Electromagnetic Fields of Moving Point Sources in the Vacuum

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Full Text

Preamble

Electromagnetic Fields of Moving Point Sources in the Vacuum

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their far fields. The results show that the velocity of electromagnetic waves and the velocity of the sources cannot be added like vectors; the velocity of electromagnetic waves from moving sources is anisotropic in vacuum; and the transverse Doppler shift is intrinsically included in the fields of moving sources and is not a pure relativity effect caused by time dilation. Since these fields are rigorous solutions of Maxwell' s equations, the findings can help us abandon long-standing misinterpretations concerning classical mechanics and classical electromagnetic theory. Although this may violate the theory of special relativity, we show mathematically that when sources move faster than light in vacuum, electromagnetic barriers and electromagnetic shock waves can be clearly predicted using the exact solutions. Since they cannot be detected by observers in the region outside their shock wave zones, an intuitive and reasonable hypothesis can be made that superluminal sources may be considered as a kind of electromagnetic black hole.

Introduction

Maxwell's theory is the foundation for handling all electromagnetic problems. In the coordinate system $(O, \mathbf{r}, t)$, Maxwell's equations in vacuum can be expressed as [?]:

$$\nabla \cdot \mathbf{E}(\mathbf{r}, t) = \frac{\rho(\mathbf{r}, t)}{\varepsilon_0}, \tag{1}$$

$$\nabla \cdot \mathbf{H}(\mathbf{r}, t) = 0, \tag{2}$$

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu_0 \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t}, \tag{3}$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \mathbf{J}(\mathbf{r}, t) + \varepsilon_0 \frac{\partial \mathbf{E}(\mathbf{r}, t)}{\partial t}, \tag{4}$$

where $\mathbf{E}$ is the electric field intensity, $\mathbf{H}$ is the magnetic field intensity, ε_0 and μ_0 are respectively the permittivity and permeability of vacuum, and $\mathbf{D}$ and $\mathbf{B}$ are the corresponding flux densities. The fields are generated by the charge density $\rho(\mathbf{r}, t)$ and the related current density $\mathbf{J}(\mathbf{r}, t)$. In vacuum, the current density is caused by the motion of the charge density, i.e., $\mathbf{J} = \rho\mathbf{v}$, where $\mathbf{v}$ is the velocity of the charges. In this paper, we consider non-relativistic electromagnetic problems and ignore the constraints from the theory of special relativity (SR). In our analysis, we remain in the same coordinate system $(O, \mathbf{r}, t)$ and do not consider observers in other inertial frames.

In Maxwell's equations, ε_0 and μ_0 are constants in vacuum; hence, $c_0 = 1/\sqrt{\varepsilon_0\mu_0}$ is also a constant. However, it is obvious that Maxwell' s theory does not state that the velocity of electromagnetic waves in vacuum is always c_0. Therefore, we temporarily set aside the relativistic rule that all objects cannot move faster than light in vacuum and consider that all causal solutions to Maxwell' s equations are physically reasonable.

The electromagnetic fields generated by point charges in vacuum are very useful because they usually have explicit expressions that can illustrate the main characteristics of electromagnetic fields. The Liénard-Wiechert potentials [?, ?, ?] are widely used in analyzing the electromagnetic fields of a moving charge; they were derived by Liénard in 1898 and Wiechert in 1900. The same techniques can be applied for deriving the fields generated by moving sources with time-varying charges, such as the Hertzian dipole [?, ?, ?, ?]. However, most published results for moving Hertzian dipoles are relativistic ones used to check relativistic behaviors. In this paper, the rigorous solutions of a moving point source with time-varying charge and that of a moving Hertzian dipole are derived based on the Liénard-Wiechert potentials. The behaviors of the far fields are analyzed and illustrated with figures. In particular, the exact relationship between the wave velocity and the velocity of the sources is provided, based on which the anisotropic property of the wave velocity and the Doppler effect are demonstrated. These findings are very important because they challenge three long-standing misinterpretations concerning classical physics. Statements can be found in most related textbooks and journal papers that, according to classical physics, the light velocity is isotropic in vacuum; the wave velocity and the velocity of the source can be added like vectors; and the transverse Doppler shift is a pure relativistic effect due to time dilation. We show that all these statements are not true. Furthermore, without the velocity limit imposed by SR, when sources move faster than light in vacuum, electromagnetic barriers and electromagnetic shock waves can be clearly predicted from the exact solutions.

