Advances in Nonlinear Optical Experiments on Simulated Gravitational Systems: Postprint

Authors: Cui Junfeng, Meng Runyu, Dong Xiaobo, Zhang Xiaoshi, Li Ruixuan, Bai Jinming

Date: 2024-03-22T00:00:00+00:00

Abstract

The nonlinear Schrödinger equation (NLSE) represents a common mathematical formulation governing the laws of numerous physical phenomena. In optical systems, the NLSE can describe the spatiotemporal evolution of laser pulses propagating in media; in astronomy, it can describe celestial and observational phenomena such as gravitational lensing and wave-like dark matter filaments (the so-called Schrödinger-Poisson equation system, SPE), and can even be ingeniously employed to analogize strong gravitational field phenomena (the so-called Einstein-Klein-Gordon system, EKG). This work provides a detailed account of the applications and advances of nonlinear optical simulation experiments in the field of astronomy from the perspectives of simulation principles, optical path design, experimental results, and astronomical interpretation; and briefly outlines the prospects for future nonlinear optical experiments simulating the evolution of various wave-like dark matter structures.

Full Text

Developments of Nonlinear Optical Experiments Emulating Gravitational Systems in Astronomy

CUI Jun-feng[1],[2], MENG Run-yu[1],[2], DONG Xiao-bo[1],[2], ZHANG Xiao-shi[3],[4], LI Rui-xuan[4],[2], BAI Jin-ming[1],[2]

[1]Yunnan Observatories, Chinese Academy of Sciences, Kunming 650216, China
[2]University of Chinese Academy of Sciences, Beijing 100049, China
[3]School of Physics and Astronomy, Yunnan University, Kunming 650500, China
[4]Aerospace Information Research Institute, Chinese Academy of Sciences, Beijing 100094, China

Abstract: The nonlinear Schrödinger equation (NLSE) represents a common mathematical formulation governing diverse physical phenomena. In optical systems, the NLSE describes the spatio-temporal evolution of laser pulses propagating in media. In astronomy, it characterizes celestial objects and observational phenomena such as gravitational lensing and wave dark matter fibers (the so-called Schrödinger-Poisson equation systems, SPE), and can even be ingeniously employed to analogize strong gravitational field phenomena (the Einstein-Klein-Gordon systems, EKG). This article comprehensively reviews the applications and progress of nonlinear optical emulation experiments in astronomy from the perspectives of simulation principles, optical design, experimental results, and astrophysical interpretation. We also briefly prospect the future prospects of conducting nonlinear optical experiments to emulate the evolution of various wave dark matter structures.

Keywords: gravitational systems; wave dark matter; analogue gravity; experimental emulations; nonlinear optics; nonlinear Schrödinger equation

1 Introduction

Analogue gravity represents an analogical research methodology that employs carefully designed non-gravitational systems (such as fluid, optical, condensed matter, and even quantum information experiments) to investigate or emulate astrophysical and cosmological gravitational phenomena that are difficult to observe or experimentally realize. The solid foundation of this analogical approach lies in the fact that gravitational phenomena and emulation experiments are described by identical mathematical equations. For instance, the nonlinear Schrödinger equation (NLSE) focused on in this paper can describe both the spatio-temporal evolution of laser pulses in optical media and astronomical objects and phenomena such as gravitational lensing and wave dark matter (waveDM), and can even be cleverly utilized to analogize strong gravitational field phenomena (see Chapter 2 for details). The significance of emulation experiments for gravity primarily encompasses two aspects: (1) Through high-precision, controllable experiments, researchers can finely probe and study effects in strong gravitational fields such as the Penrose process and Hawking radiation, particularly for astronomical phenomena where high-quality observational data are difficult to obtain. Taking Hawking radiation as an example—a quantum effect near black hole horizons—direct detection remains extremely challenging, and no convincing observational data have been obtained to date. (2) Such experiments may provide inspiring new perspectives for research objects in astrophysics and cosmology. This is because astronomical systems typically possess numerous internal degrees of freedom, and the mathematical equations describing these systems are highly nonlinear, making analytical solutions impossible and even numerical solutions extremely difficult. Therefore, we hope to understand astronomical systems through laboratory emulation experiments. Particularly, since the evolutionary timescales of astronomical objects are extremely long, making it impossible for humans to track their evolution through observations, knowledge derived from emulation experiments becomes especially important (see Section 4.2 and Chapter 5). Of course, for the same profound and complex mathematical formulation, understanding and treating it from different perspectives often brings unique conveniences in specific aspects; consequently, gravitational theory can also assist such refined laboratory experiments, meaning the analogy works bidirectionally (for example, using wormholes from gravity to understand complex quantum entanglement and guide experimental design).

Since the invention of general relativity, theoretical ideas and models have been proposed to understand curved spacetime, such as the geometric optical equations based on light propagation in media with variable refractive index (the so-called Gordon metric). Generally considered the 标志性 breakthrough of modern analogue gravity is Unruh's "acoustic black hole model" proposed in 1981, which employed analogy to "observe" Hawking radiation. Unruh used sound wave motion in fluid flow to analogize the motion of massless particles in strong gravitational fields (i.e., to characterize geodesics in curved spacetime). He specifically analyzed the identical mathematical forms possessed by these two phenomena, where the acoustic horizon in transonic water flow corresponds to a black hole's event horizon, and acoustic waves emitted near the acoustic horizon with a thermal spectrum correspond to black hole Hawking radiation. In subsequent decades, theoretical proposals for emulation experiments have emerged endlessly, with envisioned experimental methods extending from ordinary fluids to optical systems and quantum systems such as Bose-Einstein condensates (BEC). However, actual emulation experiments did not appear until the late 2010s.

