Research on Deformation Reconstruction Technology for Antenna Panels Based on Laser Mapping Methods (Postprint)

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Date: 2024-03-22T00:00:00+00:00

Abstract

This study investigates the deformation of individual panels on the main reflector of large radio telescopes by applying robotic six-degree-of-freedom (6-DOF) pose variation theory, proposing a panel deformation characterization method based on rigid-body six-degree-of-freedom deformation. Building upon this rigid-body 6-DOF pose deformation theory, an optical-method-based antenna panel deformation reconstruction technique is proposed, which rapidly and accurately reconstructs panel deformation through spot images produced by laser units affixed to the panels. The mapping relationship between laser spots and antenna panel deformation is thoroughly investigated, transforming the spatial deformation problem of antenna panels into a laser spot displacement problem within a limited region. Spot images for this scheme are studied, and effective spot image processing algorithms are provided; the combination of the image difference method with the Canny edge detection algorithm effectively addresses the issues of traditional Canny algorithms when processing small targets. Additionally, a high-precision sub-pixel centroid localization algorithm is employed to achieve sub-pixel-level positioning of laser spot points. This provides a novel and reliable image processing method for antenna panel measurement and offers new ideas and approaches for improving the accuracy and efficiency of antenna panel measurements.

Full Text

Research on Deformation Reconstruction Technology of Antenna Panel Based on Laser Mapping Method

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Abstract

This study investigates the deformation of individual panels comprising the main reflector of large radio telescopes by applying six-degree-of-freedom (6-DOF) pose transformations from robotics to characterize panel deformation. We propose a novel deformation representation method based on rigid-body 6-DOF transformations and develop an optical reconstruction technique for antenna panel deformation that rapidly and accurately reconstructs panel deformation through spot images generated by laser units fixed to the panel surface. The mapping relationship between laser spots and antenna panel deformation is thoroughly investigated, transforming the spatial deformation problem into a laser spot displacement problem within a finite region. We present an effective spot image processing algorithm that combines image differencing with Canny edge detection to overcome limitations of conventional Canny algorithms when processing small targets, alongside a high-precision sub-pixel centroid localization algorithm for sub-pixel level spot positioning. This work provides a new and reliable image processing method for antenna panel measurement and offers innovative approaches to improving measurement accuracy and efficiency.

Keywords: large radio telescopes; deformation reconstruction; image processing; sub-pixel

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1 Introduction

With the advancement of deep space exploration activities such as dark matter research, cosmic evolution studies, and radio astronomy observations, antennas —critical tools for signal transmission and reception—are continuously evolving toward larger apertures and higher frequencies. Larger apertures yield higher signal-to-noise ratios and more accurate observations of deep-space objects, but surface precision becomes increasingly difficult to maintain as aperture size grows, imposing more stringent requirements on surface accuracy. Higher frequencies also demand greater reflector surface precision, as even minor surface deformations can significantly affect reflection, causing signal loss or distortion. The Ruze formula establishes the relationship between antenna gain G and main reflector surface deformation σ as [?]: $G = G_0 \exp\left(-\left(\frac{4\pi\sigma}{\lambda}\right)^2\right)$, where G_0 is the ideal error-free reflector gain, λ is the wavelength in meters, and σ is the surface accuracy in meters or millimeters. This formula underscores the necessity of maintaining high surface precision for optimal antenna performance.

Existing antenna measurement systems face limitations in measurement speed, accuracy, and equipment requirements, particularly for large antennas or high-frequency bands demanding higher precision. Some methods require time-consuming operations like grid division or point scanning, resulting in slow measurement speeds. Phase holography [?] necessitates far-field amplitude and phase measurements, requiring numerous data points for large antennas and consequently long measurement times; moreover, it is constrained by signal sources, making comprehensive measurements at arbitrary elevation angles difficult. Laser ranging methods [?] are environmentally sensitive, only achieving accurate data under favorable weather conditions, with measurement precision rarely exceeding 0.2 mm. Some systems require expensive equipment such as LiDAR scanners and laser trackers [?], limiting their applicability and adoption. Photogrammetry [?] offers a relatively simple process, but its accuracy is heavily influenced by antenna aperture, performing poorly for larger apertures, and is thus only suitable for small antennas or those with less stringent surface error requirements. In summary, current methods for real-time measurement of large radio telescopes suffer from long measurement times, cumbersome procedures, environmental sensitivity, and high costs.