II. Fields of Moving Point Sources

Assume that a particle with harmonic charge $q(t_1) = q_0 \cos(\omega_0 t_1)$ moves along a trajectory $\mathbf{x}(t_1)$, as shown in Fig. 1 [Figure 1: see original paper]. Here, ω_0 is the oscillating angular frequency of the charge. At time t_1, the velocity is $\mathbf{v}(t_1) = \dot{\mathbf{x}}(t_1)$ and the acceleration is $\mathbf{a}(t_1) = \ddot{\mathbf{x}}(t_1)$. For brevity, we use a dot on top of a vector to denote differentiation with respect to time t_1. Denote $\mathbf{R} = \mathbf{r} - \mathbf{x}(t_1)$ as the radius vector from the source position to the observation point, and $R = |\mathbf{R}|$ as its magnitude. The fields generated by the charge at time t_1 propagate to the observation point $\mathbf{r}$ with a time delay of R/c_0. For convenience, we assume that the point source at time t is near but not fixed at the origin, so that we can reveal the temporal behavior of the far fields. In this paper, we generally use t for the fields and potentials, and t_1 for the sources if not specified otherwise.

The charge density and corresponding current density can be expressed as

$$\rho(\mathbf{r}, t) = q(t_1)\delta(\mathbf{r} - \mathbf{x}(t_1)), \quad \mathbf{J}(\mathbf{r}, t) = \rho(\mathbf{r}, t)\mathbf{v}(t_1).$$

Following exactly the same procedure used in deriving the Liénard-Wiechert potentials for a moving charge [?, ?], we can derive the Liénard-Wiechert potentials for the moving point source with harmonic time-varying charges:

$$\Phi(\mathbf{r},t)=\frac{q(t_1)}{4\pi\varepsilon_0(R-\mathbf{R}\cdot\beta)},\quad \mathbf{A}(\mathbf{r},t)=\frac{\mu_0 q(t_1)\mathbf{v}(t_1)}{4\pi(R-\mathbf{R}\cdot\beta)},$$

where $\beta=\mathbf{v}/c_0$ and t_1 is determined by the retardation condition $t_1=t-R(t_1)/c_0$. From these potentials, we can derive the fields of the moving source as [?, ?]:

$$\mathbf{E}(\mathbf{r},t)=\frac{q(t_1)}{4\pi\varepsilon_0}\left[\frac{(\mathbf{n}-\beta)(1-\beta^2)}{(R-\mathbf{R}\cdot\beta)^3}+\frac{\mathbf{n}\times[(\mathbf{n}-\beta)\times\dot{\beta}]}{c_0(R-\mathbf{R}\cdot\beta)^2}-\frac{\omega_0^2\mathbf{n}}{c_0^2(R-\mathbf{R}\cdot\beta)}\right], \tag{7}$$

$$\mathbf{H}(\mathbf{r},t)=\frac{c_0 q(t_1)}{4\pi}\left[\frac{(1-\beta^2)\beta\times\mathbf{n}}{(R-\mathbf{R}\cdot\beta)^3}+\frac{\mathbf{n}\times\{\mathbf{n}\times[(\mathbf{n}-\beta)\times\dot{\beta}]\}}{c_0(R-\mathbf{R}\cdot\beta)^2}-\frac{\omega_0^2\beta\times\mathbf{n}}{c_0^2(R-\mathbf{R}\cdot\beta)}\right], \tag{8}$$

where $\mathbf{n}=\mathbf{R}/R$. In these expressions, we denote $\beta=|\beta|$ and $n=|\mathbf{n}|$. According to the solutions of Maxwell' s equations, the field at $(\mathbf{r},t)$ generated by the pulse source at $(\mathbf{x}(t_1),t_1)$ travels with velocity c_0 in the direction $\mathbf{n}=\mathbf{R}/R$. Hence, we have $R=c_0(t-t_1)$. When $\omega_0=0$, it is straightforward to verify that (7) and (8) simplify to the fields of a moving point source with constant charge.