Analogue gravity experiments were first conducted in optical fibers (and water tanks), and within two years, BEC experiments followed. Initially, the target was the most interesting objective (basically until 2017): Hawking radiation from black holes. Naturally, the scope of analogue gravity experiments later expanded to include various astrophysical and cosmological phenomena such as properties of rotating black holes and the Penrose process for extracting black hole rotational energy. For detailed historical background, see references [1, 2, 22, 23]. In recent years, optical emulation experiments have also been conducted for weak-field astronomical objects and phenomena (e.g., gravitational lensing, tidal effects, and non-relativistic boson-star-like objects), particularly for emulating the long-timescale evolution of astronomical systems. As mentioned in the first paragraph, such emulation experiments of astronomical evolution are highly inspiring for astronomers.

This paper primarily reviews the latest research progress in laser experiments emulating gravitational systems. Additionally, for wave dark matter currently under intense discussion in the astronomy community, we briefly introduce the prospects of optical emulation experiments. Compared with fluid experiments, laser experiments offer advantages of fine control and high precision; for instance, in water tank experiments, the acoustic signal analogous to Hawking radiation is too weak compared with various water fluctuations (such as thermal noise). Compared with cold atom experiments (e.g., BEC), laser experiments are easier to implement and more cost-effective.

The structure of this paper is as follows: Chapter 2 introduces the principles of emulating gravitational systems using optical NLSE systems (the technical approach); Chapters 3 and 4 provide detailed introductions to optical experiments emulating strong and weak gravitational systems, respectively, from three aspects: experimental optical design, results, and corresponding astrophysical interpretation; Chapter 5 summarizes the entire paper and outlines prospects for using optical experiments to study the dynamical evolution of various wave dark matter structures.

2 Principles of Optical NLSE Systems for Emulating Gravitational Systems

In optical systems, assuming the light field propagates along the z-axis as $E = \psi(x, y, z)e^{i[kz-\omega t]}$, when ψ satisfies $|\partial\psi/\partial z| \ll |k\psi|$, meaning the light field envelope $\psi(x, y, z)$ varies slowly along the z-axis, its dynamical equation can be described by the NLSE, taking the form:

$$\hat{\nabla}^2\psi + \psi = 0,$$

where $\hat{\nabla}^2 = \partial_x^2 + \partial_y^2$ is the two-dimensional Laplacian operator, n_0 is the linear part of the medium's refractive index, Δn depends on the medium's nonlinear polarization intensity, and the wavenumber $k = 2\pi n_0/\lambda$. From a mathematical perspective, the NLSE generally has no analytical solutions, and numerical solutions are typically extremely complex. In recent years, with rapid development in nonlinear optics, optical experiments have become an effective method for solving or emulating the NLSE. As introduced in Chapter 1, through nonlinear optical experiments, successful laboratory emulations of astronomical objects have been conducted, targeting not only important observational SPE subjects such as gravitational lensing, gravitational redshift, and tidal effects, as well as rotating boson stars, but also hard-to-observe EKG phenomena like black hole Hawking radiation and Penrose superradiance.

To emulate gravitational systems, parameters in the optical NLSE system must be corresponded one-to-one with parameters in analogous gravitational system equations. Depending on whether the gravitational system is strong (i.e., relativistic) or weak, the specific correspondence principles (technical schemes) employed differ, which we introduce separately below.

2.1 Principles for Emulating Strong Gravitational Systems

When emulating strong gravitational systems like black holes in laser experiments, we utilize Unruh's "acoustic black hole model," treating light as a photon fluid and using the propagation of small perturbations in the photon fluid (density and phase perturbations, hereafter referred to as sound waves for convenience) to analogize the propagation of massless particles in curved spacetime.

Massless particles, i.e., test particles in strong gravitational fields, describe the metric of curved spacetime through their motion.

In curved spacetime, the equation describing the motion of general scalar particles is the relativistic (high-velocity) quantum mechanical equation with spacetime metric, namely the Einstein-Klein-Gordon (EKG) system:

$$-\Box\phi + m^2\phi = 0,$$

where the d'Alembert operator $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$, $g^{\mu\nu}$ is the spacetime metric, and m is particle mass. Historically, such scalar fields with relativistic motion velocities bound by self-gravity (Einstein's gravitational field) are called Klein-Gordon Geons. This paper essentially does not involve such EKG systems. In this section, we primarily borrow the $m = 0$ EKG system to illustrate the gravitational metric, without elaborating on the properties and motion of relativistic scalar fields; in Section 2.2 we only consider the non-relativistic limit (low velocity + Newtonian gravity version) of the EKG system, i.e., the NLSE+Poisson equations.

