Effect of Multiple Coulomb Scattering on the Beam Tests of Silicon Pixel Detectors

Authors: Li Lankun, Dong Mingyi, Dong Mingyi

Date: 2024-03-07T00:00:00+00:00

Abstract

In the research and development of silicon pixel detectors, collimated monoenergetic charged particle test beams equipped with high-resolution pixel beam telescopes are essential for detector prototype verification and performance evaluation. At low beam energies, it is imperative to account for the effects of multiple Coulomb scattering on the measurement resolution of the device under test (DUT) in order to accurately assess the performance of pixel chips and detectors. This study investigates the impact of multiple Coulomb scattering on DUT measurement resolution, particularly under low beam energy conditions. Simulations were performed using the Allpix2 software to examine the influence of multiple Coulomb scattering across various beam energies, material budgets, and telescope layouts. The simulations also determine the minimum energy threshold below which the effects of multiple Coulomb scattering become negligible. Comparison with electron beam test results conducted at DESY demonstrates good agreement between simulation and experimental data, thereby validating the reliability of the simulation approach.

Full Text

Preamble

Effect of Multiple Coulomb Scattering on the Beam Tests of Silicon Pixel Detectors

Lan-Kun Li[12], Ming-Yi Dong[23]†, Ze Gao[12], Liang-Cheng-Long Jin[24], and Shu-Jun Zhao[1]

[1]School of Physics and Microelectronics, Zhengzhou University, Zhengzhou 450001, China

[2]State Key Laboratory of Particle Detection and Electronics (Institute of High Energy Physics, CAS), Beijing 100049, China

[3]University of Chinese Academy of Science, Beijing 100049, China
[4]University of South China, Nanhua 421001, China

In the research and development of new silicon pixel detectors, collimated monoenergetic charged-particle test beams equipped with high-resolution pixel-beam telescopes are crucial for prototype verification and performance evaluation. At low beam energies, the effect of multiple Coulomb scattering on the measured resolution of the Device Under Test (DUT) must be considered to accurately evaluate the performance of pixel chips and detectors. This study investigates the effect of multiple Coulomb scattering on measured resolution, particularly at low beam energies, using Allpix^2 simulations to examine how multiple Coulomb scattering varies with beam energy, material budget, and telescope layout. The simulations also determine the minimum energy at which multiple Coulomb scattering effects become negligible. Compared with results from a five-layer detector system tested with an electron beam at DESY, the simulation results show good agreement with beam test data, confirming the reliability of the simulation approach.

Keywords: Silicon Pixel Detectors; Beam Telescope; Multiple Coulomb Scattering; Spatial Resolution

INTRODUCTION

Vertex detectors and tracking systems play critical roles in high-energy physics experiments, where detectors with high measurement accuracy are essential for identifying and reconstructing secondary particles or decay products of short-lived particles. Silicon pixel sensors [1, 2], particularly CMOS Monolithic Active Pixel Sensors (MAPS) [3] that integrate the sensor and front-end electronics readout logic on the same silicon substrate, offer high spatial resolution, high counting rates, and excellent radiation hardness. MAPS-based silicon pixel detectors have been successfully deployed in vertex detectors and tracking systems, such as the STAR experiment vertex detector at RHIC [4] and the Inner Tracking System (ITS) upgrade for the ALICE experiment at the LHC [5]. A MAPS-based vertex detector for the Circular Electron-Positron Collider (CEPC) has also been proposed, with significant progress in sensor and detector prototype development [6–9]. Beyond high-energy physics, silicon pixel detector technologies find broad applications in other fields, including photon-counting pixel detectors for High Energy Photon Sources (HEPS) [10–12] and digital tomography detectors for medical imaging [13].