When the source moves uniformly, $\mathbf{v}$ is constant, $\mathbf{a}=0$, and $\beta=\mathbf{v}/c_0$. Denote θ as the angle between $\mathbf{v}$ and $\mathbf{R}$. Solving t_1 from the retardation condition yields

$$c_0(t-t_1)=|\mathbf{r}-\mathbf{v}t_1|. \tag{9}$$

Note that t_1 must be real, and $t_1\le t$ must hold according to the causality principle. For a uniformly moving source, we can verify that there is at most one solution for t_1 satisfying these conditions if $\beta\le 1$. In other words, the fields at position $\mathbf{r}$ and time t come only from the source at time t_1 when it was at position $\mathbf{x}(t_1)=\mathbf{v}t_1$. We may consider the field generated at a point as a pulse of electromagnetic fields. The fields generated at each time instant t_1 will propagate with constant velocity c_0. However, a continuously moving point source generates continuous electromagnetic fields as it moves. All field pulses from the moving point source are superposed to form the continuous field distributions in vacuum. We can see that the behaviors of the composed fields may become quite different from those of a single electromagnetic pulse: both the wave velocity and the frequency of the composed fields will change.

At locations far from the sources, $r\gg vt$, we have

$$R\approx r-\mathbf{v}t_1\cdot\hat{\mathbf{r}},\quad \hat{\mathbf{n}}\approx\hat{\mathbf{r}}. \tag{11}$$

Consequently, $\beta \cos\theta \approx \beta \cos\theta$ and $R - \mathbf{R} \cdot \beta \approx r(1 - \beta \cos\theta)$. Substituting (11) into (7) and discarding the $1/R^2$ and $1/R^3$ terms, we obtain the approximate expression for the electric far field

$$\mathbf{E}(\mathbf{r}, t) \approx \frac{q_0}{4\pi\varepsilon_0} \frac{\omega^2}{c^2} \frac{\sin(\omega t - \mathbf{k} \cdot \mathbf{r})}{r} \hat{\theta}, \tag{12}$$

where the velocity c and angular frequency ω of the composed far field are, respectively,

$$c(\theta) = \frac{c_0}{1 - \beta \cos\theta}, \tag{13}$$

$$\omega(\theta) = \omega_0 (1 - \beta \cos\theta). \tag{14}$$

Obviously, (12) represents a spherical wave. The wave velocity c and angular frequency ω of the composed far fields are both functions of three variables: the velocity β of the source, the oscillating frequency ω_0 of the source, and the angle θ between the propagation direction of the fields and the moving direction of the source. Equations (12)-(14) are valid for fields in regions far from the point source. They do not explicitly depend on the coordinates of the point source; therefore, it is convenient to place the point source at the origin and analyze its far fields using (12)-(14).

A. Wave Velocity

The wave velocity in (13) describes the velocity of a point on the equi-phase surface of the far field moving in the direction $\hat{\mathbf{r}}$ associated with a specified θ. Denote $\mathbf{k} = (\omega/c)\hat{\mathbf{r}}$; the equi-phase surface is described by $\mathbf{k} \cdot \mathbf{r} - \omega t = \text{const}$. Therefore, c can be treated as the phase velocity of the far field of the moving source. The phase velocity of the electromagnetic far fields is dependent on the propagation direction. It is obvious that $c \geq c_0$ for all θ. The velocity reaches its maximum of $c_0/(1-\beta)$ when the observer is on the path of the source ($\theta = 0$). The wave velocity of the far field is isotropic only in the case that $\beta = 0$, i.e., the source is motionless or moves within a bounded region. To state that the wave velocity in vacuum is constant according to classical physics rules is a misinterpretation.

A typical plot of the velocity is shown in Fig. 2 [Figure 2: see original paper]. When $\beta = 0.1$, the largest wave velocity is only $1.005c_0$ by (13). It would be $1.1c_0$ if the two velocities were added directly. Equation (13) clearly shows that the propagation velocity of electromagnetic waves and the moving velocity of the sources do not satisfy the vector addition rule. To state that the two velocities are added like vectors according to classical physics rules is another misinterpretation.