We briefly introduce how acoustic perturbations in the light field correspond to the motion of massless scalar particles in strong gravitational fields. Transforming the optical NLSE equation (see Eq. (1)) into the fluid mechanics representation via the Madelung transformation $\psi = \sqrt{\rho}e^{i\phi}$, where ρ and ϕ are the amplitude and phase of the light field envelope, respectively, and assuming the relationship between refractive index and light intensity as $\Delta n = -\gamma I = -\gamma|\psi|^2$, where γ is the medium's nonlinear coefficient, Eq. (1) transforms into a series of fluid mechanics equations:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0,$$

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}|\nabla\phi|^2 + \frac{c^2}{\rho_0}\rho - \frac{1}{2k^2n_0^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} = 0.$$

Equations (3) and (4) represent the continuity equation and Euler equation of fluid mechanics, respectively, providing a fluid description of the photon system, which we henceforth call photon fluid. Here, ρ corresponds to fluid density, fluid velocity $\mathbf{v} = \nabla\phi \equiv \nabla\eta$, where η corresponds to the fluid velocity potential. Compared with classical fluids, the last term in Eq. (4) is a pressure term unique to quantum particles (strictly speaking, wave nature), generally called quantum pressure, which arises from the diffraction term in the optical equation and results from the uncertainty principle. When the density variation in the photon fluid is slow, this quantum pressure term in Eq. (4) can be neglected.

When Unruh derived the acoustic metric for sound wave motion in classical fluids, he assumed the fluid to be inviscid, barotropic, and irrotational—three conditions difficult to satisfy in ordinary fluids. However, in photon fluids, the Euler equation (4) is the momentum equation for ideal fluids (inviscid), meaning photon fluids can essentially be treated as "superfluids." Moreover, in our scenario, with small fluid density variations where quantum pressure is negligible, the pressure of the photon fluid is only a function of density, $P = c^2\gamma\rho^2/2$, making it a barotropic fluid; and in ideal fluids, the flow is potential and irrotational, a condition well satisfied (except at singular points in vortices). Therefore, photon fluids represent a more ideal experimental platform than ordinary fluids.

Considering linear perturbation treatment of micro-perturbations in photon fluids, let $\rho = \rho_0 + \epsilon\rho_1 + O(\epsilon^2)$ and $\eta = \eta_0 + \epsilon\eta_1 + O(\epsilon^2)$, where ρ_0 and η_0 are the background flow density and velocity potential, respectively. Substituting into Eqs. (3) and (4) yields the dynamic equations for perturbations (ρ_1, η_1):

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_0 \nabla \eta_1 + \rho_1 \mathbf{v}_0) = 0,$$

$$\frac{\partial \eta_1}{\partial t} + \nabla \eta_1 \cdot \mathbf{v}_0 + \frac{c^2}{\rho_0}\rho_1 = 0.$$

Substituting Eq. (6) into Eq. (5) yields an equation for η_1 alone:

$$\frac{\partial}{\partial t}\left(\frac{\partial \eta_1}{\partial t} + \mathbf{v}_0 \cdot \nabla \eta_1\right) + \nabla \cdot \left[c^2 \nabla \eta_1 - \rho_0 \mathbf{v}_0 \left(\frac{\partial \eta_1}{\partial t} + \mathbf{v}_0 \cdot \nabla \eta_1\right)\right] = 0,$$

where $c^2 \equiv \partial P(\rho_0)/\partial \rho_0 = c^2\gamma\rho_0$ is the sound speed in the fluid and $\mathbf{v}_0 = \nabla \eta_0$ is the background flow velocity. Introducing the effective metric $g_{\mu\nu}$, Eq. (7) can be rewritten as:

$$\Box \eta_1 = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \eta_1\right) = 0,$$

with the covariant form of the metric $g_{\mu\nu}$ given by:

$$g_{\mu\nu} = \frac{1}{c}\begin{pmatrix} -(c^2 - v^2) & -v_r & -v_\theta \\ -v_r & 1 & 0 \\ -v_\theta & 0 & 1 \end{pmatrix},$$

where v_r and v_θ are the radial and angular components of flow velocity, respectively, and $v_0^2 = v_r^2 + v_\theta^2$.

From Eqs. (8) and (9), after introducing the effective metric $g_{\mu\nu}$, the equation for sound wave motion in photon fluids formally matches that for massless scalar particles in gravitational fields (i.e., Eq. (2) with $m = 0$). However, two assumptions must be emphasized in the above derivation: First, the quantum pressure term in Eq. (4) must be negligible, requiring extremely slow density variation in the photon fluid—specifically, the excited sound wavelength in the photon fluid must be sufficiently long so that density changes caused by sound waves can be ignored. Second, the medium's refractive index change must satisfy $\Delta n = -\gamma|\psi|^2$, requiring the nonlinear effect to be local (called the Kerr effect in optics) and the medium to be self-defocusing, i.e., the refractive index is inversely proportional to light intensity. Only when both assumptions are satisfied can strong gravitational systems like black holes be simulated in optical experiments.

2.2 Principles for Emulating Weak Gravitational SPE Systems

In Section 2.1, we detailed the approach for emulating strong gravitational fields. In this section, we introduce weak gravitational field systems (Newtonian gravity, or flat spacetime), considering only scalar field systems with particle velocities far below light speed under Newtonian self-gravity. Such Newtonian, low-velocity scalar fields have attracted considerable attention in astronomy in recent years, for example, in wave dark matter theory, which we specifically discuss in Chapters 4 and 5.

As introduced in Section 2.1, this system represents the non-relativistic limit (low velocity + Newtonian gravity version) of the EKG system. In Newtonian gravity, Einstein's field equations reduce to Poisson's equation. When particle velocities are far below light speed, the quantum field equation changes from the Klein-Gordon equation to the Schrödinger equation, but we must still consider gravitational interactions between scalar field particles, so the Schrödinger equation includes a gravitational potential (Φ) term. This is the Schrödinger-Poisson equation system (SPE), with the specific form:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi\psi,$$

$$\nabla^2\Phi = -4\pi G\rho,$$

$$\rho = m|\psi|^2.$$

Equation (10) is the famous nonlinear Schrödinger equation (NLSE). For such weak gravitational SPE systems, the NLSE equation already exists in nonlinear optics, so the correspondence is straightforward (see the first paragraph of Chapter 2). The key is finding the optical counterpart to Poisson's equation. Here we introduce that the medium's thermo-optic effect conveniently takes the form of Poisson's equation.

