for these missing data, followed by adding noise with the average value of the second difference of clock error as the mean, then calculating the Allan deviation of each atomic clock error. Key parameters of the selected atomic clocks are shown in Table 2.

Since the DPLL steering algorithm requires two input frequency sources to have identical start and end times and equal data lengths, it is necessary to intercept

pulsar and atomic clock data with the same MJD when verifying algorithm feasibility. This paper uses three MJD ranges: 53739–54104, 54104–54469, and 54469–54834, each with a length of one year.

## 4.2. Experiment and Result Analysis

**4.2.1. Performance Analysis of EPT(1PPS) Clock Error** This section analyzes EPT performance. Considering that UTC release time lags by one year, the 1PPS clock error timescale is set to one year, aligning the initial phase of the DPLL local oscillator with the latest UTC. Since AT data begin at MJD 53739, to ensure a real-time and causal system, residuals of each pulsar after MJD 54104 are removed. J1909-3744, with the fewest residual points, is selected as the MJD benchmark to ensure authenticity of timing residuals. The MJD of the other eight pulsars is aligned to this benchmark, followed by downsampling and linear interpolation of timing residuals and errors, then calculating the  $\sigma_z(\tau)$  variance of each pulsar. To maximize utilization of pulsar long-term stability properties, EPT is weighted by the inverse of the end value of  $\sigma_z(\tau)$ . The  $\sigma_z(\tau)$  variance of nine pulsars and EPT is displayed in Figure 5 Figure 5: see original paper.

From the EPT curve, both short-term and long-term stability of EPT approach those of the best-performing pulsar, particularly on the one-year scale, where its  $\sigma_z(\tau)$  stability surpasses all individual pulsars. Therefore, weighting timing residuals can smooth uncorrelated timing noise from multiple pulsars, making EPT more stable than permitted by a single pulsar. Simultaneously, setting the pulsar timing residual length to one year ensures smaller timing error and better stability.

IPTA Data Release 2 (DR2), the pulsar dataset selected for this paper, has been traced back to TT(BIPM19). The calibration deviation of traceability is expressed as  $\Delta t$  (s), giving:

$$\text{TT(BIPM19)} = 32.184 \text{ s}$$

Thus PT can be traced back to UTC. Subsequently, the traced timing residual between MJD 53739 and MJD 54104 is intercepted.

Based on MJD benchmark pulsar J1909-3744, the number of timing residual points is 1060. Substituting  $N = 1060$  into Equation (3), timing residuals are interpolated to generate PT(1PPS) clock error, then evenly distributed random noise is added. Substituting  $M = 9$  into the ALGOS algorithm calculates the EPT(1PPS) clock error, and frequency error is obtained through differentiation. The EPT(1PPS) clock error is shown in Figure 5(b). This figure demonstrates that compared with the clock drift phenomenon in atomic clocks, pulsar clock drift is not obvious, and the UTC traceability error on a one-year scale remains in the microsecond order, indicating that long-term clock drift of atomic clocks can be calibrated by pulsar signals.

In summary, the stability and clock error of EPT(1PPS) are superior to those of a single pulsar.

When actually calculating EPT(1PPS) clock error, differences in data processing methods or systematic errors exist in the following steps:

1. When extracting pulsar timing residuals, it is necessary to use an astronomical telescope to collect TOA and ephemeris data in real time, generating .tim and .par files to calculate timing residuals [?]. Since FAST facility pulsar observation accuracy has surpassed the IPTA data source [?] used in this paper, timing residual accuracy can be improved by establishing cooperation with similar observation stations.
2. When selecting the reference pulsar by MJD and downsampling others, the IPTA dataset exhibits uneven signal acquisition intervals, incomplete frequency band coverage, and limited observation accuracy [?]. However, since telescopes continuously collect signals in real time, this step's influence on timing residuals will be reduced.
3. When tracing residuals to UTC through TT(BIPM19) and truncation, real-time TT(BIPM23) cannot be obtained, so calculations must be based on TT(BIPM21) in reality:

$$\text{TT}(\text{BIPM21}) = 32.184 \text{ s} + 0.01 \text{ MJD} + 59579 \text{ ns} + 27667.5 \text{ ns}$$

to trace TT to TAI and further to UTC. Since the difference between predicted and actual values of TT(BIPM21) ranges between -6.2 ns and -0.2 ns, error will increase slightly in actual timing systems but remains stable and controllable.

4. When linearly interpolating residuals and generating PT(1PPS) clock error, traceability error is less than 1%, corresponding to the original timing residual, making the interpolation effect from timing residual sampling uniformity more significant.
5. When generating EPT(1PPS) clock error based on residual stability weighting, calculation error of  $\sigma z(\tau)$  weights will be reduced because real-time pulsar array TOA intervals are more uniform, improving calculation accuracy.