The surface deformation of radio telescope main reflectors is complex, as they comprise multiple panels necessitating individual panel investigation. This study examines panel deformation on the Tianma telescope [?], a large dual-reflector radio telescope, and proposes an optical mapping measurement method for real-time panel deformation monitoring. This approach requires no expensive equipment or time-consuming operations, utilizing only a camera to record spot positions and algorithmic processing to rapidly complete single-panel measurements, significantly reducing system complexity, cost, and measurement time. Importantly, the method is independent of antenna pose, elevation angle, and feed constraints. By transforming the spatial deformation problem into a laser spot displacement problem within a finite region, this innovative approach addresses numerous challenges in large-aperture radio telescope panel deformation measurement and provides valuable insights for developing new surface measurement technologies.

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2 Panel Deformation Reconstruction Technology Based on Optical Mapping Method

The proposed panel deformation reconstruction technology comprises three core components: rigid-body 6-DOF theory, a mathematical model mapping panel deformation to spot displacement, and image processing techniques. The method describes and records rigid panel position and pose changes using spatial 6-DOF parameters, then establishes a mapping relationship between the panel and spots, utilizing dynamic spot position changes to map the panel' s six spatial degrees of freedom in real time. A camera serves as the vision system to capture spot positions, and our image processing algorithms extract spot information to calculate the rigid panel' s spatial 6-DOF variables.

The measurement principle is illustrated in [Figure 1: see original paper]. Laser units are installed at four corner points of an antenna panel, projecting onto a laser receiving panel to form two-dimensional spot images recorded by a high-resolution industrial camera [**?**]. When the panel is undeformed, intersection points are uniformly distributed; upon deformation, spot positions change accordingly. By deriving the mapping model between them, panel deformation can be reconstructed from receiver panel point changes. Based on this concept and the overall telescope structure, this study investigates optical mapping-based deformation reconstruction for single panels of the Tianma telescope' s main reflector.

3.1 Six-DOF Parameter Characterization and Coordinate System Establishment

In robotics, "degrees of freedom"(DOF) refers to independent motion capabilities. A robot' s DOF count determines its possible motion types and range. Six-DOF motion enables translation along x, y, and z axes and rotation about these axes, providing six independent degrees of freedom for arbitrary spatial manipulation. Similarly, assuming antenna panels are rigid bodies, 6-DOF transformation matrices can describe their spatial position and orientation to characterize deformation.

The six parameters are displacements $\Delta x, \Delta y, \Delta z$ along the x, y, and z axes, and rotations α, β, γ about these axes. These parameters characterize main reflector panel deformation and enable reverse reconstruction to accurately measure deformation under various poses.

A coordinate system is established for single-panel analysis, as shown in [Figure 2: see original paper]. The panel center serves as origin o, the axis perpendicular to the panel as the z-axis, and the panel plane as the x-o-y plane, creating a local coordinate system for individual panels.

3.2 Main Reflector Deformation-Subreflector Spot Equation

For a single panel, only the mapping relationship between panel displacement and spot position changes on the subreflector needs to be constructed. For a rigid panel undergoing 6-DOF displacement $(\Delta x; \Delta y; \Delta z; \alpha; \beta; \gamma)$, the coordinate transformation before and after deformation follows the rigid-body translation-rotation matrix:

$$\begin{pmatrix} x_d \\ y_d \\ z_d \\ 1 \end{pmatrix} = T(\Delta x; \Delta y; \Delta z; \alpha; \beta; \gamma) \begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{pmatrix}$$

where $(x_0; y_0; z_0)$ are coordinates of a point on the rigid body before deformation, and $(x_d; y_d; z_d)$ are coordinates after deformation in the local coordinate system.

The transformation matrix $T(\Delta x; \Delta y; \Delta z; \alpha; \beta; \gamma) = T_4 \cdot T_3 \cdot T_2 \cdot T_1$ is composed of three rotation matrices and one translation matrix:

$$T(\Delta x; \Delta y; \Delta z; \alpha; \beta; \gamma) = \begin{pmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & \sin\beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\beta & 0 & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

3.3 Nonlinear Equation System Optimization

The four laser beams from panel corners are treated as rigid rods (see [Figure 2: see original paper]), fixed to the rigid panel and thus undergoing identical 6-DOF displacement. Both panel and beam points can be expressed using the transformation above. For a line segment intersecting a surface, the intersection point $(x; y; z)$ satisfies:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

where $(x_1; y_1; z_1)$ and $(x_2; y_2; z_2)$ are two points on the line segment (the corner point and beam endpoint).

The subreflector surface equation in the global coordinate system is:

$$z = a_s \sqrt{\frac{x^2 + y^2}{b_s^2} + 1} + l - a_s$$

where a_s and b_s are the real and imaginary semi-axes of the subreflector, and l is the distance between main and subreflector vertices.