The thermo-optic effect refers to the absorption of photon energy by the medium in the light field, causing temperature increase and subsequent refractive index change. Here $\Delta n = \beta \Delta T$, where β is the medium's thermo-optic coefficient and $\Delta T = T - T_0$, with T_0 being the reference temperature before the light field passes through the medium. After reaching thermal steady state experimentally, heat conduction in the medium satisfies:

$$\kappa \hat{\nabla}^2 T = \kappa \hat{\nabla}^2 \Delta T = -\alpha I,$$

where κ is the medium's thermal conductivity and α is the absorption coefficient. Note that Eq. (12) assumes $\partial^2 T / \partial z^2 \ll \hat{\nabla}^2 T$, i.e., temperature change along the z-axis is neglected, with $\hat{\nabla}^2 = \partial_x^2 + \partial_y^2$ being the two-dimensional Laplacian. To correspond with Eq. (11), the steady-state heat conduction equation is rewritten as:

$$\hat{\nabla}^2 \Delta n = \frac{\alpha \beta}{\kappa} I = R |\psi|^2.$$

Equation (13) shows that the steady-state heat conduction equation is precisely Poisson's equation, allowing convenient analogy with Newtonian gravity. Comparing Eqs. (11) and (13), the refractive index change Δn analogizes to gravitational potential ϕ, and light intensity I analogizes to density ρ in gravitational systems. By appropriately adjusting these optical experimental parameters, specific SPE astronomical objects can be emulated.

2.3 Comparative Summary of Two Optical Emulation Principles

After introducing the optical emulation principles for strong and weak gravitational SPE systems, we compare their characteristics. First, strong gravitational field emulation constructs an effective curved spacetime metric using photon fluids, while weak gravitational SPE system emulation utilizes the thermo-optic effect to analogize Newtonian gravity. Second, the required optical nonlinear effects differ: for strong field emulation, the refractive index change relates to intensity as $\Delta n = -\gamma |\psi|^2$ (a local nonlinear effect for ψ), whereas weak gravitational system emulation requires $\hat{\nabla}^2 \Delta n = R |\psi|^2$, a nonlocal nonlinear effect (the refractive index change Δn is a spatial integral of ψ). Details are provided in Section 4.2. Moreover, from the expression for Δn, strong field emulation requires self-defocusing media, while SPE system emulation requires self-focusing media.

In summary, researchers adopt different experimental schemes according to the distinct characteristics of the astronomical objects to be emulated, ensuring that astronomical and optical systems satisfy the same mathematical formulation while cleverly achieving one-to-one correspondence between specific parameters of the two systems.

3 Optical Experiments Emulating Strong Gravitational Systems

In Chapter 2, we introduced the principle of emulating strong gravitational systems with optical experiments, using sound wave motion in photon fluids to analogize test particle motion in curved spacetime. In this chapter, since water flow fields are more intuitive, we first introduce how rotating black holes are emulated in water flows before transitioning to optical systems (photon fluids) to discuss research progress in optical emulation of rotating black holes and their superradiance. In water flow fields, not only sound waves but also surface gravity wave propagation equations match those for massless scalar field propagation in gravitational fields; thus gravity waves can serve as perturbations in flow fields to analogize test particle motion in gravitational fields, thereby mapping the gravitational field metric. Gravity wave experiments in water flows are easily understood, so we use them to introduce the emulation principle.

First, a stable flow field must be constructed as the Kerr metric of a rotating black hole. This flow field must satisfy several requirements mentioned in Section 2.1: inviscid potential flow, irrotational everywhere except at singularities, and absence of turbulence. Typically, a drain is placed in a water tank and given angular momentum, forming a vortex flow field near the drain—this model is called the “bathtub drain” model. Based on this model, we can calculate flow velocity $\mathbf{v}$, where radial velocity $v_r = -A/r$ and angular velocity $v_\theta = B/r$, with A and B being fixed parameters determined by drainage rate and initial angular momentum.

According to definitions of horizon and ergosphere, locations where radial flow velocity v_r equals sound speed c_s simulate the black hole’s event horizon (strictly speaking, the outer event horizon); within the horizon, the fluid’s radial velocity exceeds sound speed, so acoustic perturbations inside cannot propagate outward through the horizon. Locations where fluid velocity $\mathbf{v}$ equals sound speed c_s correspond to infinite redshift surfaces. The region between the infinite redshift surface and the horizon is called the ergosphere, where perturbations such as sound or gravity waves experience strong dragging effects co-rotating with the black hole. If they fortunately avoid falling into the horizon, they are scattered by the black hole and escape carrying higher energy—this is superradiance. In classical black hole research, the physical essence of superradiance is the Penrose process of extracting black hole rotational energy, hence superradiance is sometimes called Penrose superradiance.

Torres et al. used gravity waves in water flows to measure superradiance effects, as shown in [Figure 1: see original paper]. A steady vortex flow field (the rotating black hole analog of the “bathtub drain” model) was constructed in the center of a water tank. Gravity wave perturbations were excited from the left, and when passing the rotating black hole, scattered waves were generated that interfered with incident gravity waves, producing clear interference patterns. If the outgoing gravity wave intensity after black hole scattering exceeded the incident wave, superradiance occurred. The experiment demonstrated that when the angular momentum of incident gravity waves aligned with the central vortex (analogous to a rotating black hole), amplified outgoing waves could be produced under certain conditions.