In the R&D of new silicon pixel chips and detector prototypes, radioactive sources and cosmic rays [14–16] are commonly used to verify chip design and detector functionality. However, these laboratory tests have limitations in accurately evaluating performance due to energy constraints of radioactive sources and limited cosmic ray flux. Consequently, collimated monoenergetic charged-particle test beams are employed to comprehensively evaluate detector performance, providing more accurate measurements of spatial resolution and detection efficiency while enabling calibration and optimization of detector prototypes. The beam telescope—a necessary component in test beam systems—provides precise reference particle tracks for the Device Under Test (DUT) and must achieve higher measurement accuracy than the DUT itself. Several beam telescopes with different configurations have been used to study new detectors, including EUDET-type telescopes [17] with six pixel-detector planes equipped with MIMOSA26 chips, replicated at various facilities: the ACONITE telescope at CERN SPS with 120 GeV pions, the DATURA telescope at the DESY II Test Beam Facility with electrons up to 6 GeV, and the ANEMONE telescope at Bonn with 1–3 GeV electrons. ADENIUM represents a next-generation beam telescope demonstrator at DESY [18], while LYCORIS is a six-plane large-area strip-beam telescope installed at DESY II with a 1T solenoid magnet [19]. A CMS pixel-based telescope was constructed at the Fermilab Test Beam Facility (FTBF) [20], and a beam telescope has been proposed for 1.6 GeV proton tests at the China Spallation Neutron Source (CSNS) [21–23].

The measurement accuracy of detectors in test beam systems is strongly influenced by beam parameters and telescope performance. Particularly at low beam energies and with DUTs having large material budgets, accounting for multiple Coulomb scattering effects on measured resolution is crucial for improving the precision of pixel chip and detector prototype performance evaluation. This study investigates how multiple Coulomb scattering affects DUT measured resolution under various beam energies and material budgets, providing references for particle detector research, testing protocols, and beam telescope design. These results assist in selecting appropriate test beams based on DUT material budgets and minimizing multiple Coulomb scattering effects on resolution. Section 2 briefly introduces the beam telescope system, Section 3 presents Allpix^2 simulation studies of multiple Coulomb scattering effects under different conditions to determine the minimum energy where these effects become negligible, and Section 4 validates the simulation reliability through comparison with beam test results.

II. BEAM TELESCOPE

A. Telescope System

A beam telescope is a tracking detector system typically comprising four or more planes with silicon pixel sensors. The pixel sensors on each plane must be thin and highly granular. To minimize multiple Coulomb scattering effects, the detection area typically includes only a silicon pixel sensor and two layers of Kapton film for light shielding. A schematic of the beam telescope is shown in [Figure 1: see original paper]. The DUT is generally positioned at the center of the beam telescope, with high-energy charged particles traversing both the telescope planes and the DUT. A mechanical structure consisting of two symmetrical telescope arms is typically used to support the planes, with both arms adjustable along the beam direction to facilitate installation of DUTs of different sizes.

In beam test data analysis, detector alignment is essential for calibrating the positions of the DUT and each telescope plane by minimizing track residuals using a large number of tracks [24]. After alignment, reference tracks can be reconstructed using the minimum ^{2} matrix algorithm [25]. For low-energy beams, multiple Coulomb scattering effects on measured resolution cannot be ignored, particularly when the DUT has non-negligible material budget. Algorithms such as the Kalman Filter with a covariance matrix accounting for multiple Coulomb scattering [26, 27] or General Broken Lines [28] can be utilized for track fitting and detector alignment to mitigate these effects. To avoid algorithmic differences between simulation and data, this study employs the same alignment algorithm as in [24] and the Kalman filter track fitting from [26] for both simulation and beam test data analysis.

B. Measured Resolution

The fitting points of the DUT can be obtained via track fitting. The measured spatial resolution of the DUT, defined as the residual between fitting points and measured points, is unbiased if no DUT information is used during track fitting. The measured resolution σ includes contributions [29] from the intrinsic resolution of the DUT $\sigma\{DUT\}$*, telescope resolution* σ *, and multiple Coulomb scattering effects* $\sigma\{\mathrm{MS}\}$, as described by Eq.(1):

$$\sigma\ ^2 = \sigma\{DUT\}^2 + \sigma\ ^2 + \sigma\{\mathrm{MS}\}^2$$

Notably, multiple Coulomb scattering originates not only from the DUT but also from the telescope planes. Therefore, telescope planes are typically designed to be as thin as possible to minimize these effects.

