Machine learning computational model for hydrodynamic lubrication characteristics of a miter gate bottom pivot

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Abstract

The bottom pivot is a vital support device in the miter gate but often subject to poor lubrication and wear failures. Calculating the hydrodynamic lubrication characteristics of the bottom pivot is a complex three-dimensional (3D) problem, and most of existing models adopt simplified assumptions to reduce the calculation difficulty. To solve this issue, this work develops a 3D model to calculate the hydrodynamic lubrication characteristics of the miter gate bottom pivot. The finite difference method is used to solve the oil film thickness and pressure distribution based on the spherical coordinates Reynolds equation. The component forces in three directions are calculated from the pressure distribution and compared with the theoretical values to generate the calculation difference. Then, the genetic algorithm (GA) is used to minimize the difference to determine the optimal initial parameters for the 3D model. The analysis results show that the calculation accuracy can be significantly improved by using the optimal initial model parameters. When our initial pressure is 5.64MPa, the results meet the engineering accuracy requirements.

Full Text

Preamble

Machine Learning Calculation Model for Hydrodynamic Lubrication Characteristics of a Miter Gate Bottom Pivot

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Abstract

The bottom pivot is a vital support device in the miter gate but is often subject to poor lubrication and wear failures. Calculating the hydrodynamic lubrication characteristics of the bottom pivot is a complex three-dimensional (3D) problem, and most existing models adopt simplified assumptions to reduce computational difficulty. To address this issue, this work develops a 3D model to calculate the hydrodynamic lubrication characteristics of the miter gate bottom pivot. The finite difference method is used to solve the oil film thickness and pressure distribution based on the spherical coordinates Reynolds equation. The component forces in three directions are calculated from the pressure distribution and compared with theoretical values to generate the calculation difference. Then, the genetic algorithm (GA) is used to minimize this difference to determine the optimal initial parameters for the 3D model. The analysis results show that calculation accuracy can be significantly improved by using the optimal initial model parameters. When the initial pressure is 5.64 MPa, the results meet engineering accuracy requirements.

Keywords: miter gate; bottom pivot; hydrodynamic lubrication; genetic algorithm

Nomenclature

- h: oil film thickness
- μ: dynamic viscosity of lubricating medium
- p: oil film pressure
- U: relative movement speed of the friction pair
- t: time
- R: distance from the point to the origin
- θ, ϕ: spherical coordinates
- τ: shear stress
- $\varepsilon_x, \varepsilon_y, \varepsilon_z$: eccentricity of the spherical bearing
- $\omega_x, \omega_y, \omega_z$: relative rotational angular velocity
- τ_0: ultimate dynamic shear stress
- c: clearance of friction pair

1. Introduction

Currently, a large number of miter gates suffer from structural damage due to excessive bottom pivot wear [1]. As an important supporting component for the miter gate, the bottom pivot operates in a harsh underwater environment and is prone to wear failures. Daniel [2] found excessive wear on miter gate bottom pivots in Dutch navigation locks, while Zhao et al. [3] suggested that measuring the surface wear of bottom pivots is crucial to prevent unexpected failures and prolong service life.

Due to unique structural features, the mathematical model of the bottom pivot can be simplified as a spherical bearing model, allowing the lubrication characteristics to be analyzed through spherical bearing lubrication theory. Pylos et al. [7] predicted possible lubrication states and minimum oil film thickness for wrist implants with spherical bearing surfaces under different working conditions using elastohydrodynamic (EHD) lubrication theory. Deng et al. [8] used the Reynolds equation in spherical coordinates to analyze bearing ball lubrication. Agrawal [9] added micro-texture to the surface of hybrid spherical thrust bearings and solved the modified Reynolds equation using the finite element method. Wang and Sharma [10] solved the steady-state Reynolds equation for spherical bearings using the finite difference method and relaxation iteration method. Kumar et al. [11] used deterministic theory to analyze the influence of surface roughness on squeeze film lubrication of spherical bearings. Huang et al. [12] studied lubrication of spherical crown pit texture models under different surface texture distribution parameters using hydrodynamic lubrication theory.

From existing publications, it is evident that the finite difference method is often used for iterative solutions of the Reynolds equation, resulting in low accuracy and heavy computational time. Evolutionary algorithms can address this issue by properly optimizing the initial model parameters in the difference method. Ji et al. [13] combined radial basis functions with the NSGA-II genetic algorithm for multi-objective optimization design of turbine impellers, improving model efficiency and accuracy. Tiwari et al. [14-17] used genetic algorithms for optimum design of rolling bearings and spherical roller bearings. However, investigating the hydrodynamic lubrication characteristics of the bottom pivot using a genetic algorithm-optimized finite difference model has not been found in the literature. It is worthwhile to combine these approaches to improve calculation performance for hydrodynamic lubrication characteristics of the bottom pivot.