This forms a system of equations. For a single panel, four equation systems can be listed:

$$\frac{x_m - x_f}{x_i - x_f} = \frac{y_m - y_f}{y_i - y_f} = \frac{z_m - z_f}{z_i - z_f} \quad (i = 1; 2; 3; 4)$$

where $(x_m; y_m; z_m)$ is the deformed spot center coordinate, $(x_f; y_f; z_f)$ is a point on the laser beam (taken as the beam-subreflector intersection), and $(x_i; y_i; z_i)$ are the four panel corner coordinates.

Pre-deformation coordinates are known; post-deformation spot centroid coordinates $M(x_m; y_m; z_m)$ are obtained via sub-pixel algorithms, while other coordinates are unknown. Using the transformation matrix, $(x_f; y_f; z_f)$ and $(x_i; y_i; zi)$ can be expressed in terms of known pre-deformation coordinates and the transformation matrix, converting unknowns into the six parameters $(\Delta x; \Delta y; \Delta z; \alpha; \beta; \gamma)$.

Each corner yields two equations, so any three corners provide six equations for six unknowns—a statically determinate problem. However, real-world errors make the system inconsistent, requiring transformation into an optimization problem to find parameters minimizing equation residuals. The Levenberg-Marquardt gradient descent algorithm solves this nonlinear optimization problem.

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4 Image Processing Algorithm

Panel deformation reconstruction relies on precise spot displacement measurement. During Tianma telescope research, we found minimum spot displacements at the micron level, difficult for vision systems to resolve without sub-pixel localization. However, captured images contain noise and complex backgrounds, necessitating preprocessing via image differencing and Canny edge detection. This approach is particularly effective for small spots sensitive to minor noise, as edge detection precisely extracts spot contours, reducing noise impact on centroid localization and enabling real-time accurate extraction [**?**].

4.1 Image Difference Method

Image difference method [**?**] detects targets based on grayscale differences between images by subtracting them pixel-wise to identify changes caused by object movement, appearance, or disappearance. This method effectively eliminates backgrounds when they remain stable. For our scheme, a background reference image without laser spots is captured first, then subtracted from the target image containing spots, yielding spot position information with background removed.

Applied to our method, this effectively eliminates complex backgrounds and strong light source interference. As shown in [Figure 3: see original paper], the process involves: (a) capturing the spot image with lasers on, (b) capturing the background image without lasers, and (c) subtracting (b) from (a) to obtain clean spot images. The difference method demonstrates excellent background and interference elimination.

4.2 Spot Image Edge Processing Algorithm

While image differencing effectively filters complex backgrounds and strong light sources, weak light artifacts inevitably remain due to timing differences between reference and target image capture—preventing perfectly identical background references. These minor noise sources, often lower in grayscale than target spots and sometimes invisible to the naked eye, still affect sub-pixel centroid localization. Edge detection technology precisely identifies spot contours while filtering weak light and enhancing spots. Canny edge detection [?] is widely used, comprising five steps: Gaussian filtering, gradient calculation, non-maximum suppression, double threshold detection, and edge connection.

However, conventional Canny algorithms struggle with small targets like laser spots, as background interference hampers accurate identification. We propose a novel approach combining image differencing with Canny edge detection. The preprocessing stage uses differencing to remove background interference, making spots more prominent before Canny edge detection. This fusion algorithm ensures accurate spot identification while effectively removing background interference, improving centroid accuracy and stability. Experimental results in [Figure 5: see original paper] demonstrate that while conventional Canny performs well on large targets, our fused algorithm is better suited for laser spot edge detection.

4.3 Sub-Pixel Spot Centroid Localization

Following preprocessing, centroid localization algorithms extract spot centroids from the processed images. Sub-pixel centroid algorithms [?, ?] operate beyond integer pixel levels, achieving higher precision.

The grayscale centroid method is the simplest sub-pixel algorithm:

$$x_c = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ij} I(x_i, y_j)}{\sum_{i=1}^{M}\sum_{j=1}^{N} I(x_i, y_j)}; \quad y_c = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} y_{ij} I(x_i, y_j)}{\sum_{i=1}^{M}\sum_{j=1}^{N} I(x_i, y_j)}$$

where x_c and y_c are sub-pixel centroid coordinates, M and N are image height and width in pixels, x_{ij} and y_{ij} are pixel coordinates, and $I(x_i, y_j)$ is pixel intensity.