The telescope resolution σ can be determined when all telescope planes have the same intrinsic resolution σ_{plane} and the DUT is placed at z=0, using Eq.(2) and Eq.(3):

$$\sigma\ ^2 = \mathrm{k}\sigma\_\{\mathrm{plane}\}^2$$
$$\mathrm{k} = (\ \mathrm{N}\ \mathrm{z\_i}^2 - (\ \mathrm{N}\ \mathrm{z\_i})^2/\mathrm{N})\ /\ (\ \mathrm{N}\ \mathrm{z\_i}^2)$$

where z_i and N represent the position along the beam direction and the number of telescope planes, respectively. If the telescope planes are parallel and symmetrically distributed on both sides of the DUT, k reduces to 1/N.

When beam energy is sufficiently high, multiple Coulomb scattering effects can be ignored. Assuming the DUT and telescope planes are identical, σ_{DUT} and σ can be calculated from the measured resolution σ by combining Eq.(1), Eq.(2), and Eq.(3), yielding Eq.(4) and Eq.(5):

$$\sigma\_\{\mathrm{DUT}\} = \surd(1 + \mathrm{k}) \times \sigma$$
$$\sigma\ = \surd(\mathrm{k}/(1 + \mathrm{k})) \times \sigma$$

Based on Eq.(1), DUT measurement results are influenced by both beam telescope resolution and multiple Coulomb scattering effects. However, for silicon pixel chips used in telescopes, chip design and resolution are constrained by multiple factors and cannot be significantly improved. Therefore, evaluating and minimizing multiple Coulomb scattering effects is crucial for improving DUT measurement accuracy in beam tests.

C. Simulation Setup

We use Allpix^2 [30] to simulate the test beam system and study how multiple Coulomb scattering influences measured resolution. Allpix^2 is an open-source C++ framework built on Geant4 and ROOT, specifically designed for simulating silicon pixel detectors. It can easily simulate individual detectors and complex systems such as test beam setups, providing detailed information including pixel charge collection and digitized responses.

Unless otherwise stated, all simulations in this study employ the following configuration: the beam telescope consists of four planes divided into two upstream and two downstream planes, with the DUT positioned at the telescope center. The gap between telescope planes and the distance from the DUT to the front and rear planes are set to 20 mm. Both DUT and telescope planes use the same model: a MIMOSA28 chip [31] thinned to 50 µm with two layers of 50 µm Kapton film, resulting in a total material budget of 0.088% X_0 per plane.

The MIMOSA28 chip contains 928 (rows) × 960 (columns) pixels with a pitch of 20.7 µm. It features a 15 µm-thick high-resistivity epitaxial layer as the sensitive region, which is not fully depleted, allowing charge collection through thermal diffusion. MIMOSA28 chips are also being considered for upcoming proton beam telescopes at CSNS. In the simulation, electronic noise is set to 30 e^- based on threshold scan results, and the threshold is set to 5σ to mitigate electronic noise influence and achieve optimal resolution.

III. MULTIPLE COULOMB SCATTERING

When charged particles pass through material, they undergo many small-angle deflections known as multiple Coulomb scattering. The Highland formula [32] calculates the distribution of multiple Coulomb scattering angles $_0$, as shown in Eq.(6):

$$_0 = (13.6\ \mathrm{MeV}\ /\ (\mathrm{p}\beta \mathrm{c})) \times \sqrt{}(\mathrm{x}/\mathrm{X}_0) \times [1 + 0.038\ \ln(\mathrm{x}/\mathrm{X}_0)]$$

where p is the incident particle momentum, βc is the velocity, and Z is the atomic number. x/X_0 represents the material budget, with x being the particle track length through the material and X_0 the material radiation length. Multiple Coulomb scattering from both beam telescope planes and the DUT degrades the measured resolution of the DUT.