On the other hand, using (7), (8), and (12), we can derive that the Poynting vector corresponding to the far fields can be written as

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} = \frac{q_0^2 \omega^2}{16\pi^2 \varepsilon_0 c} \frac{\sin^2(\omega t - \mathbf{k} \cdot \mathbf{r})}{r^2} \hat{\mathbf{r}}. \tag{15}$$

Using (11), we can verify that in the far-field region, $\mathbf{S}$ is parallel to $\hat{\mathbf{r}}$. Therefore, the propagating velocity of the electromagnetic power flow is the velocity in the direction of the Poynting vector, that is,

$$c_g = c_0. \tag{16}$$

The wave velocity in the direction of the Poynting vector also represents the propagation velocity of electromagnetic energy. Obviously, although the phase velocity is angle-dependent, the propagation velocity of electromagnetic energy in the far-field region is always c_0, which is independent of the angle and, more importantly, independent of the velocity of the source.

B. Doppler Effect

The angular frequency ω of the far fields is also dependent on the propagation direction. This is the Doppler effect. We denote the normalized angular frequency as

$$s(\theta) = \frac{\omega(\theta)}{\omega_0} = 1 - \beta \cos\theta. \tag{17}$$

The relative Doppler shift is $s(\theta) - 1$. The Doppler shift is zero at the angle θ_d that satisfies $\cos\theta_d = 0$, i.e., $\theta_d = \pi/2$ or $3\pi/2$. For slowly moving sources, β is small. $s(\theta)$ can be expanded as a power series with respect to β:

$$s(\theta) \approx 1 - \beta \cos\theta + O(\beta^2). \tag{18}$$

We can verify that these results agree with classical mechanics results and the results based on SR to first order in β. Particularly, (14) shows that transverse Doppler effect clearly exists at $\theta = \pi/2$ and $\theta = 3\pi/2$. The Doppler effect is explicitly included in the wave solutions of Maxwell' s equations at locations far from the sources. It is caused by the superposition of fields radiated by the moving source at different times and is solved directly in the laboratory frame without requiring any invariant principles related to coordinate transformations. We must note that the coefficient of the second-order Doppler shift differs from that predicted by SR, which is $(1 - \beta^2)^{-0.5}$. Conventionally, the transverse Doppler effect is considered a pure relativistic effect directly related to time dilation [**?**, **?**] and has been used as strong support for the theory of special relativity. We have shown that this is once again a misinterpretation concerning classical physics. The transverse Doppler effect is definitely included in the exact solution (14) obtained using the principles of classical physics.

C. Field Patterns

The point source with harmonic charge is the simplest example for illustrating radiation properties. It can be treated as an imaginary source because its charge is not constant and may violate conventional charge conservation law. However, this is not fatal because we can combine two such sources with opposite charges to form a Hertzian dipole, in which charge is conserved. Alternatively, we may simply consider it as a point source of scalar waves. Although the expressions for wave velocity (13) and frequency (14) are derived from a uniformly moving point source with harmonic time-varying charge, they can be used to predict the propagation properties of far fields from general sources in uniform motion. For general sources, the patterns of their fields may have much more complex directivities, but the propagation velocity and Doppler effect of their far fields are the same and can be described by (13) and (14), respectively.

As an example, we have derived the far fields of a moving Hertzian dipole composed of two anti-phase harmonic charges separated by a small distance l, as shown in Fig. 3 [Figure 3: see original paper]. The electric dipole moment is $\mathbf{p}(t_1) = q(t_1)l\hat{\mathbf{p}}$, where $\hat{\mathbf{p}}$ is the polarization unit vector. The fields of the uniformly moving Hertzian dipole can be derived using exactly the same techniques as for the single harmonic charge.

We here consider the case where the dipole moves in a path perpendicular to its polarization direction, $\mathbf{v} \perp \hat{\mathbf{p}}$, as shown in Fig. 3. The scalar potential of the two charges has the same form as (5), where $R_\pm$, $\hat{\mathbf{n}}_\pm$, and $t_{1\pm}$ are respectively related to each charge. The total scalar potential is obtained by superposition. We let $l \to 0$ but keep $q_0 l = p_0$ constant. By applying vector identities in deriving (7) [**?**], we obtain the final scalar potential for the moving Hertzian dipole:

$$\Phi(\mathbf{r}, t) = \frac{p_0}{4\pi\varepsilon_0} \frac{(\hat{\mathbf{p}} \cdot \hat{\mathbf{n}})(1-\beta^2) - (\hat{\mathbf{p}} \cdot \beta)(\hat{\mathbf{n}} \cdot \beta)}{(R - \mathbf{R} \cdot \beta)^2} \cos(\omega_0 t_1). \tag{19}$$