The weighted centroid method uses grayscale values as weights:

$$x_c = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} x_{ij} \cdot G(x, y)^{\alpha}}{\sum_{i=1}^{M}\sum_{j=1}^{N} G(x, y)^{\alpha}}; \quad y_c = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N} y_{ij} \cdot G(x, y)^{\alpha}}{\sum_{i=1}^{M}\sum_{j=1}^{N} G(x, y)^{\alpha}}$$

where $G(x, y)$ is pixel grayscale value and α is a weighting coefficient emphasizing brighter pixels.

The Hessian method finds function extrema using the Hessian matrix to determine second-order partial derivatives, identifying extremum locations. For an image grayscale function $f(x_1; x_2; x_3; ...; x_n)$, the Hessian matrix is:

$$H(f) = \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{pmatrix}$$

Gaussian fitting treats spots as 2D Gaussian distributions, using least squares fitting to determine distribution parameters including center, standard deviation, and amplitude. The centroid coordinates are calculated as:

$$x_c = \frac{\sum_{i=1}^{n} l_i x_i e^{-(x_i - x_0)^2/2\sigma^2}}{\sum_{i=1}^{n} l_i e^{-(x_i - x_0)^2/2\sigma^2}}; \quad y_c = \frac{\sum_{i=1}^{n} l_i y_i e^{-(y_i - y_0)^2/2\sigma^2}}{\sum_{i=1}^{n} l_i e^{-(y_i - y_0)^2/2\sigma^2}}$$

where x_c and y_c are centroid coordinates, x_i and y_i are pixel coordinates, l_i are grayscale values, x_0 and y_0 are estimated spot centers, σ is the standard deviation, and n is the number of sampling points.

4.4 Spot Centroid Coordinate Solution

The processed images yield 2D centroid coordinates $C(x_c; y_c)$ in the image plane. The transformation to physical coordinates $S(x; y)$ on the subreflector follows:

$$C(x_c; y_c) = \frac{f}{D} \cdot S(x; y)$$

where f is the camera lens focal length and D is the working distance. The 3D coordinates $S(x_c; y_c; z_c)$ are obtained by calculating z_c from the subreflector surface equation, enabling the 6-DOF calculation process.

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5 Image Processing Algorithm Experimental Testing

To validate the localization accuracy of our image processing algorithms, we conducted experimental tests. Based on the processing flow in Section 4, necessary equipment included laser emitters, industrial cameras, lenses, translation stages, and laser rangefinders, with specifications listed in .

Table 1 Experimental Equipment Parameters

Equipment	Parameters
Industrial Camera	Resolution: 2560$×2160(5*megapixels*), *Pixelsize* : 6.5×$6.5 m, Sensor size: 4/3”
Lens	Focal length: 12 mm, Lens size: 1.2” , Model: MVL-KF1224M-25MP
Laser Rangefinder	Resolution: 25 megapixels, C-mount: HK-A5100-GM17
Translation Stage	Ranging accuracy: 1 mm, Angular accuracy: 0.1°, Translation accuracy: 10 m, DOF: Single

Prior to experiments, the camera and lens were assembled, calibrated, and pre-heated. The laser emitter was fixed to the translation stage to control spot movement without relative motion. A laser rangefinder was positioned parallel to the camera to determine object distance (camera-to-receiver distance), as shown in [Figure 6: see original paper]. The procedure involved: (1) capturing a background image without laser spots, (2) capturing the initial spot image with lasers on, and (3) translating the spot a known distance and capturing again. Processing these images per Section 4 yielded pre- and post-translation centroid coordinates, from which sub-pixel spot displacement was calculated.

Since actual displacement was known from the translation stage, the difference between calculated and actual distances determined localization accuracy. Varying object distances and translation distances generated extensive data, divided into two random groups for mean and standard deviation analysis, with results in .

Table 2 Edge Detection and Centroid Localization Results

Method	Group 1 Mean (pixel)	Group 1 Std Dev (pixel)	Group 2 Mean (pixel)	Group 2 Std Dev (pixel)
First-order centroid	0.0064	0.0231	0.0371	0.0125
Second-order centroid	0.0231	0.0371	0.0125	0.0064
Third-order centroid	0.0371	0.0125	0.0064	0.0231

The results demonstrate significant advantages, with localization errors controlled at ideal sub-pixel levels and minimal fluctuation between groups, validating experimental accuracy and method stability. Multiple data points achieved errors as low as 0.03 pixel (even 0.02 pixel), confirming the algorithm’s precision. First-order centroid method proves most suitable for small-target localization in our scheme, maintaining accuracy around 0.0125 pixel with edge preprocessing.