As shown in Eq.(6), both beam energy and material budget influence the multiple Coulomb scattering angle $_0$. Additionally, for a given $_0$, the effect of multiple Coulomb scattering depends on the distance between measurement points. Therefore, three factors influence multiple Coulomb scattering effects: beam energy, material budget, and telescope layout.

A. Beam Energy

To study the impact of beam energy on the multiple Coulomb scattering angle and consequently on the simulated resolution (σ), we simulated electron beam energies from 0.5 GeV to 30 GeV, with the DUT identical to the telescope plane model (material budget 0.088% X_0). Based on Eq.(6), at sufficiently high beam energy (e.g., 120 GeV), multiple Coulomb scattering effects become negligible. Therefore, 120 GeV electrons serve as the reference case without multiple Coulomb scattering effects.

[Figure 2: see original paper] shows the simulated resolution as a function of beam energy. Increasing beam energy significantly improves resolution in the low-energy regime. When beam energy becomes sufficiently large, the simulated resolution approaches the 120 GeV electron value and no longer improves. The multiple Coulomb scattering contribution at different energies can be calculated using Eq.(1). For beam energies above 12 GeV, σ_{MS} is less than 0.5 μm, causing less than 1% degradation compared to the 120 GeV electron case.

B. Material Budget

Beam telescopes are designed to minimize material budget, making the DUT material budget the primary factor influencing multiple Coulomb scattering effects. To study this impact, we varied the DUT material budget from 0.106% X_0 to 0.53% X_0 by increasing the equivalent silicon thickness from 100 μm to 500 μm (without Kapton films), while keeping the telescope plane material budget constant at 0.088% X_0.

[Figure 3: see original paper] shows the simulated resolution as a function of material budget for 5 GeV and 120 GeV electrons. With 120 GeV electrons, the simulated resolution remains nearly constant as material budget increases because multiple Coulomb scattering effects are negligible. However, with 5 GeV electrons, doubling the material budget increases the multiple Coulomb scattering contribution by approximately 25% and degrades the simulated resolution by about 5%.

Based on these results, the simulated resolution with 120 GeV electrons (approximately 4.18 μm) serves as a reference for studying multiple Coulomb scattering effects across different material budgets and beam energies. This allows estimation of the minimum energy where multiple Coulomb scattering becomes negligible for various DUT material budgets. We varied the DUT material budget from 0.106% X_0 to 1.06% X_0 at beam energies from 5 to 50 GeV. [Figure 4: see original paper] shows the multiple Coulomb scattering contribution as a function of beam energy for different DUT models. Increasing beam energy significantly reduces multiple Coulomb scattering effects. When σ{*MS*} *falls below*

0.5 μm, the DUT resolution is dominated by simulation randomness and statistical errors, causing fluctuations in the $\sigma\{\mathrm{MS}\}$ curves at high beam energies, as seen in Figures 2 and 4. [Figure 5: see original paper] shows the minimum energy where multiple Coulomb scattering effects can be ignored as a function of DUT material budget. The minimum energy increases with DUT material budget.

C. Telescope Layout

Telescope arms can be adjusted along the beam direction to modify the distance between the DUT and telescope planes, enabling testing of various device sizes. Layout modifications influence multiple Coulomb scattering effects even when the scattering angle $_0$ remains constant. Furthermore, changing the number of beam telescope planes affects the factor 'k' in Eq.(2) and the multiple Coulomb scattering contribution. To study these effects, we simulated 4-layer, 5-layer, and 6-layer telescopes with different layouts. For the 5-layer telescope, three planes were placed before and two after the DUT. In these simulations, the DUT was identical to the telescope plane model.

The DUT resolution is strongly influenced by the distance between the DUT and the nearest telescope planes. As shown in Figure 6: see original paper, simulation results indicate that reducing the gap between the DUT and telescope planes within the 10–100 mm range improves the simulated resolution. Increasing the track length leads to larger residuals between fitted and measured points, thereby degrading resolution. However, the DUT-telescope distance is limited by the DUT size. For DUTs of varying dimensions, minimizing this gap is crucial for achieving optimal resolution.