3.1 Optical Emulation of Rotating Black Holes

Compared with water flows, stable flow fields in optics are less intuitive but can still reference the "bathtub drain model" by measuring radial and angular velocities of photon fluids. Following Section 2.1, photon fluid velocity $\mathbf{v} = \nabla\phi$, where ϕ is the phase of the light field pulse envelope. Therefore, to construct a stable vortex flow field (optical vortex) in optical experiments, phase must be introduced.

This experimental concept was initially proposed by Marino and realized by Vocke et al., as shown in [Figure 2: see original paper]. After passing through a phase mask, the laser beam produces multiple diffraction orders, and the first-order diffracted light (vortex beam with topological charge $l = 1$) is extracted through a pinhole. Finally, the beam is split into two paths: one measuring intensity distribution and the other measuring spatial frequency spectrum. For spatial frequency spectrum measurement, the camera is placed at the focal plane of a lens, which performs a two-dimensional Fourier transform of the light field, producing an image of the spatial frequency spectrum of the light field before the lens.

The wavefront of an optical vortex exhibits a spiral shape, carrying a phase factor $e^{il\theta}$, where l is the topological charge number and θ is the azimuthal angle. The phase changes by $2\pi l$ when the optical vortex rotates once around its axis. Due to this special topological structure, a phase singularity exists at the vortex center, creating a shadow region (zero light field intensity). [Figure 3: see original paper] illustrates three common methods for generating optical vortices: (1) spiral phase plate—a transparent plate gradually thickening along the central azimuthal angle like a rotating step; (2) phase pattern loaded onto a spatial light modulator (SLM); and (3) computed holography, the method used by Vocke et al. The hologram can be printed onto a holographic plate to create a grating (phase mask) or loaded onto an SLM.

Vocke et al. measured the intensity distribution of the beam in the near field, i.e., photon fluid density ρ, to calculate the sound speed $c_s^2 = c^2\gamma\rho_0$ at each point in the flow field. A scanner (small aperture in the diagram) before the far field scanned along the beam diameter to measure the spatial frequency spectrum (K_x, K_y) at each point, obtaining flow velocity via:

$$\mathbf{v}(r) = \nabla\phi(r) = \frac{1}{k}\begin{pmatrix} K_x(r) \\ K_y(r) \end{pmatrix}.$$

By experimentally measuring sound speed and flow velocity at each point, the emulated curved spacetime structure can be determined, identifying the horizon and ergosphere of the rotating black hole, as shown in [Figure 4: see original paper]. After constructing the effective spacetime structure with photon fluids, a natural question arises: how to simulate the superradiance process of rotating black holes? This research area developed rapidly after Vocke et al., with Braidotti et al. first reporting superradiance measurements in 2022 (see Section 3.2).

3.2 Optical Measurement of Penrose Superradiance from Rotating Black Holes

The Penrose process for rotating black holes was briefly introduced earlier. Braidotti et al. input a weak signal light (acting as sound waves in photon fluid) together with pump light into the medium, as shown in [Figure 5: see original paper]. After interaction, a new idler light was generated and trapped within the ergosphere, while the signal light successfully escaped. In optical systems, the process of signal light extracting energy from pump light is also called optical parametric amplification, but this process has strict phase-matching requirements. Specifically, as pump light continuously excites signal and idler light at each location, if the generated signal light is phase-mismatched with existing signal light, its intensity may decrease. Only when newly generated signal light is phase-synchronized with previous signal light can the signal be ultimately amplified. Various phase-matching methods exist; in rotating black hole experiments, it is called geometry-induced phase matching, where idler and signal light are spatially separated, corresponding to the Penrose process, and the phase-matching condition also aligns with the Zel'dovich-Misner condition for superradiance.

The superradiance measurement results are shown in [Figure 6: see original paper]. Here l, n, and q represent the topological charge numbers of pump, probe, and idler light, respectively, while R_N is the reflection coefficient measuring whether superradiance occurs ($R_N > 1$ indicates superradiance). [Figure 6b: see original paper] shows spatial distributions for two different parameter sets: when superradiance occurs, signal and idler light are clearly separated; when no superradiance occurs, they overlap.

4 Optical Experiments Emulating Weak Gravitational SPE Systems

This chapter introduces two optical experiments emulating weak gravitational systems: Bekenstein et al. used interactions between Gaussian and Airy beams to simulate gravitational lensing effects produced by massive celestial bodies; Roger et al. emulated the evolution of rotating "boson stars" (non-relativistic) through light vortex evolution in media.

4.1 Gravitational Lensing Effects

4.1.1 Experimental Scheme To simulate gravitational lensing, tidal forces, gravitational redshift and blueshift effects produced by massive celestial bodies (such as massive stars, galaxies, and clusters), Bekenstein et al. designed laser experiments. Since these gravitational effects belong to weak-field phenomena not requiring full Einstein field equations, they adopted the SPE system simulation method based on thermo-optic effects introduced in Section 2.2.

Using a continuous-wave laser at 488 nm, the output Gaussian beam was split into two paths. When one path passed through a spatial light modulator, it generated a special beam called an “accelerating beam,” which then interacted with the other high-intensity Gaussian beam in the medium (optical glass SF11) to simulate various gravitational effects, as shown in [Figure 7: see original paper].