This work develops a genetic algorithm-optimized finite difference model to investigate the hydrodynamic lubrication characteristics of the bottom pivot. First, the force on the bottom pivot is calculated through theoretical analysis. Then, initial model parameters are used to solve the Reynolds equation in spherical coordinates to obtain the film thickness and pressure distribution of the bottom pivot. The calculated results are compared with theoretical values, and the difference is used as feedback information to optimize the initial parameters using the genetic algorithm. Changes in oil film and pressure of the pivot after parameter optimization are studied under different operating conditions. This paper proposes a method combining the finite difference method and genetic algorithm to solve the Reynolds equation. During the solution process, there is no need to non-dimensionalize the Reynolds equation; only actual parameters need to be substituted. The model can be applied to solve the Reynolds equation in different cases, with differences less than $-31.010\times$.

The remainder of this paper is organized as follows: the theoretical model of the pivot is established in Section 2, and the finite difference model is introduced in Section 3. Section 4 presents the genetic algorithm optimization procedure. Numerical calculation results are analyzed in Section 5, and main conclusions are drawn in Section 6.

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2. Mathematical Model of a Bottom Pivot

The general form of the Reynolds equation is [18]:

$$\frac{\partial}{\partial t}(h) + \nabla \cdot \left(\frac{h^3}{12\mu}\nabla p\right) = \nabla \cdot \left(\frac{h}{2}U\right)$$

where h is the oil film thickness, μ is the dynamic viscosity of the lubricating medium, p is the oil film pressure, U is the relative movement speed of the friction pair, and t is time. Replacing rectangular coordinates (x, y, z) with spherical coordinates (R, θ, ϕ) yields:

$$\begin{cases} x = R\sin\theta\cos\phi \\ y = R\sin\theta\sin\phi \\ z = R\cos\theta \end{cases}$$

where R is the distance from the point to the origin, θ is the angle between the point and the z-axis, and ϕ is the angle between the projection of this point in the XY plane and the x-axis.

The bottom pivot is lubricated with grease, and its lubricating medium can be considered as a Bingham fluid, so the shear stress can be expressed as:

$$\tau = \tau_0 + \eta\frac{\partial v}{\partial s}$$

where τ_0 is the ultimate dynamic shear stress.

Due to high viscosity of the lubricant grease and small gap between the bottom pivot components, the grease flow can be considered as laminar. The classical Reynolds equation can be derived from the Navier-Stokes equations if ignoring fluid film curvature, inertial forces, and slip boundary conditions. The Reynolds equation for incompressible fluid in spherical coordinates can be obtained as follows [18]:

$$\frac{\partial}{\partial\theta}\left(\frac{h^3}{\mu}\sin\theta\frac{\partial p}{\partial\theta}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial\phi}\right)$$
$$=12R\sin\theta\left(U_\theta\frac{\partial h}{\partial\theta}+U_\phi\frac{\partial h}{\partial\phi}+h\frac{\partial U_\theta}{\partial\theta}+h\frac{\partial U_\phi}{\partial\phi}\right)$$
$$+12R\sin\theta\cos\theta\,U_R+12R\sin^2\theta\frac{\partial h}{\partial t}$$

where R is the radius of the base pivot, U_θ, U_ϕ, U_R are the linear velocities of the fluid in respective directions.

Using $\varepsilon_x, \varepsilon_y, \varepsilon_z$ to represent the eccentricity of the spherical bearing and $\omega_x, \omega_y, \omega_z$ to represent the relative rotational angular velocity, we obtain:

$$h=c(1-\varepsilon_x\sin\theta\cos\phi-\varepsilon_y\sin\theta\sin\phi-\varepsilon_z\cos\theta)$$

where c represents the clearance of the friction pair.