Figure 6: see original paper shows the simulated resolution as a function of the distance between telescope planes. Minimizing the inter-plane distance reduces multiple Coulomb scattering effects. Theoretically, increasing the number of telescope planes improves telescope resolution, as shown in Eq.(5). However, this also increases the total material budget of the beam telescope. With 5 GeV electrons, the simulated resolution with a 4-layer telescope is essentially identical to that with a 6-layer telescope.

IV. COMPARISON BETWEEN SIMULATION AND TEST RESULTS

To validate simulation accuracy, we compared simulation results with beam test data from a five-ladder detector system [33, 34], as shown in [Figure 7: see original paper]. In the beam test system, each ladder consists of MIMOSA28 chips thinned to 50 μm, a flex cable, and a 1.8 mm thick carbon fiber and foam composite support, yielding a material budget of approximately 0.35% X_0. In this system, the first and last two layers serve as the telescope, while the middle ladder acts as the DUT. Tests were performed at the T24 beamline at DESY with beam energies from 1 to 6 GeV. The simulation adopted the methods described previously, with models identical to the actual ladders, and used the same algorithms for test data analysis as in the simulation.

Figure 7: see original paper shows the measured resolution as a function of threshold from 4.45σ to 8.2σ. The threshold range was determined from threshold scans; thresholds that are too low allow fake hits from electronic noise to impact track fitting and reconstruction. The simulation results are slightly smaller than the beam test measurements, primarily because factors such as beam energy dispersion and slight variations in noise between different chips were not included in the simulation.

Figure 7: see original paper shows the measured resolution as a function of beam energy from 1 to 6 GeV. Based on test results, the multiple Coulomb scattering contribution at 5 GeV is approximately 2.7 µm due to the relatively large material budget of the ladders. The simulation and beam test results show good consistency across different thresholds and beam energies.

V. CONCLUSION

Using Allpix^2 simulations, we studied how multiple Coulomb scattering affects DUT measured resolution under various beam energies, material budgets, and telescope layouts. Simulation results are consistent with beam test data, confirming the reliability of the simulation approach. Multiple Coulomb scattering effects depend on three main factors: beam energy, material budget, and telescope layout. Increasing beam energy or reducing material budget effectively mitigates these effects. For 5 GeV beams and DUT material budgets from 0.1% X_0 to 0.5% X_0, the multiple Coulomb scattering contribution is approximately 1–2 µm. At sufficiently high beam energies, DUT resolution becomes independent of energy because multiple Coulomb scattering effects become negligible. When the DUT material budget is below 0.2% X_0, the minimum energy where multiple Coulomb scattering can be ignored is approximately 20 GeV, increasing with larger material budgets. Additionally, DUT resolution is significantly influenced by the distance between the DUT and the nearest telescope planes; reducing this gap to within 10–100 mm improves measured resolution. To accurately evaluate DUT performance across different sizes, minimizing the DUT-telescope gap is essential. These results provide valuable references for selecting appropriate test beam energies for DUTs with different material budgets and for designing test beam telescopes.

VI. ACKNOWLEDGEMENTS

The measurements leading to these results were performed at the Test Beam Facility at DESY Hamburg (Germany), a member of the Helmholtz Association (HGF). This study was supported by the National Natural Science Foundation of China (Grant No.11875274 and No.U1232202).