The laser output beam is a particular solution under resonator boundary conditions, with a Gaussian transverse amplitude distribution, hence called a Gaussian beam. After entering the medium, thermo-optic effects cause higher refractive index at the center and lower at the edges, producing self-focusing. Meanwhile, Gaussian beams experience diffraction, causing beam width to increase continuously. When self-focusing exactly cancels diffraction, the Gaussian beam propagates stably through the medium. Accelerating beams, also known as Airy beams, exhibit self-bending propagation without diffraction. Due to these advantages, their trajectory and structural changes can well reflect gravitational effects without diffraction interference.

4.1.2 Experimental Results In the emulation experiment, the high-intensity Gaussian beam analogizes massive celestial bodies, while the Airy beam analogizes test photons in gravitational fields. The thermal effect produced by the former influences the trajectory and structure of the Airy beam, analogizing gravitational effects. The Airy beam self-bends during propagation (blue line in [Figure 7b: see original paper]), but under Gaussian beam influence (red line), its bending decreases (green line), analogizing gravitational lensing. As laser output energy increases, the outgoing Airy beam deflects more toward the Gaussian beam side, indicating stronger lensing effects ([Figure 8a: see original paper]).

More interestingly, the Airy beam structure changes with interaction ([Figure 8b: see original paper]), becoming more significant at higher intensities—this analogizes tidal effects from massive bodies. Since different parts of the Airy beam are at different distances from the Gaussian beam, they experience different gravitational fields, analogous to celestial bodies near massive objects experiencing tidal deformation.

Additionally, by controlling the incident Airy beam trajectory to approach or recede from the Gaussian beam and measuring changes in the outgoing wavevector k_z, Bekenstein et al. found k_z increases when approaching and decreases when receding. In gravitational systems, photons leaving a massive body's potential well experience gravitational redshift (frequency decrease), while approaching photons experience blueshift (frequency increase). The k_z change in optical experiments resembles photon frequency changes, with refractive index change Δn analogizing gravitational potential ϕ, i.e., $\Delta\omega/\omega = \Delta n(x_2) - \Delta n(x_1)$, thus k_z changes emulate gravitational redshift/blueshift phenomena.

4.2 Optical Emulation of Rotating "Boson Star" Evolution

In astronomy, boson stars consist entirely of bosons. While ordinary stars maintain stability through radiation pressure from nuclear fusion resisting gravitational collapse, boson stars are Bose-Einstein condensates (BEC)—macroscopic quantum states. Due to the uncertainty principle (wave nature), BEC exhibits quantum pressure (or diffraction effect, see Chapter 2). Additionally, repulsive self-interactions between bosons may exist, and both effects can balance boson star self-gravity.

Boson stars can rotate (possess angular momentum). Roger et al. modeled rotating boson stars as: (1) the entire star being a vortex with a central phase singularity, and (2) treating the boson star as the non-relativistic limit of the EKG system, i.e., the SPE system. We will comment on these modeling approaches at the end of Section 4.2.

4.2.1 Experimental Scheme Since optical experiments can only study light field evolution in the transverse plane over time (with the z-axis as propagation direction serving as the time axis), both the NLSE and Poisson equation become two-dimensional systems (see Chapter 2). For two-dimensional Poisson systems, the temperature change (or refractive index change, corresponding to gravitational potential) takes a logarithmic form that does not approach zero as $r \to \infty$, requiring integration to boundaries in numerical solutions. Thus in experiments, boundary conditions significantly affect gravitational potential integration regardless of how distant the light field boundaries are—making boundary condition handling problematic in 2D systems compared to 3D (see Section 2.4 of reference [3]). A conventional approach ensures the beam intensity distribution is much smaller than the field boundary, allowing a screening term in Poisson's equation:

$$\hat{\nabla}^2\phi - \frac{1}{\sigma^2}\phi = -f(\mathbf{r}),$$

making the integral converge, where σ is the nonlocal interaction length, generally taken as $\sigma \approx D/2$ with D being the transverse medium length.

Roger et al. adopted this boundary treatment, proposing a "distributed loss model" (DLM) incorporating boundary effects:

$$(\rho_0 C)\frac{\partial \Delta T}{\partial t} = \kappa\hat{\nabla}^2(\Delta T) + \alpha I - \frac{\kappa}{\sigma^2}\Delta T,$$

where σ has the same meaning as in Eq. (15). Comparing the two equations reveals that DLM provides an intuitive understanding of Poisson's equation with screening term. In experiments, ensuring the input beam's spatial spectrum satisfies $|K_\perp| \gg 1/\sigma$ (i.e., the beam's spatial energy distribution is not too extensive) makes DLM the solution of the 2D Poisson equation for $r < \sigma$.

The optical principles and techniques in Roger et al.'s experiment are identical to the optical vortex experiment introduced in Section 3.1, differing only in the emulation object and correspondence scheme. The vortex generation technique also uses holographic gratings, with nonlinear medium being lead-doped specialty glass SF6. The optical design is shown in [Figure 9: see original paper].

4.2.2 Experimental Results The evolution of optical vortices in the medium is shown in [Figure 10: see original paper], with the horizontal axis serving as the time axis. [Figure 10b: see original paper] shows spatial frequency spectrum evolution. Throughout the experiment, the beam always satisfied the condition $(\sigma k_\perp)^2 \gg 1$ described in Section 4.2.1. It should be noted that real-time monitoring of vortex evolution in the medium is impossible; only post-propagation results can be monitored, and medium length cannot be varied continuously. Roger et al. used varying input beam power, taking power p (horizontal axis) as a proxy for propagation distance z (i.e., time).