The miter gate structure is shown in Figure 1 [Figure 1: see original paper], and the oil film distribution is depicted in Figure 2 [Figure 2: see original paper]. The bearing capacity of the bottom pivot is calculated as follows:

Based on Figure 2, Eq. (3) is processed and simplified to obtain Eq. (9):

$$\begin{cases}F_x=\int_0^{2\pi}\int_0^{\pi/2}Rp\sin\theta\cos\phi\sin\theta\,d\theta\,d\phi\\F_y=\int_0^{2\pi}\int_0^{\pi/2}Rp\sin\theta\sin\phi\sin\theta\,d\theta\,d\phi\\F_z=\int_0^{2\pi}\int_0^{\pi/2}Rp\cos\theta\sin\theta\,d\theta\,d\phi\end{cases}$$

For convenience of calculation, Eq. (9) can be reduced as:

$$A\frac{\partial^2p}{\partial\theta^2}+B\frac{\partial^2p}{\partial\phi^2}+C\frac{\partial p}{\partial\theta}+D\frac{\partial p}{\partial\phi}+E=0$$

The coefficients in Eq. (10) are described as:

$$\begin{cases} A = \frac{h^3}{\mu}\sin\theta \\ B = \frac{h^3}{\mu\sin\theta} \\ C = \frac{3h^2}{\mu}\frac{\partial h}{\partial\theta}\sin\theta + \frac{h^3}{\mu}\cos\theta \\ D = \frac{3h^2}{\mu\sin\theta}\frac{\partial h}{\partial\phi} \\ E = -12R\sin\theta\left(U_\theta\frac{\partial h}{\partial\theta} + U_\phi\frac{\partial h}{\partial\phi} + h\frac{\partial U_\theta}{\partial\theta} + h\frac{\partial U_\phi}{\partial\phi}\right) - 12R\sin\theta\cos\theta\, U_R - 12R\sin^2\theta\frac{\partial h}{\partial t} \end{cases}$$

3. Numerical Analysis

The finite difference method is used to solve the Reynolds equation, and the difference relationship is described in Figure 3 [Figure 3: see original paper]. The solution area is divided into 90 meshes in the θ direction and 360 meshes in the ϕ direction, with step length $\Delta\theta = \frac{\pi/2}{90}$ along the θ direction and step length $\Delta\phi = \frac{2\pi}{360}$ along the ϕ direction.

According to the difference relationship in Figure 3, the partial derivatives of pressure at point (i, j) can be expressed as:

$$\begin{cases} \frac{\partial p}{\partial\theta} = \frac{p_{i+1,j} - p_{i-1,j}}{2\Delta\theta} \\ \frac{\partial p}{\partial\phi} = \frac{p_{i,j+1} - p_{i,j-1}}{2\Delta\phi} \\ \frac{\partial^2 p}{\partial\theta^2} = \frac{p_{i+1,j} - 2p_{i,j} + p_{i-1,j}}{\Delta\theta^2} \\ \frac{\partial^2 p}{\partial\phi^2} = \frac{p_{i,j+1} - 2p_{i,j} + p_{i,j-1}}{\Delta\phi^2} \end{cases}$$

Substituting Eq. (12) into Eq. (11), the relation between $p_{i,j}$ of each node and p of adjacent nodes can be obtained as:

$$C_N p_{i,j+1} + C_S p_{i,j-1} + C_E p_{i+1,j} + C_W p_{i-1,j} + G = p_{i,j}$$

where:

$$\begin{cases} C_N = \frac{B}{\Delta\phi^2} + \frac{D}{2\Delta\phi} \\ C_S = \frac{B}{\Delta\phi^2} - \frac{D}{2\Delta\phi} \\ C_E = \frac{A}{\Delta\theta^2} + \frac{C}{2\Delta\theta} \\ C_W = \frac{A}{\Delta\theta^2} - \frac{C}{2\Delta\theta} \\ K = -\left(\frac{2A}{\Delta\theta^2} + \frac{2B}{\Delta\phi^2}\right) \\ G = -\frac{E}{K} \end{cases}$$

After discretization, Jacobi iteration is used to solve the differential equation:

$$p_{i,j}^{k+1} = (1-\alpha)p_{i,j}^{k} + \alpha p_{i,j}^{k+1}$$

where $p_{i,j}^k$ is the current pressure, $p_{i,j}^{k+1}$ is the new pressure, $p_{i,j}^{k+1}$ is the pressure obtained by Eq. (13), and α is a decimal between 0 and 1.

The boundary is determined by the following well-known scheme:

$$p_{i,j} = 0 \quad \text{if} \quad p_{i,j} < 0$$

Adopting the discriminant method of relative convergence for the Reynolds equation, we derive:

$$\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}|p_{i,j}^{k+1} - p_{i,j}^{k}|}{\sum_{i=1}^{n}\sum_{j=1}^{m}|p_{i,j}^{k+1}|} \leq \varepsilon$$

where n is the number of grids divided by rows, m is the number of grids divided by columns, and ε is a small constant; in this work $\varepsilon = 1.0 \times 10^{-4}$.