Overall, EPT(1PPS) clock error calculation can be made more accurate through these steps.

**4.2.2. Performance Analysis of ACE(1PPS) Clock Error** This section analyzes ACE performance. According to the quadratic error model of Equation (4), clock errors are fitted with results shown in Table 3, enabling complete clock interpolation based on fitted parameters.

Since this paper primarily utilizes the short-term stability of atomic clocks, the starting point of Allan deviation for each clock is substituted into the ALGOS algorithm, yielding normalized weights for Cs1350104, Cs1351463, and H1400711

as 0.0509, 0.0356, and 0.9135 respectively. After generating ACE(1PPS) clock error through weighting, it can be plotted together with the three atomic clock errors as shown in Figure 6(a), with ACE(1PPS) clock error and Allan deviations of the three atomic clocks displayed together in Figure 6(b) [Figure 6: see original paper].

In Figure 6(a), due to the high weight of H1400711, the ACE(1PPS) clock error closely follows it, and the relative UTC deviation of ACE is smaller than that of cesium clocks in the short term and hydrogen masers in the long term. The quadratic clock error model remains dominant in ACE, but the random noise term determines the magnitude of the Allan deviation curve shown in Figure 6(b). This demonstrates that ACE short-term stability is better than cesium clocks, while its long-term stability surpasses that of hydrogen masers, making it superior overall to any single atomic clock.

When actually calculating ACE(1PPS) clock error, differences in data processing methods or systematic errors exist in the following steps:

1. In selecting measured atomic clock errors, data from the National Time Service Center (NTSC) can be used instead [?], improving time service stability and system robustness by increasing the number of atomic clocks. NTSC provides more public timing data, and even if 1PPS signals cannot be read directly, 1PPS clock error for each clock can be approximated through matrix operations. Therefore, errors in actual timing systems will be reduced and system robustness improved.
2. When using the quadratic model to fit clock error and interpolation, if the 1PPS signal can be read directly, this step can be skipped, avoiding simulation errors in the statistical sense. Otherwise, algorithm steps and errors remain unchanged.

Overall, ACE(1PPS) clock error calculation in practical engineering will be more accurate than the results presented in this paper.

**4.2.3. DPLL Steering Performance Analysis** This section analyzes PT steering AT performance based on DPLL. First, Allan deviations of ACE(1PPS) and EPT(1PPS) are fitted separately based on power-law spectrum parameters. Fitting parameters are shown in Table 4, with fitting curves displayed in Figure 7 Figure 7: see original paper and (b). Second, single sideband phase noise of these two frequency sources is calculated based on power-law spectrum parameters, as shown in Figure 7(c). The intersection of phase noise is calculated to obtain loop parameter K. Finally, based on this parameter, clock errors of ACE(1PPS), EPT(1PPS), and (ACE+EPT)(1PPS) before and after DPLL steering are plotted in Figure 7(d), where the EPT(1PPS) clock error represented by the green line is the pulsar clock error curve from Figure 5(b). The short-term stability of the blue and red lines is good, with their second-order differences further compared and verified in Figure 8 Figure 8: see original paper-(c).

From the power-law spectrum parameter fitting results in Table 4, ACE(1PPS) clock error contains a small amount of frequency random walk noise ( $h_2$ ), which significantly reduces long-term atomic clock stability. However, EPT(1PPS) clock error contains only  $h_0$  and  $h_2$ , so its long-term stability is much better than ACE(1PPS). Since Figures 7(a) and (b) complete accurate fitting of the frequency source, the single sideband phase noise calculated from ACE(1PPS) and EPT(1PPS) power-law spectrum parameters is accurate. Taking the intersection of the two curves in Figure 7(c) yields:

It can be seen from Figure 7(d) that the (ACE+EPT)(1PPS) clock error shown by the red line overcomes the clock drift defect of ACE, reduces short-term clock error fluctuation of EPT, and realizes steering of ACE(1PPS) by EPT(1PPS).

In Figure 8, Figure 8(a) representing the second-order difference of ACE(1PPS) clock error is close to Figure 8(b) representing the second-order difference of (ACE+EPT)(1PPS) clock error, demonstrating that the DPLL algorithm preserves the high-frequency portion well. Figure 8(d) is obtained from their difference. Clock error is reduced by about two orders of magnitude, with curve trend similar to the second-order difference of EPT(1PPS) frequency difference, showing that the algorithm simultaneously preserves the low-frequency portion of EPT(1PPS) clock error.

Figure 8(b) can be enlarged as shown in Figure 9 Figure 9: see original paper, then the Allan deviation of ACE(1PPS) clock error, EPT(1PPS) frequency difference, and (ACE+EPT)(1PPS) clock error is calculated, as depicted in Figure 9(b).