The most interesting result is that even at high intensity (high density), the vortex undergoes alternating contraction and expansion oscillations without collapsing due to self-focusing, as seen in the real-space intensity distribution $I(r)$ evolution over time in [Figure 10a: see original paper]. This thermo-optic self-focusing corresponds to Jeans collapse under self-gravity in astronomical scenarios (see Poisson equation analysis in Sections 2.2 and 4.2.1). Roger et al. attribute the absence of collapse to two mechanisms: First, quantum pressure (or diffraction term) introduced at the beginning of Section 4.2, which balances self-gravity and maintains boson star stability—this mechanism is independent of angular momentum. Second, the topological nature of the optical vortex itself, with its central phase singularity preventing wave collapse.

We conclude Section 4.2 with the following critiques: (1) Their emulation object is problematic. Such optical experiments can only simulate "2D space + 1D time" systems because in conventional optical experiments, photons cannot be relatively "stationary," and the beam propagation direction (spatial axis z) corresponds to the time axis of the emulated system. Roger et al. claimed to emulate boson stars, but these are three-dimensional gravitational systems, making their "emulation correspondence" inappropriate. The corresponding astronomical objects for 2D gravity could be dark matter filamentary structures, which we briefly introduce in Section 5.2. In fact, in this research group's later work (Vocke et al.'s experiment introduced in Section 3.1), they explicitly stated that optical vortex experiments can only emulate 2D gravitational objects (or systems describable by 2D gravity). (2) Whether boson stars in astronomical contexts should be described by EKG or SPE will be discussed in Chapter 5. (3) Their two explanations for the absence of vortex collapse are problematic. The first mechanism—quantum pressure (or diffraction term)—cannot always balance self-gravity; theoretical analysis of SPE systems has shown this, though the community remains uncertain about the final product of such collapse (a nonlinear evolution process). Their second mechanism is invalid because, as Feynman pointed out in 1955, vortices in macroscopic quantum states like boson stars are quantized and cannot be single large vortices. In summary, Roger et al.'s rotating boson star model is incorrect, particularly in using the 2D Poisson equation to force-fit a 3D gravitational system, causing confusion in parts of their paper. This experiment should only be understood at the level of "single optical vortex."

5 Comments and Outlook

5.1 Review of Strong Gravitational Field Emulation Experiments

As introduced in Chapter 1, both theoretical discussions of emulation schemes and actual experiments began with the most important problem—black hole Hawking radiation—before expanding to various aspects of strong gravitational fields. Since strong field research involves quantum effects near black hole horizons and the reconciliation of quantum field theory with general relativity, it has attracted many physicists (theoretical and experimental). Emulation methods are not limited to optics, including quantum entanglement experiments and cold atom experiments mentioned in Chapter 1 and reviewed by Zhang et al. [2], so we will not provide extensive summary and outlook analysis here—readers are referred to references [2, 23]. Optical emulation experiments are covered in Chapter 3.

5.2 Review of SPE Optical Emulation and an Outlook: Wave Dark Matter

Regarding weak gravitational field SPE system optical emulation experiments, we introduced them in Chapter 4, a field that has only recently emerged (within the last decade). Compared with strong field effects, SPE systems have more connections with traditional astronomical observations (e.g., gravitational lensing, gravitational redshift associated with optical imaging and spectroscopic observations), so we provide further commentary here. Particularly, the experiments introduced in Section 4.2 are more applicable to the currently astronomy-hot topic: wave dark matter (waveDM). Below we specifically comment on and prospect the astronomical significance of the experiments introduced in Section 4.2.

Roger et al. used NLSE-described optical vortices to emulate rotating "boson star" evolution. In real astronomical scenarios, boson stars are generally compact celestial bodies bound by their own strong gravitational fields, i.e., the Klein-Gordon Geon introduced in Chapter 2, and should therefore be described by the EKG equation.

The astronomy community has very active discussions about the SPE version of Klein-Gordon Geons—this is wave dark matter. Dark matter is a fundamental problem in contemporary astrophysics and particle physics. The current standard cosmological model, the "dark energy + cold dark matter" model (ΛCDM), can explain large-scale structures from cosmic microwave background to galaxy distribution down to ~1 Mpc scales. However, several "small-scale challenges" persist at galaxy and sub-galaxy scales. Over the past decade, the wave dark matter model has attracted widespread attention, typically referring to dark matter particles with masses below 30 eV. Taking characteristic galaxy-scale velocities of ~250 km/s, their de Broglie wavelength can exceed the mean interparticle spacing, allowing excellent classical wave description—hence "wave dark matter." Compared with CDM, waveDM's greatest advantage is potentially resolving galaxy-scale small-scale challenges. Schive et al. found through high-precision numerical simulations that under identical initial conditions, large-scale structures in waveDM match CDM results, but at galaxy scales, almost every waveDM halo develops a core with approximately constant central density, potentially solving CDM's "core-cusp problem" ([Figure 11: see original paper]).

Astronomical dark matter structures exhibit diverse morphologies from cosmic web to galaxy scales. In ~10 Mpc cosmic web structures, there are sheet-like structures whose gravitational effects on neighboring matter can be described by 1D Poisson equations (1D gravitational systems), and filamentary structures whose gravitational effects can be approximated as infinite mass rods—i.e., 2D Poisson equations. At galaxy cluster ($ $1 Mpc) and galaxy scales, various dark matter halos, subhalos, and substructures exist, generally requiring 3D Poisson equations.