The pressure on the bottom pivot edge is set to zero. In the initial condition, once the pressure is determined, the array of the NumPy library in Python is applied to calculate the component force of each pressure point by multiplying the elements, and then summing the component forces in each direction as the total component force in that direction.

Let us assume that there is only one load in the Z direction and only one rotational angular velocity around the Z-axis. At the beginning of the calculation, the film thickness distribution matrix is calculated by Eq. (5), and the obtained film thickness is substituted into Eq. (11) to calculate the values of A, B, C, D, E. These computed values are then substituted into Eqs. (13) and (14) to calculate the pressure distribution matrix. The iterative process of Eq. (13) is repeated, and convergence is judged using Eq. (15). After convergence, a stable pressure distribution matrix is obtained. During the iterative process, to remove negative pressure, any calculated negative pressure is reset to zero to meet Reynolds boundary conditions after several iterations in this study. The calculation procedure is shown in Figure 4 [Figure 4: see original paper].

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4. Model Initial Parameters Optimization

Since the component forces calculated by the finite difference method are affected by initial model parameters such as initial pressure and initial eccentricity, the genetic algorithm (GA) is employed to optimize these parameters. Due to complex coupling effects, the relationship between initial pressure and force components is non-linear, requiring a proper objective function for GA optimization. In this study, the objective function is selected as:

$$f'(x) = \min|f_z - f_z'|$$

In the above formula, f_z represents the theoretical force in the vertical direction and f_z' represents the force calculated by the finite difference method.

A random number generating function is used to create 12-bit binary random data to represent the GA initial "genes". If *num* is the actual number converted from binary to decimal, it will be mapped into a specified range by:

$$X = X_{\min} + \frac{num}{2^{12} - 1} \times (X_{\max} - X_{\min})$$

When iterating the difference equation, to obtain a good "gene", evaluation information (i.e., the fitness function) is needed, which is expressed as:

$$g(x) = \frac{1}{|f(x) - f'(x)| + 0.0001}$$

After the evaluation information is determined according to Eq. (19), possible solutions will be selected. Solutions closer to the theoretical value are more likely to be retained, with higher corresponding selection probabilities. Parameter settings of the GA and the pivot are shown in the following tables.

Table 1 . GA settings

Parameter	Value
Population	200
Probability of mutation	0.1
The crossover probability	0.9
Evolution generations	40

Table 2 . Bottom pivot material and iterative parameters

Parameter	Value
Base pivot mushroom head material	40 Cr
Bottom pivot tile material	QT600-3
Quality of gate	850 t
Lubricant viscosity	-
Elastic modulus of bearing bush	169 GPa
Bearing bush Poisson's ratio	-
Tensile strength of bearing bush	$ $600 MPa
Initial eccentricity	0, 0.1, 0.1
The iteration error	1.0×10^{-4}
Maximum search times	-

5.1. The Initial Pressure

According to the above solving process, the authors wrote a Python program to optimize several working conditions. Taking a specific type of gate as an example, the initial parameter settings of the simulation are shown in Table 1 and Table 2. Three-dimensional, two-dimensional, spherical, and rectangular images of oil film thickness and pressure distribution are shown as follows:

Figure 5 [Figure 5: see original paper]. Three-dimensional schematic diagram of oil film thickness and pressure distribution

Figure 5 shows that the oil film thickness gradually decreases along the direction. Since the initial parameters are preset, the oil film thickness along the direction will first decrease and then increase. The normalized results of the maximum stress are shown in Figure 6 [Figure 6: see original paper].

Figure 6. Schematic diagram of the changing trend of normalized pressure and film thickness

According to Eq. (13), during each iteration, the pressure at the center point in the grid plane is affected by surrounding points. Therefore, different initial pressures will affect the calculation results. Other parameters are shown in Table 2. The initial pressure is set from 1 MPa, 2 MPa, 3 MPa… to 10 MPa to observe the impact of different initial pressures on the results, as shown in the following figure:

Figure 7 [Figure 7: see original paper]. The effect of the initial pressure on the result of the force-splitting calculation

As can be seen from Figure 7, the relationship between initial pressure and components is approximately linear, and the increase of initial pressure affects different directional components differently. Therefore, when calculating the bearing capacity of the bottom pivot, choosing an appropriate initial pressure is conducive to improving calculation speed.