The vector potential for the Hertzian dipole is derived similarly:

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0 p_0}{4\pi} \frac{\hat{\mathbf{p}} \times (\hat{\mathbf{n}} \times \beta)}{(R - \mathbf{R} \cdot \beta)^2} \cos(\omega_0 t_1). \tag{20}$$

Note that the vector potential includes contributions not only from the two moving charges but also from the moving dipole moment. The fields can be derived from them rigorously using the same formulae as in [**?**, **?**]. Keeping the main terms that include the first order of $1/R$, the far fields of the uniformly moving Hertzian dipole can be approximately expressed as

$$\mathbf{E}(\mathbf{r},t) \approx \frac{p_0\omega^2}{4\pi\varepsilon_0 c^2}\frac{\sin(\omega t - \mathbf{k}\cdot\mathbf{r})}{r}\left[\hat{\mathbf{p}} - (\hat{\mathbf{p}}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}\right], \tag{21}$$

where ω and c are given by (14) and (13), respectively. The propagation property is exactly the same as for the uniformly moving single harmonic charge, except for a directivity factor related to the dipole polarization. For $\beta = 0$, we can verify that (21) reduces to

$$\mathbf{E}(\mathbf{r},t) = \frac{p_0\omega_0^2}{4\pi\varepsilon_0 c_0^2}\frac{\sin(\omega_0 t - k_0 r)}{r}\sin\theta\,\hat{\theta}, \tag{22}$$

which is the far electric field of a fixed Hertzian dipole.

We use three types of sources to illustrate field properties at $\beta = 0$, 0.5, and 0.9. The electric fields for the time-invariant charge and harmonic time-varying charge are calculated using (7). The fields of the Hertzian dipole are calculated using (21). For motionless sources, results are shown in Fig. 4. The static charge simply generates a static field, as shown in Fig. 4(a). For sources with time-varying charges, the wave velocity is c_0 in all directions, as shown in Figs. 4(b) and 4(c).

The electric fields of moving sources are shown in Figs. 5 and 6, where red arrows indicate moving directions. When a static charge moves, it generates a time-varying field in space with a spectrum dependent on the moving velocity, and its field patterns contract along the moving direction. For moving sources with time-varying charges, the anisotropic property of the wave velocity and the Doppler effect are clearly demonstrated in their fields.

The radiation directivity of the Hertzian dipole is obviously different from that of the single moving harmonic charge. When the source velocity is close to c_0, very strong radiations with very high frequencies can be observed in the moving direction of the Hertzian dipole, while radiations with lower frequencies can be observed in the opposite direction. For $\beta = 0.9$, the wave velocity is shown in Fig. 7(a). It is exactly c_0 at $\theta = \pm\pi/2$. The normalized frequency is plotted in Fig. 7(b). In this case, the red-shift area is much smaller than the blue-shift area.

The normalized electric field is plotted in Fig. 8(a). The radiation pattern is quite different from that of the motionless Hertzian dipole shown in Fig. 8(b). These results show that both the radiation pattern and signal frequency of an antenna implemented on a fast-moving platform may suffer severe distortions and need careful compensation.

III. Cherenkov Radiation and Electromagnetic Shock Waves

As we can see, the fields we obtained are exact solutions to Maxwell' s equations. Since Maxwell' s theory itself does not impose any restrictions on the velocity of electromagnetic waves, formulae (7)-(14) are valid for all β, including $\beta \geq 1$. Therefore, we temporarily set aside the velocity limitation and analyze the properties of causal solutions to Maxwell' s equations when sources move faster than light in vacuum. However, we must observe the causality principle that fields cannot be generated by sources in the future. Explicitly, it is required that t_1 should be real and $t_1 \leq t$.