REFERENCES

[1] R. He, X. Y. Niu, Y. Wang et al., Advances in nuclear detection and readout techniques. Nucl. Sci. Tech. 34, 205 (2023). https://doi.org/10.1007/s41365-023-01359-0
[2] S. Seidel, Silicon strip and pixel detectors for particle physics experiments. Phys. Rep. 828, 1–34 (2019). https://doi: 10.1016/j.physrep.2019.09.003
[3] G. Deptuch, J. D. Berst, G. Claus et al., Design and testing of monolithic active pixel sensors for charged particle tracking. IEEE. T. Nucl. Sci. 49, 601-610 (2002). https://doi: 10.1109/TNS.2002.1003683
[4] L. Greiner, E. Anderssen, H. S. Matis et al., A MAPS based vertex detector for the STAR experiment at RHIC. Nucl. Instrum. Meth. A 650, 68–72 (2011). https://doi: 10.1016/j.nima.2010.12.006
[5] P. Yang, G. Aglieri, C. Cavicchioli et al., MAPS development for the ALICE ITS upgrade. J. Instrum. 10, C03030 (2015). doi: 10.1088/1748-0221/10/03/C03030
[6] L. J. Chen, H. B. Zhu, X. C. Ai et al., Characterization of the first prototype CMOS pixel sensor developed for the CEPC vertex detector. Radiat. Detect. Technol. Methods. 3, 45 (2019). https://doi.org/10.1007/s41605-019-0124-0
[7] S. Dong, P. Yang, Y. Zhang et al., Design and characterisation of JadePix-3 CMOS pixel sensor. Nucl. Instrum. Meth. A 1048, 167967 (2023). https://doi.org/10.1016/j.nima.2022.167967
[8] W. Wei, T. Y. Wu, Z. J. Liang et al., Characterization of a CMOS Pixel Sensor prototype for the CEPC vertex detector. Nucl. Instrum. Meth. A 1056, 168601 (2023). https://doi.org/10.1016/j.nima.2023.168601
[9] T. Y. Wu, S. Q. Li, W. Wei et al., Beam test of a 180 nm CMOS Pixel Sensor for the CEPC vertex detector. Nucl. Instrum. Meth. A 1059, 168945 (2024). https://doi.org/10.1016/j.nima.2023.168945
[10] Z. j. Li, Y. C. Jia, L. F. Hu et al., Study of silicon pixel sensor for synchrotron radiation detection. Chinese. Phys. C 40, 036001 (2016). doi: 10.1088/1674-1137/40/3/036001
[11] Y. Ding, J. Zhang, W. Wei et al., The development and application of the test system for the silicon pixel modules in HEPS-BPIX. Radiat. Detect. Technol. Methods. 5, 53-60 (2021). https://doi.org/10.1007/s41605-020-00217-6
[12] J. Zhang, H. X. Li, W. Wei et al., HEPS-BPIX2: The hybrid pixel detector upgrade for high energy photon source in China. Nucl. Instrum. Meth. A 958, 162488 (2020). https://doi.org/10.1016/j.nima.2019.162488
[13] Y. L. Chen, H. K. Wang, S. Y. Zhang et al., Hi' CT: a pixel sensor-based device for ion tomography. Nucl. Sci. Tech. 34, 111 (2023). https://doi.org/10.1007/s41365-023-01251-x
[14] L. Ma, X. Dong, H. Qiu et al., Alignment calibration and performance study of the STAR PXL detector. Nucl. Sci. Tech. 28, 25 (2017). https://doi: 10.1007/s41365-016-0177-4
[15] R. Nouicer, Y. Akiba, R. Bennett et al., Status and performance of new silicon stripixel detector for the PHENIX experiment at RHIC: Beta source, cosmic-rays and proton beam at 120 GeV. J. Instrum. 4, P04011 (2009).