5.2. Eccentricity

In the actual process, the bottom pivot is affected by external load and its own gravity, resulting in different eccentricities that cause the lubricating oil film to produce a wedge effect, affecting the regular operation of the bottom pivot. According to Equation (4), the oil film thickness at any point in the bottom pivot is related to its eccentricity. To analyze the influence of film thickness on lubrication performance, different eccentricity ratios are selected to observe changes in force components in each direction, as shown in the figures below:

Figure 8 [Figure 8: see original paper]. Influence of eccentricity in the X direction on component force under different initial pressures

Figure 9 [Figure 9: see original paper]. Influence of eccentricity in the Y direction on component force under different initial pressures

Figure 10 [Figure 10: see original paper]. Influence of eccentricity in the Z direction on component force under different initial pressures

It can be seen from Figures 8, 9, and 10 that when eccentricity in the X direction increases, the bearing capacity in the X and Z directions increases while the bearing capacity in the Y direction decreases. The increase in eccentricity in the Y direction also leads to increased bearing capacity, and the greater the initial pressure, the more obvious the impact on bearing capacity. When eccentricity in the Z direction increases, the bearing capacity in the X direction increases while the bearing capacity in Y and Z directions decreases.

5.3. Rotational Angular Velocity

When the spherical bearing does not rotate, there is only a static pressure effect. When it rotates, a dynamic pressure effect is produced due to the existence of a wedge-shaped oil film, which affects bearing capacity. Under the condition that other parameters remain unchanged, the rotating speeds are set to 3°/s, 6°/s, 9°/s, 12°/s, and 15°/s. The bearing capacity is shown in the figure below:

Figure 11 [Figure 11: see original paper]. Influence of rotational speed on component force under different initial pressure

As shown in Figure 11, with increasing rotating speed, the bearing capacity in the X direction also increases, and the greater the initial pressure, the more obvious the improvement. However, the Y direction shows the opposite trend. When the initial pressure in the Z direction is less than 4 MPa, higher speed results in greater bearing capacity, but this decreases when the initial pressure is greater than 4 MPa.

5.4. Optimization Process Analysis

Even if the pressure obtained by the difference method converges, the obtained solution is not necessarily the fundamental solution to the problem, so feedback processes are needed. The genetic algorithm parameters are: 200 population size, 40 evolution generations, and crossover probabilities of 0.6 and 0.9 respectively, with the goal of minimizing the difference between the carrying capacity and the theoretical value in the Z direction. The variation process of the initial pressure in the difference method across evolution times is shown in the following figure:

Figure 13 [Figure 13: see original paper]. Relationship between evolution times and initial pressure

It can be seen from Figure 13 that the initial pressure obtained when the crossover probability is 0.9 gradually tends to be stable after 30 evolutions. In contrast, when the crossover probability is 0.6, it does not stabilize after 30 evolutions. Therefore, this paper adopts a crossover probability of 0.9 for optimization.

Figure 14 [Figure 14: see original paper]. Comparison diagram of optimization results

As can be seen from Figure 14, the difference between calculated values and theoretical values varies significantly across different schemes. Schemes 1 and 2 are results obtained after genetic algorithm optimization, while schemes 3 to 10 are results from manually setting the initial value. The figure demonstrates that the genetic algorithm achieves higher accuracy without requiring multiple manual trials. After GA optimization, when the initial pressure is about 5.64 MPa, the ratio of the difference between the estimated value and the theoretical value to the theoretical value is less than $-31.010\times$.

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6. Conclusions

A numerical calculation model of hydrodynamic lubrication characteristics for miter gate bottom pivot is proposed in this paper. The effects of initial pressure, eccentricity, and rotating speed on oil film thickness and pressure distribution are analyzed. In the iterative process, increasing initial pressure significantly improves bearing capacity in the z-axis direction. Increases in eccentricity in the X and Y directions increase the component forces along these directions, with greater initial pressure resulting in faster component increase. Meanwhile, increasing Z-direction eccentricity reduces bearing capacity. Additionally, component forces in the X and Z directions increase with rotation speed, while components in the Y direction decrease. Finally, the genetic algorithm is used to optimize the initial pressure, significantly reducing the error between theoretical and calculated values so that the calculated results and external load meet engineering accuracy requirements. Through the finite difference method and genetic algorithm calculation, the oil film thickness distribution and pressure distribution are consistent with expectations. Because the load on the bottom pivot is large and vertical, the film thickness at the center of the ball is the thinnest and the pressure in this area is the largest. More effective optimization algorithms can be employed. When more data are available, machine learning and deep learning can be used to train larger models to improve accuracy and adaptability.

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Note: Figure translations are in progress. See original paper for figures.