Without loss of generality, we choose the position of the source at time t as the origin of the coordinate system for simplicity; that is, we set $\mathbf{x}(t) = 0$. To guarantee that t_1 is real, we derive from (9) that the range of the angle should satisfy

$$\cos\theta \leq \frac{1}{\beta}, \tag{23}$$

which defines a biconical region. To meet the causality condition $t_1 \leq t$, we check from (9) that

$$t_1 = t - \frac{r}{c_0} \frac{\sqrt{1-\beta^2 \sin^2\theta}}{1-\beta^2} \leq t. \tag{24}$$

Denote the critical angle as

$$\theta_c = \arccos(1/\beta). \tag{25}$$

It is obvious that electromagnetic fields only exist in the conical region defined by $\theta_c \leq \theta \leq \pi - \theta_c$. At the edge of the conical zone, $\theta = \theta_c$ or $\theta = \pi - \theta_c$, the amplitude of the fields tends to become infinitely large for point sources. For general sources, the amplitude is finite but may be unusually large. The electromagnetic fields at the edge form a kind of electromagnetic barrier of very large amplitude.

For a moving source with constant charge, Cherenkov radiation occurs when $\beta \geq 1$ [**?**]. It is conventionally believed that Cherenkov radiation cannot occur in vacuum since sources cannot move faster than light in vacuum. However, it can occur in a medium because sources can move faster than light in that medium. Cherenkov radiation was discovered in 1934 in experiments bombarding water with γ-rays, where bluish light was clearly observed.

When the velocity of sources with harmonic time-varying charges increases to $\beta = 1$, then $\theta_c = \pi/2$. The wave fronts of the electromagnetic waves are in the same plane as the source. The source crosses the electromagnetic barrier [**?**, **?**], similar to the sonic barrier that supersonic planes encounter when crossing the sound speed. The patterns of the electric fields are shown in Fig. 9. It is reasonable to predict that electromagnetic shock waves may be observed in media.

For superluminal sources, $\beta > 1$ and $\theta_c < \pi/2$. The wave fronts lag behind the source. The fields are confined to the conical zone and cannot surpass the source. The solutions describe electromagnetic shock waves, just like sonic shock waves. For point sources, at the edges of the shock waves, the fields tend to be infinitely large and form conical-shaped electromagnetic barriers. The electric field distributions for uniformly moving point sources of the three types are shown in Fig. 10.

At the electromagnetic barriers, not only are the amplitudes of the fields very large, but the frequencies and velocities of the fields also tend to become very large, as we can verify from (13) and (14). Superluminal sources radiate high-energy rays with velocities much larger than c_0 at the electromagnetic barriers. An observer will experience an electromagnetic boom when swept by the electromagnetic barriers of superluminal sources. However, for an observer outside the shock wave zone of a superluminal source, the superluminal source is invisible. We may consider the superluminal source as an electromagnetic black hole to such an observer.

The wave velocity, normalized frequency, and field amplitude in the shock wave region are plotted in Fig. 11 [Figure 11: see original paper]. As can be seen, fields in the middle area of the conical region have the largest red-shift and propagate with velocities slightly larger than c_0.

IV. Conclusions

It is often stated in textbooks that, according to classical physics rules, the wave velocity and the velocity of sources are added like vectors; the light velocity in vacuum is constant in all directions and independent of the light sources; and the transverse Doppler shift is a pure relativistic effect. These are three long-standing misinterpretations concerning classical physics. The exact solutions of electromagnetic fields of uniformly moving point sources in vacuum reveal that far fields are spherical waves with velocity depending on propagation direction. The wave phase velocity and the moving velocity of the source satisfy $c = c_0/(1 - \beta \cos\theta)$. This relationship clearly demonstrates that the two velocities cannot be added like vectors under classical physics principles. Meanwhile, the propagation velocity of electromagnetic energy in the far-field region is independent of the moving velocity of the sources. Moreover, the transverse Doppler effect is intrinsically included in the solutions.

Although superluminal sources have not been officially confirmed [**?**, **?**, **?**], this does not discourage us from performing mathematical analysis on causal solutions of fields from sources moving faster than light in vacuum. Electromagnetic shock waves are naturally introduced according to the solutions, together with a reasonable hypothesis that superluminal sources can be considered as electromagnetic black holes to observers staying in regions outside the shock wave zone.

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Note: Figure translations are in progress. See original paper for figures.