https://doi: 10.1088/1748-0221/4/04/P04011
[16] C. H. Chen, Z. K. Li, X. H. Wang et al., Development of high performance PIN-silicon detector and its application in radioactive beam physical experiment. Acta Phys. Sin. 72, 122902 (2023). http://dx.doi.org/10.7498/aps.72.20230213
[17] C. Hu-Guo, J. Baudot, G. Bertolone et al., First reticule size MAPS with digital output and integrated zero suppression for the EUDET-JRA1 beam telescope. Nucl. Instrum. Meth. A 623, 480-482 (2010). https://doi: 10.1016/j.nima.2010.03.043
[18] Y. Liu, C. Q. Feng, I. M. Gregor et al., ADENIUM –A demonstrator for a next-generation beam telescope at DESY. J. Instrum. 18, P06025 (2023). https://doi: 10.1088/1748-0221/18/06/P06025
[19] J. Brau, M. Breidenbach, D. R. Freytag et al., Lycoris–A large-area, high resolution beam telescope. J. Instrum. 16, P10023 (2021). https://doi: 10.1088/1748-0221/16/10/P10023
[20] S. Kwan, C. M. Lei, D. Menasce et al., The pixel tracking telescope at the Fermilab Test Beam Facility. Nucl. Instrum. Meth. A 811, 162-169 (2016). https://doi: 10.1016/j.nima.2015.12.003
[21] S. Wang, S. X. Fang, S. N. Fu et al., Introduction to the overall physics design of CSNS accelerators. Chinese. Phys. C 33, 1 (2009). https://doi: 10.1088/1674-1137/33/S2/001
[22] J. Wei, H. S. Chen, Y. W. Chen et al., China spallation neutron source: design, R&D, and outlook. Nucl. Instrum. Meth. A 600, 10-13 (2009). https://doi.org/10.1016/j.nima.2008.11.017
[23] S. N. Fu, H. S. Chen, Y. B. Chen et al., CSNS Project Construction. J. Phys. Conf. Ser. 1021, 012002 (2018). 10.1088/1742-6596/1021/1/012002
[24] V. Karimaki, A. Heikkinen, T. Lampen et al., Sensor alignment by tracks. Paper presented at the 2003 Conference for Computing in High-Energy and Nuclear Physics, CA, USA, 24-28 Mar 2003. https://doi: 10.48550/arXiv.physics/0306034
[25] G. Lutz, Optimum track fitting in the presence of multiple scattering. Nucl. Instrum. Meth. A 273, 349-361. (1988). https://doi: 10.1016/0168-9002(88)90836-4
[26] E. J. Wolin, L. L. Ho, Covariance matrices for track fitting with the Kalman filter. Nucl. Instrum. Meth. A 329, 493-500 (1993). https://doi: 10.1016/0168-9002(93)91285-U
[27] R. Frühwirth, T. Todorov, M. Winkler, Estimation of detector alignment parameters using the Kalman filter with annealing. J. Instrum. 29, 561 (2003). https://doi: 10.1088/0954-3899/29/3/309
[28] C. Kleinwort, General Broken Lines as advanced track fitting method. Nucl. Instrum. Meth. A 673, 107-110 (2012). https://doi: 10.1016/j.nima.2012.01.024
[29] A. Bulgheroni, EUDET-JRA1 Consortium, Results from the EUDET telescope with high resolution planes. Nucl. Instrum. Meth. A 623, 10.1016/j.nima.2010.03.015. (2010). https://doi:
[30] S. Spannagel, K. Wolters, D. Hynds et al., Allpix^2: A modular simulation framework for silicon detectors. Nucl. Instrum. Meth. A 901, 164-172 (2018). https://doi: 10.1016/j.nima.2018.06.020

[31] I. Valin, C. Hu-Guo, J. Baudot et al., A reticle size CMOS pixel sensor dedicated to the STAR HFT. J. Instrum. 7, C01102 (2012). https://doi: 10.1088/1748-0221/7/01/C01102

[32] V. L. Highland, Some practical remarks on multiple scattering. Nucl. Instrum. Methods 129, 497-499 (1975). https://doi: 10.1016/0029-554X(75)90743-0

[33] M. Y. Dong, X. D. Ju, X. C. Tian et al., Development of MAPS-based detector ladders for the BESIII inner tracker upgrade. Nucl. Instrum. Meth. A 924, 287-292 (2019). https://doi: 10.1016/j.nima.2018.06.032

[34] C. Y. Qu, M. Y. Dong, J. Baudot et al., Performance study of a MAPS detector prototype based on test beam. Nucl. Instrum. Meth. A 986, 164810 (2021). https://doi: 10.1016/j.nima.2020.164810