In Figure 9(a), the (ACE+EPT)(1PPS) clock error over one year is less than 3.60 ns. In Figure 9(b), the Allan deviation of (ACE+EPT)(1PPS) clock error first follows ACE(1PPS), then is biased by EPT(1PPS) and always follows its slope, with post-steering stability better than both signal sources after one day. As steering time continues to increase, (ACE+EPT)(1PPS) becomes increasingly stable, with one-year stability two orders of magnitude better than ACE(1PPS) before steering.

Although Equation (3) makes it difficult for interpolated EPT data to maintain the short-term stability of the original residual, causing the short-term Allan variance of EPT to not reflect the true value, the Allan variance of (ACE+EPT)(1PPS) follows ACE(1PPS) on short-term scales and the ACE(1PPS) clock difference over the steering period. The effect of imprecise and poor Allan variance of EPT can be ignored.

**4.2.4. Performance Analysis of Local Cesium Clock Calibration** This section performs performance analysis of local atomic clock calibration based on the (ACE+EPT)(1PPS) DPLL steering output, enabling the steering output to have a physical clock carrier and be applicable to actual timing systems. Since the total error of the timing system constructed in this paper equals the steering error plus the error caused by local atomic clock self-timekeeping, selecting

an atomic clock with the smallest daily drift and performing DPLL steering operations daily can generate the best-performing 1PPS timing signal.

For the steering error of 3.60 ns in Figure 9(a) (this value is the maximum on the vertical axis), the cesium clock Cs1351463 with the slowest clock accumulation among the three atomic clocks (producing a maximum clock error of 4.73 ns in one day) is selected. Adding the clock error and daily offset error together, the clock error of Cssteered(1PPS) is less than 8.33 ns in one year.

Two additional independent experiments were performed for added confidence. Considering that pulsar timing residuals represent the biggest variable in steering experiments, the MJD of nine pulsars was directly replaced with 54104–54469 and 54469–54834 in repeated experiments, with loop parameters unchanged. All variables involved in atomic clocks and pulsars across different years are summarized in Table 5, and clock errors and Allan deviations after EPT steering ACE are redrawn as shown in Figure 10 [Figure 10: see original paper].

As seen from Table 5, atomic clock weights differ slightly across the three-year timespans because atomic clock model parameters are relatively fixed while noise is random. The main pulsar and number of residual points remain almost unchanged, demonstrating system robustness brought by MJD homogenization. Figures 10(a) and (c) show the maximum (ACE+EPT)(1PPS) error is 4.74 ns, so the daily drift of Cssteered(1PPS) is less than 9.47 ns over 3 yr, which is three orders of magnitude higher than the microsecond-level daily drift from cesium clock independent timing before calibration. Figures 10(b) and (d) show that after replacing the input sequence, the Allan deviation of the output still steadily follows the short-term stability of ACE(1PPS) and long-term stability of EPT(1PPS), affirming that DPLL-based steering is insensitive to initial loop parameters and the system is robust.

In summary, the annual stability of AT based on the DPLL steering algorithm improves by two orders of magnitude, and the daily drift accuracy of a local cesium clock calibrated by this signal improves by three orders of magnitude. In actual time service systems, local cesium clock calibration cannot be compensated by clock speed and drift because the clock calibration is quadratic, so calibration is realized completely after a period of time. According to algorithm runtime, clock error can be calibrated by adjusting the clock reading at the previous moment, or by using the phase microstepping method. Additionally, data length and calibration period can be slowly adjusted with input data to maintain a high-precision time reference.

## 5. Conclusion

To improve the long-term stability and timing accuracy of AT, this paper proposes a pulsar-time-steering algorithm based on DPLL. Pulsar signals and atomic clock signals serve as the steering frequency source and local oscillator frequency source of DPLL respectively. The final generated steered frequency source not only avoids frequency drift of atomic clock signals but also reduces

large frequency jitter in pulsar timing residuals in the short term. Experimental results show that based on automatic adjustment of loop parameters, the long-term stability of AT after steering follows the EPT(1PPS) signal, with daily drift of (ACE+EPT)(1PPS) less than 4.74 ns over 3 yr, representing a three-order-of-magnitude improvement in timing accuracy compared with uncalibrated systems. After using (ACE+EPT)(1PPS) to calibrate the local cesium clock, daily clock drift remains below 9.47 ns. By applying this algorithm to atomic clocks with smaller power-law spectrum noise, or introducing low-frequency filtering methods to generate EPT with higher long-term stability [?], even higher timing accuracy can be obtained. The algorithm in this paper can be applied to power grid timing and other fields requiring high precision and high stability. In the next step, we only need to replace the historical data used in this paper with real-time atomic clock error (updated every five days, for example) and real-time pulsar timing residuals to complete real-time calibration of the local atomic clock and improve timing accuracy.

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