Current astronomical research on wave dark matter structures is primarily based on NLSE numerical simulations, with no emulation experiments conducted. However, numerical simulation capabilities are limited. First, complex dynamical processes like wave dark matter turbulence are difficult to reproduce numerically; wave collapse, for example, exceeds NLSE's validity range. Second, SPE solutions are more computationally expensive than pure gravitational Poisson equations in CDM, limiting simulation scale ("particle number") and precision, preventing realistic description of actual dark matter scenarios.

Therefore, the optical emulation experiments in Section 4.2 provide an excellent research approach. For 3D gravitational wave dark matter halos, as mentioned at the end of Section 4.2, conventional optical experiments cannot emulate them and require alternative experimental schemes. However, for 2D gravitational wave dark matter filaments and 1D gravitational wave dark matter sheets, conventional optical experiments are excellent tools for emulating their dynamical evolution. Particularly, for recently observed Mpc-scale rotating dark matter filamentary structures, the "single optical vortex" experiments introduced in Section 4.2 are precisely their optical counterparts.

References

[1] Barceló C, Liberati S, Visser M. Living Rev Relativ, 2011, 14: 1
[2] Zhang B C, Cai Q Y, Zhan M S. Sci Sin-Phys Mech Astron, 2014, 44: 879
[3] Paredes A, Olivieri D N, Michinel H. Physica D: Nonlinear Phenomena, 2020, 403: 2301
[4] Hui L. ANNU REV ASTRON ASTR, 2021, 59: 247
[5] Jafferis D, Zlokapa A, Lykken J D, et al. Nature, 2022, 612: 51
[6] Gordon W. Ann Phys, 1923, 72: 421
[7] Unruh W G. Phys Rev Lett, 1981, 46: 1351
[8] Philbin T G, Kuklewicz C, Robertson S, et al. Science, 2008, 319: 1367
[9] Rousseaux G, Mathis C, Mässa P, et al. New J Phys, 2008, 10: 3015
[10] Lahav O, Itah A, Blumkin A, et al. Phys Rev Lett, 2010, 105: 2401
[11] Belgiorno F, Cacciatori S L, Clerici M, et al. Phys Rev Lett, 2010, 105: 0901
[12] Weinfurtner S, Tedford E W, Penrice M C J, et al. Phys Rev Lett, 2011, 106: 1302
[13] Faccio D. Contemporary Physics, 2012, 53: 97
[14] Steinhauer J. Nature Physics, 2015, 12: 929
[15] Muñoz de Nova J R, Golubkov K, Kolobov V I, et al. Nature, 2019, 569: 688
[16] Torres T, Patrick S, Coutant A, et al. Nature Physics, 2017, 13: 833
[17] Vocke D, Maitland C, Prain A, et al. Optica, 2018, 5: 1099
[18] Prain A, Maitland C, Faccio D, et al. Phys Rev D, 2019, 100: 4037
[19] Solnyshkov D D, Leblanc C, Koniakhin S V, et al. Phy Rev B, 2019, 99: 4511
[20] Braidotti M C, Faccio D, Wright E M. Phys Rev Lett, 2020, 125: 3202
[21] Braidotti M C, Prizia R, Maitland C, et al. Phys Rev Lett, 2022, 128: 3901
[22] Barceló C. Nature Physics, 2019, 15: 210
[23] Almeida C R, Jacquet M J. European Physical Journal H, 2023, 48:15
[24] Marino F. Phys Rev A, 2008, 78: 3804
[25] Fang Z X, Zhao J, Xiao Z Z, et al. Laser & Optoelectronics Progress, 2022, 59: 1
[26] Kaup D J. Phys Rev, 1968, 172: 1331
[27] Penrose R. Riv Nuovo Cimento, 1969, 1: 252
[28] Zel'dovich Y B. Sov Phys JETP, 1972, 35: 1085
[29] Misner C. Phys Rev Lett, 1972, 28: 994
[30] Bekenstein R, Schley R, Mutzafi M, et al. Nature Physics, 2015, 11: 872
[31] Roger T, Maitland C, Wilson K, et al. Nature communication, 2016, 7: 1
[32] Liebling S L, Palenzuela C. Living Reviews in Relativity, 2023, 26: 1
[33] Feng S, Winful H G. Optics Letter, 2001, 26: 485
[34] Shen Y, Wang X, Xie Z, et al. Light: Science & Applications, 2019, 8: 90

[35] Goodman J. Introduction to Fourier Optics. Fourth ed. New York: Freeman Company Inc, 2017: 140
[36] Feynman R P. Application of quantum mechanics to liquid Helium. New York: Intersience Publishers Inc, 1955
[37] Schützhold R, Unruh W G. Phys Rev D, 2002, 66: 4019
[38] Schunck F E, Mielke E W. Class Quantum Grav, 2003, 20: 301
[39] Schive H Y, Chiueh T, Broadhurst T. Nature Physics, 2014, 10: 496
[40] Bullock J S, Boylan-Kolchin M. ANNU REV ASTRON ASTR, 2017, 55: 343
[41] Chavanis P. Phys Rev D, 2016, 94: 3007
[42] Binney J, Tremaine S. Galactic Dynamics. Second ed. Princeton: Princeton University Press, 2008: 400
[43] Wang P, Libeskind N, Tempel E, et al. Nature Astronomy, 2021, 5: 839

Note: Figure translations are in progress. See original paper for figures.