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6 Single Panel Deformation Simulation and Measurement Experiment

To validate the proposed measurement technique and mapping relationship, we simulated realistic gravitational deformation of a Tianma telescope main reflector panel using finite element methods [**?**], then reconstructed the deformation using our Matlab-based measurement method. Comparing results with finite element simulations evaluated the optical mapping method’ s error level. To control variables and scientifically validate the 6-DOF mapping approach, spot displacement measurement errors were not introduced.

The Tianma telescope’ s main reflector consists of 1,008 solid panels arranged in 14 rings. We selected a random panel from ring 9, extracting its finite element coordinates at elevation angles of 45° and 15° under gravitational deformation as ground truth. Using our measurement method, we: (1) calculated pre-deformation corner coordinates and laser beam equations, (2) solved for beam-subreflector intersection spots, (3) determined post-deformation spot positions to obtain accurate displacement, (4) reconstructed panel deformation by solving for 6-DOF parameters per Section 3, and (5) calculated post-deformation corner coordinates for comparison with finite element results. Detailed data appear in and .

Table 3 Gravitational Deformation Test Results at 45° Elevation

Corner	Real Coordinates (m)	Spot Displacement (mm)	Measured Results (m)	Error (m)
1	(-16.58084588, -17.37749485, -18.23593996)	-1.83$\times10^{-5}$	(-16.5808673, -17.37748224, -18.23594944)	2.14$\times10^{-5}$
2	(-19.11210009, -16.58084588, -17.37749485)	9.48$\times10^{-6}$	(-19.1120818, -16.5808673, -17.37748224)	-2.10$\times10^{-5}$
3	(-18.23593996, -19.11210009, -16.58084588)	-3.29$\times10^{-6}$	(-18.23594944, -19.1120818, -16.5808673)	1.16$\times10^{-5}$
4	(-17.37749485, -18.23593996, -19.11210009)	-1.26$\times10^{-5}$	(-17.37748224, -18.23594944, -19.1120818)	-1.53$\times10^{-5}$

Table 4 Gravitational Deformation Test Results at 15° Elevation

Corner	Real Coordinates (m)	Spot Displacement (mm)	Measured Results (m)	Error (m)
1	(-16.58040933, -17.37721667, -18.23546282)	1.17×10^{-5}	(-16.580420993, -17.37714276, -18.23550228)	1.23×10^{-5}
2	(-19.11171886, -16.58040933, -17.37721667)	5.84×10^{-6}	(-19.11163669, -16.580420993, -17.37714276)	1.93×10^{-5}
3	(-18.23546282, -19.11171886, -16.58040933)	2.07×10^{-6}	(-18.23550228, -19.11163669, -16.580420993)	8.22×10^{-5}
4	(-17.37721667, -18.23546282, -19.11171886)	-7.39×10^{-5}	(-17.37714276, -18.23550228, -19.11163669)	3.95×10^{-5}

The error results demonstrate that our mapping method achieves expected performance for rigid panel pose measurement, with errors ranging from several to tens of micrometers compared to finite element simulations. This confirms that the optical mapping method accurately measures large radio telescope panel deformation across six degrees of freedom, including translation and rotation. Furthermore, single-panel deformation reconstruction completes within 2 seconds after system installation, which is significant for local antenna measurement and evaluation, providing reliable means for real-time monitoring and adjustment.

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7 Conclusion

This paper proposes a novel optical method for antenna panel deformation reconstruction, employing rigid-body 6-DOF theory to characterize panel deformation and successfully deriving the mapping relationship between laser spots and panel deformation. Combining this mapping with 6-DOF theory enables deformation reconstruction of rigid antenna panels. We also present effective spot image processing algorithms that address conventional Canny algorithm limitations with small targets by integrating image differencing with Canny edge detection, ensuring accurate spot identification while removing background interference. High-precision sub-pixel centroid localization algorithms achieve sub-pixel level spot positioning, further enhancing accuracy and efficiency.

Future work may optimize this method by decoupling relationships between all antenna panels for full-aperture real-time measurement, improving algorithm precision and speed, studying error compensation methods, and expanding applicability to more antenna types and scenarios, contributing further to antenna technology development.

Acknowledgments: We thank Dr. Fu Li from Shanghai Astronomical Observatory, Chinese Academy of Sciences for providing Tianma finite element model data; the SMC Technology Center at Shanghai Jiao Tong University for laboratory and equipment support; and Li Wang and Tu Biao for their assistance in panel research.

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Note: Figure translations are in progress. See original paper for figures.